Organic Farming Transitions:
A Dynamic Bioeconomic Model

Michael A. Meneses,
Clare L. Casteel, Miguel I. Gómez, David R. Just,
Ravi Kanbur, David R. Lee, and C.-Y. Cynthia Lin Lawell

Abstract

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model accounts for newly documented interrelationships among synthetic compound use, soil health, and crop yields. In particular, new insights from soil science show that the use of synthetic compounds can be harmful to beneficial soil microbes that improve agricultural yields by enhancing crop nutrient use, stress tolerance, and pest resistance. We characterize and solve for a "fully informed" farmer's optimal synthetic compound use strategy, and for whether and how a farmer should transition from conventional to organic farming. These solutions are compared to those from a "misinformed" model in which the farmer is not aware of the interactions between synthetic compound use, soil health, and crop yields, allowing us to assess how gaining knowledge of these interactions might be expected to change farmers' synthetic compound use strategies and, ultimately, their decisions around adopting organic management. We identify and discuss agricultural and economic conditions under which farmers can be expected to voluntarily reduce their reliance on synthetic compounds, and possibly even adopt organic management, upon learning of the benefits associated with stewardship of their soil's microbiome.

Keywords: organic farming, dynamic bioeconomic model, soil health
***JEL* Codes:** Q12, Q15, Q24
This draft: April 17, 2024

*Meneses: Cornell University; mam789@cornell.edu. Casteel: Cornell University; clc269@cornell.edu. Gómez: Cornell University; mig7@cornell.edu. Just: Cornell University; drj3@cornell.edu. Kanbur: Cornell University; sk145@cornell.edu. Lee: Cornell University; drl5@cornell.edu. Lin Lawell: Cornell University; clinlawell@cornell.edu. We thank Shadi Atallah, Apurva Borar, Nano Barahona, Peter Blair Henry, Jacob Bradt, Stephanie Brockmann, Vincenzina Caputo, Bingyan Dai, Will Davis, Eric Denemark, Avanti Dey, Victor Simoes Dornelas, Yurou He, John Hubbard, Jenny Ifft, Kelsey Jack, Ted Jaenicke, Kaleb Javier, Francesco Lissoni, Tong Liu, Trevon Logan, Khyati Malik, Conrad Miller, Rosina Moreno, José Nuño-Ledesma, Jess Rudder, Michelle Segovia, Michael Toman, Harold van Es, Joakim Weill, and Lawrence White for detailed and helpful comments and discussions. We benefited from comments from participants at the Ph.D. Excellence Initiative Summer Research Workshop at New York University; an Association of Environmental and Resource Economists (AERE) Session at the Eastern Economics Association (EEA) Annual Conference (AERE@EEA); the Heartland Environmental and Resource Economics Workshop; the American Economic Association (AEA) Summer Mentoring Pipeline Conference (SMPC); the Agricultural and Applied Economics Association (AAEA) Annual Meeting; the California Department of Pesticide Regulation (DPR) Pesticide Use Report (PUR) Analysis Workgroup; the Southern Economic Association (SEA) Annual Meeting; the Interdisciplinary PhD Workshop in Sustainable Development (IPWSD) at Columbia University; the Economics of Science and Innovation Summer School at the University of Barcelona; the Ph.D. Applied Macroeconomics guest lecture at Cornell University; and the Voices of Quebec Labrador Foundation panel 'The Rise of Young Professionals'. We received funding from the U.S. Department of Agriculture (USDA) National Institute of Food and Agriculture (NIFA), Cornell Graduate School Conference Travel Grants, Cornell TREESPEAR Research Grants, and the Cornell DEEP-GREEN-RADAR Khaled H. Kheiravar Memorial Scholarship. Gómez, Just, Kanbur, Lee, and Lin Lawell are Faculty Fellows at the Cornell Atkinson Center for Sustainability. All errors are our own.

1 Introduction

Conventional agriculture has been criticized for its adverse effects on natural resources and the environment – including biodiversity loss, water pollution, and other forms of pollution posing threats to public health and climate stability - many of which are due in part to the prevalent use of synthetic fertilizers and pesticides that characterizes conventional management practices. Organic farming – wherein farmers do not use synthetic fertilizers, pesticides, herbicides, or fungicides to grow their produce – is widely considered to be a far more sustainable alternative to conventional food production (Varanasi, 2019). This paper combines insights from economics and the natural sciences to study and inform farmer transitions from conventional to organic management.

Soil microbes benefit agricultural production and improve agricultural yields by enhancing crop nutrient use, stress tolerance, and pest resistance (Lori et al., 2017). New insights from soil science show that the use of synthetic fertilizers and pesticides can be harmful to these beneficial soil microbes (Blundell et al., 2020; Hussain et al., 2009; Kalia and Gosal, 2011; Lo, 2010; Lori et al., 2017). Thus, while using synthetic fertilizers and pesticides may have the initial effect of increasing crop yields, over time these synthetic compounds exert an indirect negative effect on crop yields through their negative effects on soil health. This insight has implications for a farmer's optimal synthetic fertilizer and pesticide strategy, and for whether and how a farmer should transition from conventional to organic farming.

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model of crop production accounts for the newly documented interrelationships among synthetic compound use, soil health, and crop yields. By more accurately capturing the important biological processes at play, our model yields a solution that more accurately captures a farmer's optimal synthetic compound use and organic production strategy.

Our objectives are the following. First, we characterize a farmer's optimal trajectory of synthetic compound use over time, given the harmful effects that these compounds have on soil bacteria, and given the beneficial effects of soil bacteria on crop yields. Second, we examine a farmer's decision of whether to adopt organic certification by lowering their use of synthetic compounds to meet certification requirements. The solution to our model describes the feasibility and optimality of organic production, and how a farmer can best make the transition from conventional to organic management, given the way that soil health will respond to the transition. Third, in order to assess how knowledge about soil microbiomes and the interrelationships among synthetic compound use, soil health, and crop yields may affect farmers' decisions about transitioning from conventional to organic management, we compare the optimal synthetic compound use and organic production strategy determined by our model with the synthetic compound use and organic production strategy predicted by a model in which farmers are not aware of these interrelationships.

Formally, the dynamic optimization problem faced by the farmer is to choose a pesticide and fertilizer input trajectory to maximize the present discounted value of their entire stream of profits from crop production. Crop yields, and therefore profits, are a function of pesticide and fertilizer input use, as well as of soil bacteria. Soil bacteria populations, in turn, are a function of per-period chemical use as well as the stock of synthetic residues that have built up in the farm's soils from past chemical use.

We characterize and solve for a "fully informed" farmer's optimal synthetic compound use strategy, and for whether and how a farmer should transition from conventional to organic farming. These solutions are compared to those from a "misinformed" model in which the farmer is not aware of the interactions between synthetic compound use, soil health, and crop yields, allowing us to assess how gaining knowledge of these interactions might be expected to change farmers; synthetic compound policies and, ultimately, decisions around adopting organic management.

We find that when farmers account for soil bacteria, some may transition to organic management "accidentally" as their optimal trajectories eventually take them toward the certification threshold. This can happen even in the absence of an organic price premium. Others will have discrete "jump" transitions that are induced purely by the organic price premium.

When farmers do not account for soil bacteria, however, they never make an "accidental" transition to organic, and will instead disinvest as fast as possible until the stock of synthetic compounds in the soil reaches the maximum chemical stock capacity of the soil. If farmers who do not account for soil bacteria do transition to organic management, it will be a "jump" transition, and can only be induced by a price premium.

Results also show that the "fully informed" farmer's behavior contrasts starkly to that of the "misinformed" farmer, whose erroneous solution leads them to always use as much synthetic compounds as possible, and to therefore never transition to organic management in the absence of an organic price premium. Further, because the "misinformed" farmer misses the source of value in organic farming generated coming from improved stewardship of beneficial soil microbes (and rather only anticipates the value associated with the organic price premium) the "misinformed" farmer may incorrectly hold off on adopting organic management until they face an organic price premium higher than is needed for organic management to be optimal. This mistake in the "misinformed" farmer's adoption choice may lead to losses in both private and social welfare relative to the "fully informed" scenario.

We find and describe several empirically relevant conditions under which learning about the biological interactions of interest will lead a farmer to discontinue their use of chemical inputs entirely. Learning about the benefits of soil microbe stewardship may prompt some farmers to transition to organic management even in the absence of an organic price premium. In other cases, learning about the importance of soil microbes may result in reduced reliance on chemical inputs.

Our research will help farmers improve decision-making around synthetic compound use and organic production, and has the potential to improve soil bacteria stewardship, crop yields, farmer profits, agricultural sustainability, greenhouse gas mitigation, biodiversity, resilience of the organic farming system, the protection of water and other resources, the provision of ecosystem services, and public and environmental health.

Our study will also shed light on the importance to farmers of optimally acting upon an accurate understanding of the role that soil bacteria play in crop production and the sensitivity of that bacteria to the application of synthetic compounds. This will help open the way to educational extension programs that could result in improved yields for conventional farmers, organic farmers, and farmers transitioning to organic farming.

2 Literature Review

Transitioning to organic farming entails the discontinuation of pesticide use, a change that may impact farm profits. The relationship between pesticide use and farm profit has been the subject of many studies. Chambers et al. (2010) shows pesticide use as increasing returns to quasi-fixed factors of production like capital and land. In contrast, Jacquet et al. (2011) use a mathematical programming model to determine whether pesticide use can be reduced without affecting farmer income and find that a up to a 30 percent reduction is possible. In the long run, pesticide use may even negatively affect profits due to their effects on soil productivity through soil health. Sexton et al. (2007) acknowledge the effect that pesticide use can have on soil health through its impact on soil microbiomes. Kalia and Gosal (2011) also document the damaging effects that the application of pesticides in conventional farming has on soil microorganisms that benefit plant productivity. Jaenicke and Lengnick (1999) estimate a soil-quality index consistent with the notion of technical efficiency. The literature is thus inconclusive about the long-term effects of pesticide use on profits once effects on soil health are accounted for.

The dynamic response of soil health and productivity to the sorts of changes in pesticide use entailed by transitions to organic farming is still not well accounted for in economic assessments of the profitability of transitioning to organic farming. Stevens (2018) argues that optimal control models may be well suited for studying the economics of soil management. In this paper we argue further that dynamic optimization and dynamic programming may help shed light on the optimal rate of transition from conventional to organic farming, by allowing us to better capture the countervailing and dynamic effects that pesticide use has on profits through its effect on pest pressure and soil health.

Multiple studies have applied the dynamic optimization and programming toolkits to the study of optimal agricultural management practices. Jaenicke (2000) develops a dynamic data envelopment analysis (DEA) model of crop production to investigate the role soil capital plays in observed productivity growth and the crop rotation effect. Yeh et al. (2024) develop a novel dynamic bioeconomic analysis framework that combines numerical dynamic optimization and dynamic structural econometric estimation, and apply it to analyze the optimal management strategy for Spotted Wing Drosophila, a pest affecting soft-skinned fruits. Wu (2000) develops a dynamic model and solves for the optimal time path for herbicide application. Dynamic models have also been developed to study agricultural productivity (Carroll et al., 2019), agricultural groundwater management (Sears et al., 2019, 2024), agricultural disease control (Carroll et al., 2024a), pollination input decisions by apple farmers (Wilcox et al., 2024), supply chain externalities (Carroll et al., 2024b), optimal bamboo forest management (Wu et al., 2024), fisheries management (Conrad et al., 2024; Shin et al., 2024), and grapes (Sambucci et al., 2024).

Delbridge and King (2016) use dynamic programming to address the question of why so few farmers choose to transition to organic farming. They model the decision to transition to organic production as a dynamic programming problem where the transition involves sunk costs, and find the slow uptake of organic farming may be partially driven by the option value generated by the sunk costs associated with organic transition. Other studies have sought to incorporate transition dynamics, such as the empirically documented initial decrease in crop yields associated with conventional to organic transitions, into profitability assessments of organic farming. Dabbert and Madden (1986) find in their multi-year simulation of a 117-hectare crop-livestock farm that the initial decrease in crop yields during an organic transition results in a 30 percentage point decrease in income in the first year of transition. The biological underpinnings of this initial decrease in productivity, and their response to farmer control variables are not made explicit.

The current study is unique in that it approaches the matter of finding an optimal transition trajectory from a bioeconomic perspective, informing its net benefit function with insights from soil sciences on how soils respond to organic management. Blundell et al. (2020) find that organic management is associated with decreased pest pressure on tomato plants. This effect is driven by an accumulation of salicylic acid in plant tissue, and is likely mediated by soil microbe communities. Lori et al. (2017) find that organic management is associated with increased microbial abundance and activity. Our net benefit function captures such soil health effects on crop productivity and farmer costs during the organic transition. We are not aware of any other studies that use a bio-economic dynamic programming approach to solving for a farm's optimal trajectory for transitioning from conventional to organic farming.

Another unique aspect of our study is that we use our model to determine the value to farmers of better understanding (and acting upon their understanding of) both how soil bacteria respond to synthetic fertilizers and pesticides, and what those responses mean for crop yields. Murphy et al. (2020) find that farmers in developing countries usually do not have sufficient information about their soil nutrient levels to make profit-maximizing decisions about fertilizer usage, and that there can be potentially large net benefits to providing farmers with soil information.

3 Dynamic Bioeconomic Model

3.1 Model

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model of crop production accounts for the newly documented interrelationships among synthetic compound use, soil health, and crop yields.

We model the farmer's field-level organic transitions decision-making problem as an infinite horizon dynamic optimization problem. We assume for the initial analysis that the farmer optimizes for each of their fields independently.

The crop production function for a field's crop output $y(t)$ at time t is given by $\tilde{f}(b(t), c(t); X)$, where crop output is a function of beneficial soil microbes (or bacteria) $b(t)$, chemical (or synthetic compound) inputs $c(t)$, and other human and natural inputs X. Soil microbes $b(t)$ have a non-negative effect on crop production: $\frac{\partial \tilde{f}(b(t),c(t);X)}{\partial b} \geq 0$.

The biological production function for soil microbes $b(t)$ is given by $\tilde{g}(C(t), c(t); X)$, where the prevalence of beneficial soil microbes $b(t)$ decreases with the total stock of synthetic compounds in the soil $C(t)$, and with greater per-period chemical (or synthetic compound) input use $c(t)$: $\frac{\partial \tilde{g}(C(t),c(t);X)}{\partial C} \leq 0$, $\frac{\partial \tilde{g}(C(t),c(t);X)}{\partial c} \leq 0$. The total stock of synthetic compounds in the soil $C(t)$ only affects crop production through its effects on soil bacteria $b(t)$, so that $C(t)$ only appears in the crop production function $\tilde{f}(\cdot)$ through its role in the soil microbe production function $g(\cdot)$. The other human and natural inputs X includes soil characteristics, which are important to how beneficial bacteria $b(t)$ respond to total synthetic compound stocks $C(t)$ and to synthetic compound use $c(t)$, and which are also important to the rate $\mu(X)$ at which stocks of synthetic compounds in soils $C(t)$ decompose on their own.

The chemical input use $c(t)$ at any point in time does not exceed an upper bound $\bar{c}$, which may depend on the total stock of synthetic compounds present in the soil $C(t)$, and which may represent, for example, the maximum recommended dose for any given application, the maximum chemical input dose that is not lethal to the crop and/or to humans, the maximum chemical dose above which consumers will no longer purchase the crop, or the maximum chemical input flow at any point in time that does not destroy the farmer's land and soil. We assume the upper bound $\bar{c} > 0$.

As long as the chemical input use $c(t)$ at any point in time does not exceed the threshold $\bar{c}$ the marginal product of chemical input use $c(t)$, conditional on soil microbes $b(t)$, is non-negative: $\frac{\partial \tilde{f}(b(t),c(t);X)}{\partial c} \geq 0$. When accounting for both the direct effect of chemical inputs $c(t)$ on output $y(t)$ conditional on soil microbes $b(t)$, as well as the indirect effect of chemical inputs $c(t)$ on output $y(t)$ through their harmful effects on soil microbes $b(t)$, however, it is possible that the unconditional marginal product of chemical input use $c(t)$, $\frac{d\tilde{f}(\cdot)}{dc}$, may have be non-positive.

The farmer's control variables are the amounts of chemical inputs $c(t)$, which include synthetic fertilizer and pesticides (including herbicides, insecticides, fungicides, etc.), at each time t.

We assume for the time being that the other human and natural inputs X (which may include capital, labor, soil characteristics, and land quality) are exogenous, taken as given, and fixed. We can therefore think of the other human and natural inputs X as a farm-field fixed effect that may lead to different soil microbe biological production functions and output production functions, for example by shifting the marginal products of the inputs to soil microbe production and the marginal products of the inputs to crops production. By assuming these other human and natural inputs X are fixed, we are assuming for the time being for this stylized theory model that the time scale for any changes in these inputs is much longer than the relevant time scale for chemical input $c(t)$ decisions and for organic transitions.

The state variable is the stock of synthetic chemicals $C(t)$ that is present in the farm's soil at time t. The stock of synthetic chemicals increases with chemical input use $c(t)$ and decays at a constant rate $\mu(X) \geq 0$ that may depend on the soil characteristics and other human and natural inputs X:

$$\dot{C}(t) = c(t) - \mu(X)C(t). \tag{1}$$

Thus, the use of synthetic compounds $c(t)$ not only has harmful contemporaneous effects on soil microbes $b(t)$, but also has harmful effects on soil microbes $b(t)$ over time by increasing the stock of synthetic chemicals $C(t)$ that is present in the farm's soil, since both the total stock of synthetic compounds in the soil $C(t)$ and the per-period chemical input use $c(t)$ have harmful effects on soil microbes $b(t)$. As a consequence, while using synthetic fertilizers and pesticides $c(t)$ may have the initial direct effect of increasing crop yields $y(t)$, over time these synthetic compounds $c(t)$ exert an indirect negative effect on crop yields $y(t)$ through their negative effects on soil health.

Let $\overline{C}$ denote the maximum chemical stock capacity of the soil; if the stock of synthetic chemicals $C(t)$ ever exceeds this upper bound $\overline{C}$, the land and soil is destroyed forever and cannot ever be used for agricultural production again.

The national organic certification threshold for the stock of synthetic chemicals present in a farm's soil is given by C_{org}, where $0 \leq C_{org} < \overline{C}$. For the majority of our analysis, we assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils until they are pristine, such that $C_{org} = 0$.

The spot price of the crop in the organic market is P_{org}. The spot price of the conventionally grown crop is P_{con}. We normalize the unit price of chemical input $c(t)$ to be 1.

The optimal transition trajectory can be described by the solution to the following dynamic optimization problem:

$$\begin{aligned}
\max_{\{c(t)\}} \int_{t=0}^{\infty} \big((P_{con} \cdot \mathbf{1}\{C(t) > C_{org}\} &+ P_{org} \cdot \mathbf{1}\{C(t) \leq C_{org}\}) \cdot \tilde{f}(b(t), c(t); X) - c(t) \big) e^{-\rho t} dt \\
s.t. \quad & \dot{C}(t) = c(t) - \mu(X)C(t) \\
& b(t) = \tilde{g}(C(t), c(t); X) \\
& 0 \leq c(t) \leq \bar{c} \\
& 0 \leq C(t) \leq \overline{C} \\
& C(0) = C_0 \, ,
\end{aligned} \tag{2}$$

where $\mathbf{1}\{x\}$ is an indicator function that is equal to 1 if the condition x is true, and 0 otherwise; ρ is the interest rate; and C_0 is the initial stock of synthetic compounds in the soil.

Following Weitzman (2003), to facilitate analysis and economic interpretation, we convert our problem to prototype economic control problem form. We do this by first defining the stock of clean soil, $K(t)$, to be

$$K(t) = \overline{C} - C(t). \tag{3}$$

Net investment in clean soil stock, $I(t)$, is given by:

$$I(t) \equiv \dot{K}(t) = -\dot{C}(t). \tag{4}$$

Synthetic compound input use $c(t)$ in terms of $K(t)$ and $I(t)$ is therefore given by the following function $\tilde{c}(\cdot)$:

$$c(t) = \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t). \tag{5}$$

The constraint that $c(t) \geq 0$ can be rewritten as:

$$\mu(X)(\overline{C} - K(t)) \geq I(t) \tag{6}$$

The constraint that $c(t) \leq \bar{c}$ can be rewritten as:

$$\mu(X)(\overline{C} - K(t)) - \bar{c} \leq I(t). \tag{7}$$

We assume that $\bar{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$ and $\bar{c} > 0$ when $\mu(X) = 0$. Applying these assumptions to Equation (7), we obtain the following lower bound for net investment:

$$\begin{cases} I(t) \geq -\mu(X)K(t) & \text{if } \mu(X) > 0 \\ I(t) > \mu(X)(\overline{C} - K(t)) & \text{if } \mu(X) = 0 \end{cases} \tag{8}$$

The organic certification threshold in terms of clean soil capital is given by:

$$K_{org} = \overline{C} - C_{org}. \tag{9}$$

In terms of clean soil capital, our assumption $C_{org} = 0$ that organic certification requires that a farmer fully remediate their soils can be rewritten as $K_{org} = \overline{C}$.

We assume that the crop production function $\tilde{f}(\cdot)$ is given by:

$$\tilde{f}(b, c; X) = \alpha_b(X)b + \alpha_c(X)c + A_y(X) \tag{10}$$

where, $\forall X$, $\alpha_b \geq 0$, $\alpha_c \geq 0$, and $A_y \geq 0$.

We define a crop production function $f(\cdot)$, which is the crop production function $\tilde{f}(\cdot)$ in terms of

K and I. The crop production function $f(\cdot)$ in terms of K and I is given by:

$$\begin{aligned} f(K, I; X) &= \tilde{f}(b, \tilde{c}(K, I); X) \\ &= \alpha_b(X)b + \alpha_c(X)c(K, I) + A_y(X) \end{aligned} \tag{11}$$

Let the soil microbe production function $\tilde{g}(\cdot)$ be given by:

$$\begin{aligned} \tilde{g}(C, c; X) =& \gamma_c(X)c + \frac{1}{2}\gamma_{cc}(X)c^2 + \gamma_K(X)\left(\overline{C} - C(t)\right) + A_b(X) \\ =& \gamma_c(X)c + \frac{1}{2}\gamma_{cc}(X)c^2 + \gamma_K(X)K + A_b(X) \end{aligned} \tag{12}$$

where, $\forall X$, $\gamma_c \leq 0$, $\gamma_{cc} \leq 0$ (i.e., convex costs to synthetic compound use), $\gamma_K \geq 0$, and $A_b \geq 0$.

We define a soil microbe production function $g(K, I; X)$, which is the soil microbe production function $\tilde{g}(\cdot)$ in terms of K and I as follows:

$$g(K, I; X) = \gamma_c(X)\tilde{c}(K, I) + \frac{1}{2}\gamma_{cc}(X)\tilde{c}(K, I)^2 + \gamma_K(X)K + A_b(X). \tag{13}$$

Let's define the national organic certification threshold in terms of the stock of clean soil K_{org} as:

$$K_{org} = \overline{C} - C_{org}. \tag{14}$$

The initial stock of clean soil K_0 is given by:

$$K_0 = \overline{C} - C_0. \tag{15}$$

The per period net gain function $G(K, I)$ is therefore given by:

$$G(K, I) = (P_{con} \cdot \mathbf{1}\{K < K_{org}\} + P_{org} \cdot \mathbf{1}\{K \geq K_{org}\}) \cdot f(b, \mu(X)(\overline{C} - K) - I; X) - \left(\mu(X)(\overline{C} - K) - I\right), \tag{16}$$

and the farmer's problem can be re-written in prototypical economic form as follows:

$$
\begin{aligned}
\max_{\{I(t)\}} \int_0^{\infty} & \big((P_{con} \cdot \mathbf{1}\{K(t) < K_{org}\} + P_{org} \cdot \mathbf{1}\{K(t) \geq K_{org}\}) \cdot f(b(t), \tilde{c}(K(t), I(t)); X) \\
& \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad - \tilde{c}(K(t), I(t)) \big) \cdot e^{-\rho t} dt \\
s.t. \quad & \dot{K}(t) = I(t) \qquad \qquad : p(t) \\
& b(t) = g(K(t), I(t); X) \\
& \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t) \\
& \mu(X)(\overline{C} - K(t)) - \bar{c} \leq I(t) \leq \mu(X)(\overline{C} - K(t)) \\
& 0 \leq K(t) \leq \overline{C} \\
& K(0) = K_0 \,,
\end{aligned} \tag{17}
$$

where the co-state variable $p(t)$ is the marginal value to the farmer's optimal program of an extra unit of clean soil.

3.2 What makes this optimal control problem novel and challenging to solve

The partial derivatives near the national organic certification threshold are tricky to calculate, since they involve derivatives of indicator functions. The indicator function $\mathbf{1}\{K \geq K_{org}\}$ for satisfying the national organic certification threshold is the Heaviside function $H(K - K_{org})$, where $H(x) = 1$ if $n \geq 0$ and $H(x) = 0$ if $x < 0$. The derivative of the Heaviside function $H(x)$ is the Dirac delta function $\delta(x)$:

$$
\frac{dH(x)}{dx} = \delta(x), \tag{18}
$$

which unfortunately is tricky to work with and interpret (it's a function that spikes at zero).

So instead of trying to take a derivative of an indicator function, we analyze each stage of the dynamic bioeconomic model separately, and then consider possible transitions from conventional to organic management. The first stage is conventional agriculture with prices P_{con}. The second stage is organic agriculture with prices P_{org}. The second stage is reached if organic certification requirement $K(t) \geq K_{org}$ is satisfied.

4 Optimal Solution for Each Stage

We first describe behavior *within* each stage $j \in \{con, org\}$. For each stage $j \in \{con, org\}$, we solve for stationary rate of return on capital (clean soil stock) $R_j(K)$; determine whether there is a stationary solution $\hat{K}_j$; characterize direction and speed of net investment $I(t)$; and solve for the optimal trajectories $I^*(t)$ and $K^*(t)$ using the Maximum Principle.

4.1 Optimal control problem for each stage $j \in \{con, org\}$

For each stage $j \in \{con, org\}$, the farmer's dynamic optimization problem is given by:

$$\begin{aligned}
\max_{\{I(t)\}} \int_0^\infty & \big(P_j \cdot f\left(g(K(t), I(t); X), \tilde{c}(K(t), I(t)); X\right) - \tilde{c}(K(t), I(t))\big) \cdot e^{-\rho t} dt \\
s.t. \quad & \dot{K}(t) = I(t) \qquad : p(t) \\
& \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t) \\
& \mu(X)(\overline{C} - K(t)) - \bar{c} \leq I(t) \leq \mu(X)(\overline{C} - K(t)) \\
& 0 \leq K(t) \leq \overline{C} \\
& K(0) = K_{0j} \,.
\end{aligned} \tag{19}$$

The Hamiltonian is then:

$$H_j = G_j(K, I) + \rho I(t), \tag{20}$$

where

$$G_j(K, I) = P_j \cdot f\left(g(K, I), c(K, I); X\right) - c(K, I; X). \tag{21}$$

Given our functional form assumptions, the per-period net gain (or profits) $G_j(K, I)$ for each stage $j \in \{con, org\}$ is given by:

$$\begin{aligned}
G_j(K, I) = & P_j \cdot \Bigg(\alpha_b \bigg(\gamma_c \left(\mu\left(\overline{C} - K\right) - I\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_K K + A_b \bigg) \\
& \qquad + \alpha_c \left(\mu\left(\overline{C} - K\right) - I\right) + A_y \Bigg) \\
& - \left(\mu\left(\overline{C} - K\right) - I\right)
\end{aligned} \tag{22}$$

where the convex costs to synthetic compound use, as measured by the parameter $\gamma_{cc} \leq 0$, introduces non-linear investment costs. We will have a most rapid approach (MRA) policy if $\gamma_{cc} = 0$ since then $G(K, I)$ is linear in net investment I.

The solution to the farmer's dynamic optimization problem for each stage $j \in \{con, org\}$ satisfies the FOCs of the Maximum Principle:

$$[\#1]: \quad \frac{\partial H_j}{\partial I} = 0 \tag{23}$$

$$[\#2]: \quad \dot{p}(t) = -\frac{\partial \widetilde{H_j}}{\partial K}(K^*(t), p(t)) + \rho p(t) \tag{24}$$

$$[\#3]: \quad \lim_{t\to\infty} p(t)K(t)e^{-\rho t} = 0 \tag{25}$$

The FOCs of the Maximum Principle are both necessary *and sufficient* for optimality since our per-period net gain function $G_j(K, I)$ is concave and the control set is convex for each stage $j \in \{con, org\}$.

4.2 Characterizing the Optimal Solution for each stage $j \in \{con, org\}$

The present discounted value (PDV) of entire stream of marginal net benefit (MNB) of an additional unit of synthetic compound $c(t)$ today is given by:

$$P_j \cdot \alpha_c - \left(-P_j\alpha_b\left(\gamma_c + \gamma_{cc}c(t)\right) + \frac{P_j\alpha_b\gamma_K}{\mu+\rho} + 1 \right), \tag{26}$$

each term of which is explained in detail in Figure 1. Owing to the convex costs of synthetic compounds on soil microbe production, as measured by the parameter $\gamma_{cc} \leq 0$, the PDV of the entire stream of marginal net benefits is decreasing in synthetic compound use $c(t)$. Since $c(t) = \mu(X)(\overline{C} - K(t)) - I(t)$ is decreasing in $K(t)$, the PDV of the entire stream of marginal net benefits is increasing in $K(t)$.

The optimal unconstrained amount of synthetic compound $c(t)$ to apply at any time t is the synthetic compound input level c_j^{**} at which the present discounted value (PDV) of entire stream of marginal net benefit (MNB) of an additional unit of synthetic compound c today is 0. In other words, at the optimal unconstrained amount of synthetic compound c_j^{**}, the PDV of the entire stream of marginal benefits of an additional unit of synthetic compound $c(t)$ today is exactly offset by the PDV of the entire stream of marginal costs of an additional unit of synthetic compound $c(t)$ today. This optimal unconstrained amount of synthetic compound c_j^{**} is a constant that is a function of parameters but not of $K(t)$ and is given by:

$$c_j^{**} = -\frac{P_j \cdot \left(\alpha_c + \alpha_b\left(\gamma_c - \frac{\gamma_K}{\mu+\rho}\right)\right) - 1}{P_j\alpha_b\gamma_{cc}}. \tag{27}$$

As long as the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is greater than 0 (i.e., as long as $c(t) < c_j^{**}$, since the PDV of the entire stream of MNB is decreasing in $c(t)$), we would want to increase the amount of synthetic compound we use today, and will continue to do so until either (1) the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is 0 (i.e., until $c(t) = c_j^{**}$); or (2) we hit the upper bound $\bar{c}$ for synthetic compound use. If we are constrained by the upper bound $\bar{c}$ for synthetic compound use from increasing synthetic compound use $c(t)$ any further even though the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0(i.e., if $c_j^{**} > \bar{c}$, then the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is the smallest positive value that it can be.

Figure 1: PDV of entire stream of marginal net benefit of additional unit of synthetic compound $c(t)$ today

$$P_j \cdot \underbrace{\underbrace{\alpha_c}_{\substack{\geq 0\\ \text{direct effect}\\ \text{of } c(t)\\ \text{on crop output}}}}_{\substack{\text{PDV of entire stream}\\ \text{of marginal benefit}\\ \text{of additional unit}\\ \text{of synthetic compound today}}} - \underbrace{\left(\underbrace{\underbrace{-P_j \underbrace{\alpha_b \left(\underbrace{\gamma_c}_{\text{linear}} + \underbrace{\gamma_{cc} c(t)}_{\text{nonlinear}}\right)}_{\substack{\leq 0\\ \text{indirect effect of } c(t)\\ \text{on crop output}\\ \text{via their direct negative effect}\\ \text{on soil microbes}}}}_{\substack{\geq 0\\ \text{indirect marginal cost of } c(t)\\ \text{via its direct negative effect}\\ \text{on soil microbes}}} + \underbrace{\frac{P_j \overbrace{\alpha_b \gamma_K}^{\substack{\text{indirect effect of}\\ \text{clean soil stock}\\ \text{on crop output}\\ \text{via its direct positive effect}\\ \text{on soil microbes}}}}{\mu + \rho}}_{\substack{\geq 0\\ \text{PDV of entire stream of indirect marginal cost}\\ \text{of unit of synthetic compound today}\\ \text{via its indirect negative effect on soil microbes}\\ \text{thru its negative effect on stock of clean soils}\\ \text{(stock effect)}}}}_{\substack{\text{PDV of entire stream of indirect marginal costs}\\ \text{of additional unit of synthetic compound today}\\ \text{via its negative effects on soil microbes}}} + \underbrace{\underbrace{1}_{\substack{\geq 0\\ \text{unit price}\\ \text{of } c(t)}}}_{\substack{\text{direct marginal cost}\\ \text{of unit of}\\ \text{synthetic compound today}}} \right)}_{\text{PDV of entire stream of marginal costs of additional unit of synthetic compound today}}$$

With the farmer's problem now in prototypical economic control form, we can solve for the stationary rate of return on capital, $R_j(K)$ for each stage $j \in \{con, org\}$. The stationary rate of return on capital $R_j(K)$ is the per-period rate of return on the clean soil capital stock K from increasing net investment I a tiny bit this period from a net investment level of $I = 0$. In other words stationary rate of return on capital $R_j(\overline{K})$ is the per-period yield from increasing the clean soil capital stock K from a stationary state $\overline{K}$, to a stationary state $\overline{K} + \epsilon$, and is given by (Weitzman, 2003):

$$R_j(K) = -\frac{\frac{\partial G_j(K,0)}{\partial K}}{\frac{\partial G_j(K,0)}{\partial I}}. \tag{28}$$

Solving for the stationary rate of return $R_j(K)$ on clean soil capital for each stage $j \in \{con, org\}$, we obtain:

$$R_j(K) = -\mu + \frac{\gamma_K}{\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}. \tag{29}$$

The slope of $R_j(K)$ is:

$$R_j'(K) = \frac{\gamma_K}{\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}\gamma_{cc}\mu \tag{30}$$

which we can sign as follows:

$$R_j'(K) = \frac{\underbrace{\gamma_K}_{\geq 0}}{\underbrace{\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \dfrac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}_{\geq 0}}\underbrace{\gamma_{cc}}_{\leq 0}\underbrace{\mu}_{\geq 0} \leq 0 \tag{31}$$

The stationary solution $\hat{K}_j$ is the clean soil stock at which the stationary rate of return on the clean soil capital stock, $R_j(\hat{K}_j)$ is equal to the rate of the return on the best alternative investment (i.e., the bank), ρ (Weitzman, 2003):

$$R_j(\hat{K}_j) = \rho \tag{32}$$

Plugging in Equation (29) for the stationary rate of return on the clean soil capital stock, $R_j(\cdot)$ into Equation (32), we obtain the following condition for the stationary solution $\hat{K}_j$:

$$P_j\alpha_c = -P_j\alpha_b\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right)\right) + \frac{P_j\alpha_b\gamma_K}{\mu + \rho} + 1 \tag{33}$$

The intuition for the condition for the stationary solution $\hat{K}_j$ is presented in Figure 2.

Figure 2: Condition for stationary solution $\hat{K}_j$ for each stage $j \in \{con, org\}$

$$\underbrace{P_j \cdot \underbrace{\alpha_c}_{\substack{\geq 0 \\ \text{direct effect} \\ \text{of synthetic compounds} \\ \text{on crop output}}}}_{\substack{\text{PDV of entire stream of marginal benefit} \\ \text{of additional unit of synthetic compound today}}} = \underbrace{\underbrace{\underbrace{-P_j \underbrace{\alpha_b \left(\underbrace{\gamma_c}_{\text{linear component}} + \underbrace{\gamma_{cc}\, \mu \underbrace{\left(\overline{C} - \hat{K}_j \right)}_{\equiv \hat{c}_j}}_{\text{nonlinear component}} \right)}_{\substack{\leq 0 \\ \text{indirect effect of synthetic compounds} \\ \text{on crop output} \\ \text{via their direct negative effect on soil microbes}}}}_{\substack{\geq 0 \\ \text{indirect marginal cost} \\ \text{of additional unit of synthetic compound today} \\ \text{via its direct negative effect on soil microbes}}} + \underbrace{P_j \frac{\overbrace{\alpha_b \gamma_K}^{\substack{\geq 0 \\ \text{indirect effect of clean soil stock} \\ \text{on crop output} \\ \text{via its direct positive effect on soil microbes}}}}{\mu + \rho}}_{\substack{\geq 0 \\ \text{PDV of entire stream of indirect marginal cost} \\ \text{of unit of synthetic compound today} \\ \text{via its indirect negative effect on soil microbes} \\ \text{thru its negative effect on stock of clean soils} \\ \text{(stock effect)}}}}_{\substack{\text{PDV of entire stream of indirect marginal costs} \\ \text{of additional unit of synthetic compound today} \\ \text{via its negative effects on soil microbes}}} + \underbrace{\underbrace{1}_{\substack{\geq 0 \\ \text{unit price} \\ \text{of synthetic compound}}}}_{\substack{\text{direct marginal cost} \\ \text{of unit of synthetic compound today}}}}_{\substack{\text{PDV of entire stream of marginal costs} \\ \text{of additional unit of synthetic compound today}}} \tag{34}$$

As seen in Equation (34) in Figure 2, at the stationary solution $\hat{K}_j$ for a given stage $j \in \{con, org\}$, the present discounted value (PDV) of the entire stream of marginal benefits from applying an additional unit of synthetic compound today equals the present discounted value of the entire stream of marginal costs of applying an additional unit of synthetic compound today.

The present discounted value of the entire stream of marginal benefits from applying an additional unit of synthetic compound today, which is given by $P_j \alpha_c$, comes from the direct effect of chemical input use $c(t)$ on crop output $y(t)$ today and therefore on crop revenue today. Using an additional unit of synthetic compound today does not yield any future marginal benefits.

The present discounted value of the entire stream of marginal costs of applying an additional unit of synthetic compound today, which is given by the right-hand side of Equation (34), consists of several components. Applying an additional unit of synthetic compound today incurs both direct and indirect marginal costs. The direct marginal cost of an additional unit of synthetic compound today is simply the unit price of chemical inputs $c(t)$, which we normalize to 1, and which is only incurred today. The indirect marginal cost of an additional unit of synthetic compound today comes from the negative effects of synthetic compounds on soil microbes $b(t)$ and their resulting negative effects on crop output $y(t)$ and therefore on crop revenue. There are two channels through which synthetic compounds have negative effects on soil microbes. First, applying an additional unit of synthetic compound today has a direct negative effect on soil microbes today through its direct negative effect on soil microbe production today. Second, applying an additional unit of synthetic compound today has an indirect negative effect on soil microbes by decreasing the stock of clean soils $K(t)$ today, which may last for multiple periods, and which in turn has a negative effect on soil microbe production over multiple periods of time; we call the PDV of the entire stream of indirect marginal costs of applying an additional unit of synthetic compound today via their indirect negative effect on soil microbes through their negative effect on stock of clean soils the 'stock effect'. Thus, the present discounted value of the entire stream of marginal costs from applying an additional unit of synthetic compound today comes from the direct marginal cost of purchasing synthetic compounds today, 1; the indirect marginal cost of applying an additional unit of synthetic compound today via their direct negative effect on soil microbes today, $-P_j \alpha_b \left(\gamma_c + \gamma_{cc} \mu \left(\overline{C} - K\right)\right)$; and the PDV of the entire stream of indirect marginal costs of applying an additional unit of synthetic compound today via their indirect negative effect on soil microbes through their negative effect on stock of clean soils (the stock effect), $\frac{P_j \alpha_b \gamma_K}{\mu + \rho}$.

The optimal choice of synthetic compound $c(t)$ at any time t is when the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound $c(t)$ today is 0 (or as small a non-negative number as possible).

Thus, when the stationary solution exists, the optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j \equiv \mu \left(\overline{C} - \hat{K}_j\right)$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

If $(\rho + \mu) \gamma_{cc} \mu \neq 0$, $\hat{K}_j$ is given by:

$$\hat{K}_j = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{35}$$

Note that our solution for $\hat{K}_j$ may be negative or positive depending on model parameters. If our solution for $\hat{K}_j$ is negative then our non-negativity constraint on $K(t)$ will bind. Importantly, we note that under this formulation of the model it is possible for (A.29) to be positive even in the case of special interest in which $\mu > 0$.

We conduct comparative statics for $\hat{K}_j$ in Appendix A.3.

The stationary solution $\hat{K}_j$ is a function of prices P_j. $\frac{\partial \hat{K}_j}{\partial P_j}$ is given by:

$$\frac{\partial \hat{K}_j}{\partial P_j} = \frac{1}{\underbrace{\alpha_b}_{\geq 0}\underbrace{\gamma_{cc}}_{\leq 0}\underbrace{\mu}_{\geq 0}\underbrace{P_j^2}_{>0}} \leq 0 \tag{36}$$

Thus, the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j.

Since the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j and $P_{con} < P_{org}$, the stationary solution for the organic stage 2, if it exists, is less than the stationary solution for the conventional stage 1, if it exists.

Note that $K(t)$ is constrained such that $K(t) \in [0, \overline{C}]$.

It is therefore possible that $\hat{K}_j$ is not feasible because $\hat{K}_j$ is not within the set of feasible K. In other words, it is possible that $\hat{K}_j$ is not feasible because either $\hat{K}_j < 0$ or $\hat{K}_j > \overline{C}$).

Since our analysis using the stationary rate of return on capital $R_j(K)$ makes the assumption of the prototype economic control model that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain). We cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R(K)$ and ρ to describe the optimal solution $\frac{\partial G(K,I)}{\partial I} < 0$. As shown in Appendix A.1, $\frac{\partial G(K,I)}{\partial I} < 0$ when $K \leq \tilde{K}_j$, where $\tilde{K}_j$ is defined as the stock of clean soils at which $\frac{\partial G(K,0)}{\partial I} = 0$. A farmer with $K \leq \tilde{K}_j$ would invest in the stock of clean soil, not disinvest, since there is no trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well. Thus, for $K \leq \tilde{K}_j$, the farmer will invest in clean soil. Also as shown in Appendix A, $\hat{K}_j \geq \tilde{K}_j$. Thus, since for $K \leq \tilde{K}_j$, the farmer will invest in clean soil, this means that for $K_{0j} \leq \tilde{K}_j$, if the stationary solution $\hat{K}_j$ exists, the farmer will continue to invest in clean soil until he reaches the stationary solution $\hat{K}_j$.

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own), then $R_j(K)$ is a constant (that does not depend on K) and $\hat{K}_j$ will not exist (nor will $\tilde{K}_j$).

We discuss how prices P_j affect the optimal solution in Appendix A.2.

4.3 Optimal Trajectories for Stage j When $\hat{K}_j$ Exists

We now solve for the farmer's optimal stage j trajectories.

In Appendix A.4, we start by solving for the unconstrained solution for each stage j by using second-order Taylor series approximations of the net gain function $G(K, I)$. Since the net gain function $G(K, I)$ is quadratic, these second-order Taylor series approximations and the solutions derived using them are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K, I)$ is an exact second-order Taylor series expansion of the net gain function $G(K, I)$.

In Appendix A.5, we then solve for the constrained optimal solution for each stage j by solving for an exact solution via direct derivation.

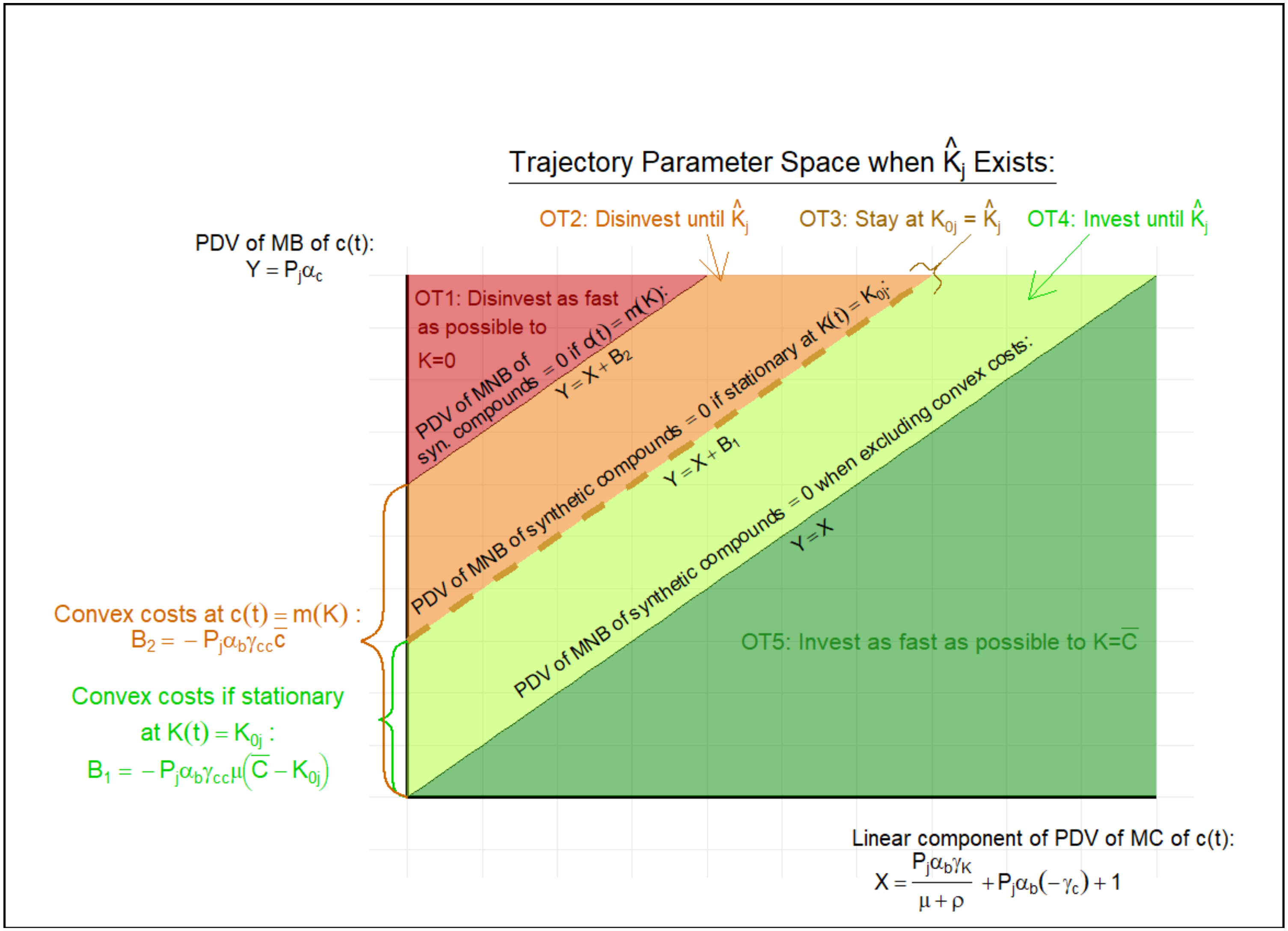
There are five possible types of optimal trajectories that arise when $\hat{K}_j$ exists, depending on the parameters. In order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least synthetic compound use and the most investment in the stock of clean soils, these five types of optimal trajectories are as follows:

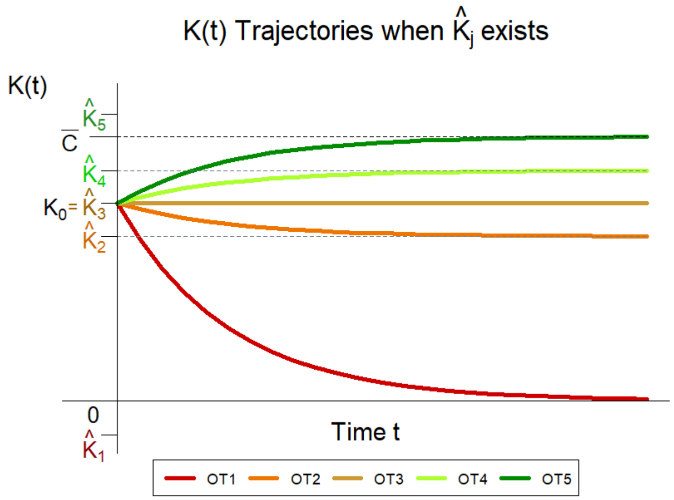
- **Optimal Trajectories 1 [OT1]**: Disinvest as fast as possible until $K = 0$ by always applying $c(t) = \bar{c}$
- **Optimal Trajectories 2 [OT2]**: Disinvest to $\hat{K}_j < K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 3 [OT3]**: Stay at initial clean soil stock $K_0 = \hat{K}_j$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 4 [OT4]**: Invest until $\hat{K}_j > K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 5 [OT5]**: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

Figure 3 presents the parameter spaces for each of the five types of optimal trajectories when $\hat{K}_j$ exists. Figure 4 plots examples of each of the five types of optimal trajectories that arise when $\hat{K}_j$ exists.

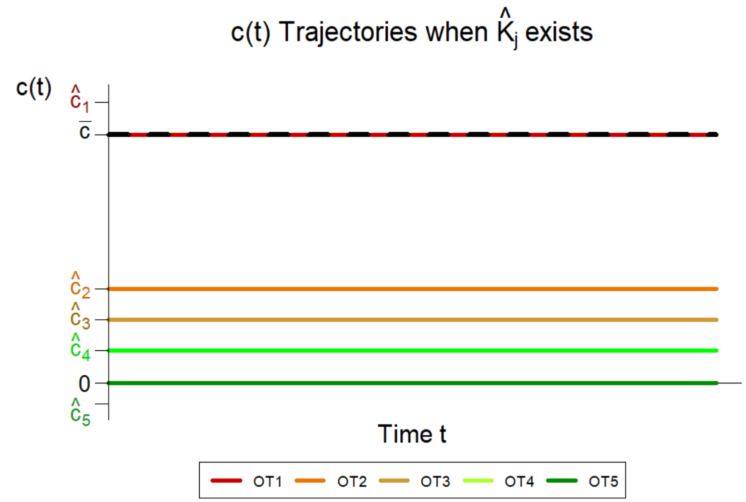
As seen in Figure 4, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the order of the optimal trajectory type by per-period yield reverses, and the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

Figure 3: Parameter Space for Optimal Trajectories When $\hat{K}_j$ Exists



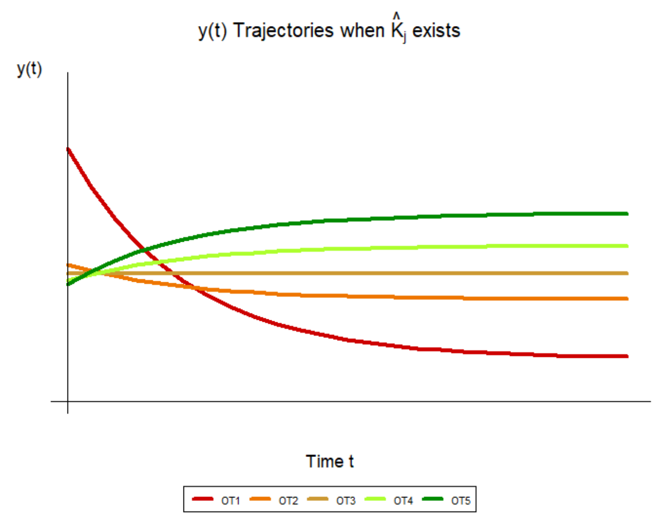


(a) Clean soil stock $K(t)$



(b) Synthetic compound use $c(t)$

(c) Microbes $b(t)$



(d) Yield $y(t)$

Figure 4: Optimal Trajectories When $\hat{K}_j$ Exists

Note: We assume $\overline{c} = \mu\overline{C}$.

4.3.1 Optimal Trajectories 1: Disinvest as fast as possible to $K = 0$

When $\hat{K}$ exists and is negative (i.e., $\hat{K}_j < 0$), the lower-bound constraints on net investment I *always* bind (for all t), which means that the upper bound constraint on synthetic compound use always binds (i.e., $c^{**} > \bar{c}$). Thus, when $\hat{K}_j < 0$, the farmer's optimal solution is to disinvest as fast as possible until $K = 0$. The optimal synthetic compound use $c(t)$ is to apply the maximum amount possible $\bar{c}$ every period until we reach $K = 0$, at which point we stay at $K = 0$ (i.e., by applying $c(t) = \mu\overline{C}$ each period).

$\hat{K}_j < 0$ occurs when the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is positive even when the farmer uses the maximum permissible dose of a synthetic compound:

$$P_j\alpha_c + P_j\alpha_b\gamma_{cc} \cdot \bar{c} + P_j\alpha_b\gamma_c - P_j\alpha_b\frac{1}{(\rho+\mu)} \cdot \gamma_K - 1 \geq 0 \tag{37}$$

When $\hat{K}_j$ exists, our lower corner solutions for $K(t)$, $I(t)$, $c(t)$, a $C(t)$, $b(t)$, and $y(t)$ are as follows:

$$K(t) = K(0) \cdot e^{-\mu \cdot t}\, \forall\, t \tag{38}$$

$$I(t) = -\mu K(t)\, \forall\, t \tag{39}$$

$$c(t) = \bar{c} = \mu\overline{C}\, \forall\, t \tag{40}$$

$$C(t) = \overline{C} - K(0) \cdot e^{-\mu \cdot t}\, \forall\, t \tag{41}$$

Given $\tilde{g}(C(t), c(t)) = \gamma_c c + \frac{1}{2}\gamma_{cc}c^2 + \gamma_K\left(\overline{C} - C(t)\right) + A_b$:

$$b(t) = \max\{\gamma_c\bar{c} + \frac{1}{2}\gamma_{cc}\bar{c}^2 + \gamma_K \cdot \left(K(0) \cdot e^{-\mu \cdot t}\right) + A_b,\, 0\}\, \forall\, t \tag{42}$$

Given $f(c(t), b(t)) = \alpha_c c(t) + \alpha_b b(t) + A_y$:

$$y(t) = \alpha_c \cdot \bar{c} + \alpha_b \cdot b(t)_{LC} + A_y\, \forall\, t \tag{43}$$

4.3.2 Optimal Trajectories 2: Disinvest until $\hat{K}_j$

This case occurs when $\hat{K}_j \in [0, \overline{C}]$ *and* $K_{0j} > \hat{K}_j$.

In this case the optimal solution is to disinvest until $\hat{K}_j$.

The optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

$$c_j^*(t) = \hat{c}_j = \mu\left(\overline{C} - \hat{K}_j\right) \tag{44}$$

At $\hat{c}_j$, PDV of MNB $= 0$.

This case occurs when the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is:

- ≥ 0 when the clean soil stock is less than the initial clean soil stock
- ≤ 0 when clean soil stock is less than 0

In this case the optimal solution approaches $\hat{K}_j$ at a moderate speed. The optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

$$c_j^*(t) = \mu\left(\overline{C} - \hat{K}_j\right) \tag{45}$$

$$I_j^*(t) = \mu\left(\hat{K}_j - K(t)\right) < 0 \tag{46}$$

$$K_j^*(t) = \hat{K}_j + \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{47}$$

$$C_j^*(t) = \overline{C} - \hat{K}_j - \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{48}$$

$$b_j^*(t) = \left(\gamma_c \mu\left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b + \gamma_K\left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}\right) \tag{49}$$

$$y_j^*(t) = \alpha_b\left(\gamma_c \mu\left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b\right) \tag{50}$$

$$+ \alpha_c \mu\left(\overline{C} - \hat{K}_j\right) + A_y + \alpha_b \gamma_K\left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{51}$$

4.3.3 Optimal Trajectories 3: Stay at initial clean soil stock and do not invest or disinvest

If $K_{0j} = \hat{K}_j$, we always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock.

In this case it is optimal to stay at initial clean soil stock and not to invest or disinvest. In other words, in each period our chemical input use $c(t)$ should exactly offset the stock of chemicals in the soil decays on its own so that the stock of chemicals in the soil stays constant, and therefore the clean soil stock stays constant at its initial value.

Thus, the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

Thus, for OT3, the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

$$K(t) = K_{0j} \,\forall t \tag{52}$$

$$I(t) = 0 \,\forall t \tag{53}$$

$$c(t) = \mu C(0) \,\forall t \tag{54}$$

$$C(t) = \overline{C} - K_{0j} = C(0) \,\forall t \tag{55}$$

$$b(t) = \left(\gamma_c \mu \left(\overline{C} - K_{0j}\right) + \frac{1}{2}\gamma_{cc}\mu^2 \left(\overline{C} - K_{0j}\right)^2 + \gamma_K K_{0j} + A_b\right) \forall t \tag{56}$$

$$\tilde{f}(t) = \alpha_b \left(\gamma_c \mu \left(\overline{C} - K_{0j}\right) + \frac{1}{2}\gamma_{cc}\mu^2 \left(\overline{C} - K_{0j}\right)^2 + \gamma_K K_{0j} + A_b\right) + \alpha_c \mu \left(\overline{C} - K_{0j}\right) + A_y \,\forall t \tag{57}$$

4.3.4 Optimal Trajectories 4: Invest until $\hat{K}_j$

This case occurs when $\hat{K}_j \in [0, \overline{C}]$ *and* $K_{0j} < \hat{K}_j$.

In this case the optimal solution is to invest until $\hat{K}_j$.

The optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

$$c_j^*(t) = \hat{c}_j = \mu \left(\overline{C} - \hat{K}_j\right) \tag{58}$$

At $\hat{c}_j$, PDV of MNB = 0.

This case occurs when PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is:

- ≥ 0 when clean soil stock $K(t)$ is between the initial clean soil stock K_{0j} and the maximum level of clean soil stock $\overline{C}$, at which point there are no convex costs of synthetic compounds on soil microbes
- ≤ 0 when clean soil stock $K(t)$ is less than the initial clean soil stock K_{0j}

$$K_{0j} < \hat{K}_j \leq \overline{C} \tag{59}$$

In this case the optimal solution approaches $\hat{K}_j$ at a moderate speed. The optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

$$c_j^*(t) = \mu \left(\overline{C} - \hat{K}_j\right) \tag{60}$$

$$I_j^*(t) = \mu \left(\hat{K}_j - K(t)\right) > 0 \tag{61}$$

$$K_j^*(t) = \hat{K}_j + \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{62}$$

$$C_j^*(t) = \overline{C} - \hat{K}_j - \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{63}$$

$$b_j^*(t) = \left(\gamma_c \mu \left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc} \left(\mu \left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b + \gamma_K \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}\right) \tag{64}$$

$$y_j^*(t) = \alpha_b \left(\gamma_c \mu \left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc} \left(\mu \left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b\right) \tag{65}$$

$$+ \alpha_c \mu \left(\overline{C} - \hat{K}_j\right) + A_y + \alpha_b \gamma_K \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{66}$$

4.3.5 Optimal Trajectories 5: Invest as fast as possible until $K = \overline{C}$

If $\hat{K}_j > \overline{C}$, upper-bound constraints on net investment I *always* bind (for all t). In this case the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. It is optimal not to use any synthetic compounds $c(t)$ at all.

If $\hat{K}_j > \overline{C}$, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is negative even when there are no convex costs of synthetic compounds on soil microbes:

$$\Rightarrow P_j \alpha_c < -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} - 1 \tag{67}$$

When $\hat{K}_j > \overline{C}$, the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. It is optimal not to use any synthetic compounds $c(t)$ at all.

$$c_j^*(t) = 0 \tag{68}$$

$$K^*(t)_j = K(t)_{UC,j} = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{69}$$

$$I^*(t)_j = I(t)_{UC,j} = \mu \cdot \left(\overline{C} - K(t)_{UC,j}\right) \tag{70}$$

$$C^*(t)_j = \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{71}$$

$$b^*(t)_j = \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) \tag{72}$$

$$y^*(t)_j = \alpha_b \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) + A_y \tag{73}$$

4.4 Optimal Trajectories for Stage j When $R(K)$ is Constant Because $\gamma_{cc} = 0$

If $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) then $R_j(K)$ is a constant (that does not depend on K).

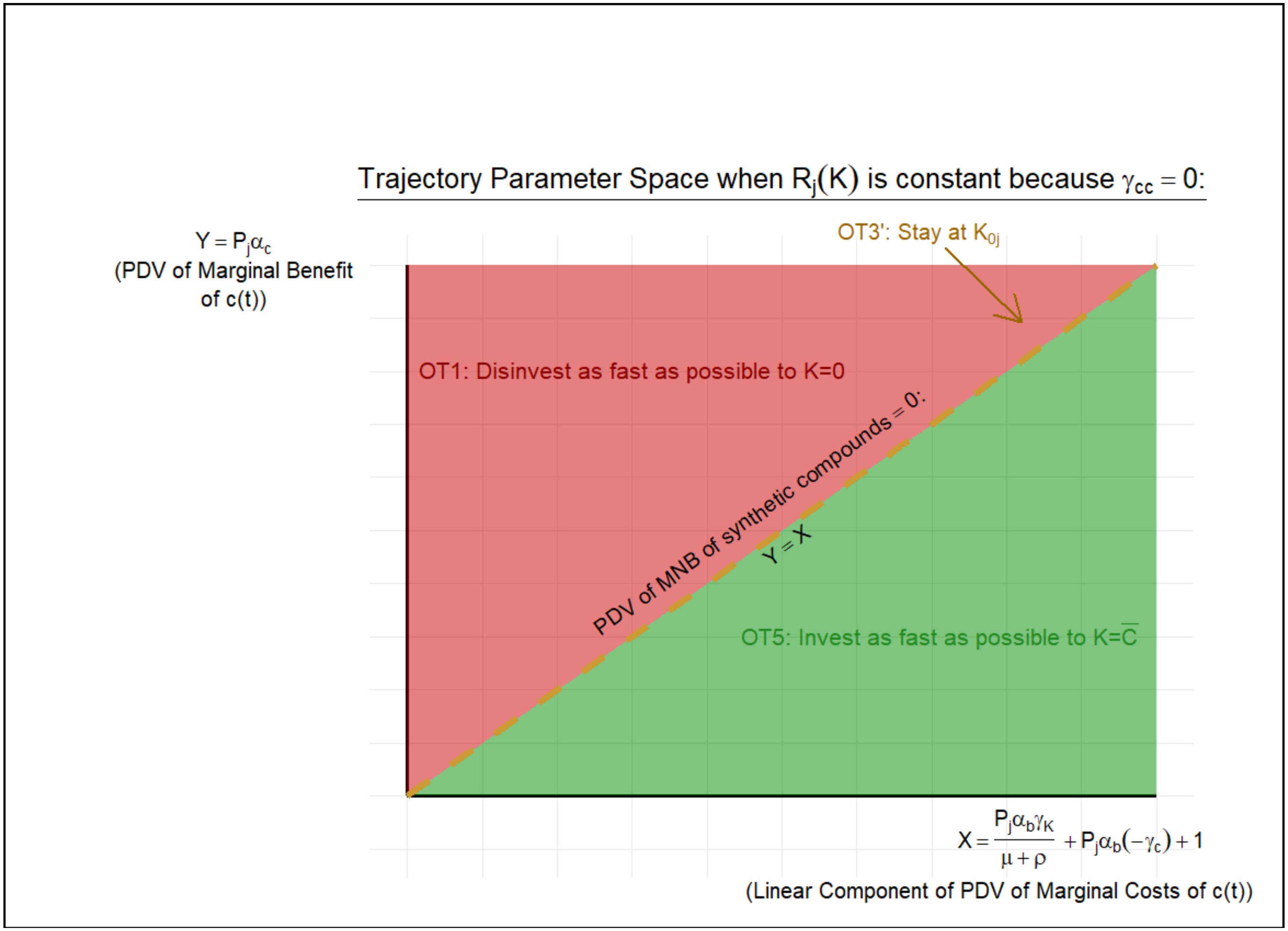
There are three possible types of optimal trajectories that arise when $R(K)$ is constant because $\gamma_{cc} = 0$, depending on the parameters. In order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least syntheic compound use and the most investment in the stock of clean soils, these three types of optimal trajectories are as follows:

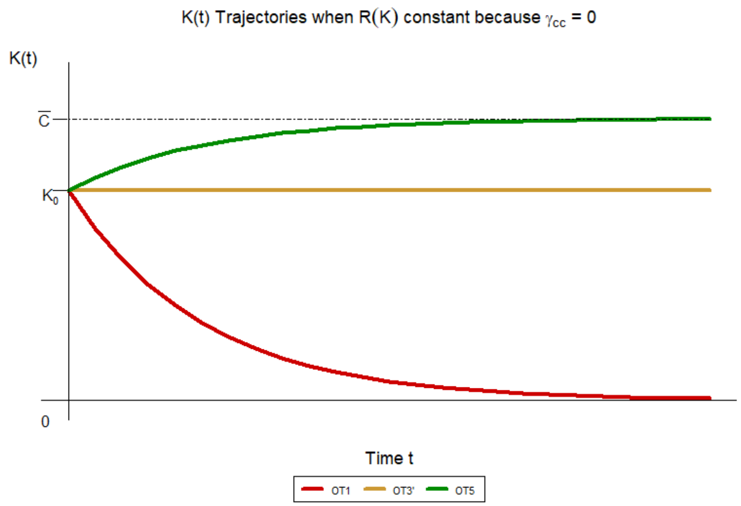
- **Optimal Trajectories 1 [OT1]**: Disinvest as fast as possible until $K = 0$ by always applying $c(t) = \overline{c}$
- **Optimal Trajectories 3' [OT3']**: Stay at initial clean soil stock K_0 by always applying the amount of synthetic compounds that exactly offsets how much the initial stock of chemicals decays on its own
- **Optimal Trajectories 5 [OT5]**: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

Figure 5 presents the parameter spaces for each of the three types of optimal trajectories when $R(K)$ is constant because $\gamma_{cc} = 0$. Figure 6 plots examples of each of the 3 types of optimal trajectories that arise when $R(K)$ is constant because $\gamma_{cc} = 0$.

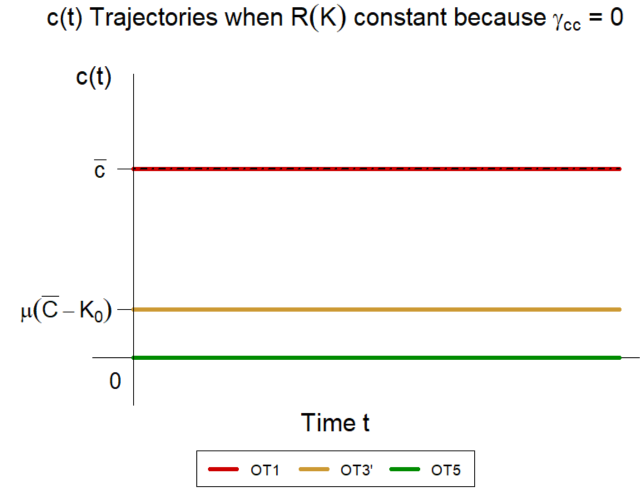
As seen in Figure 6, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the order of the optimal trajectory type by per-period yield reverses, and the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

Figure 5: Parameter Space for Optimal Trajectories When $R(K)$ is Constant Because $\gamma_{cc} = 0$

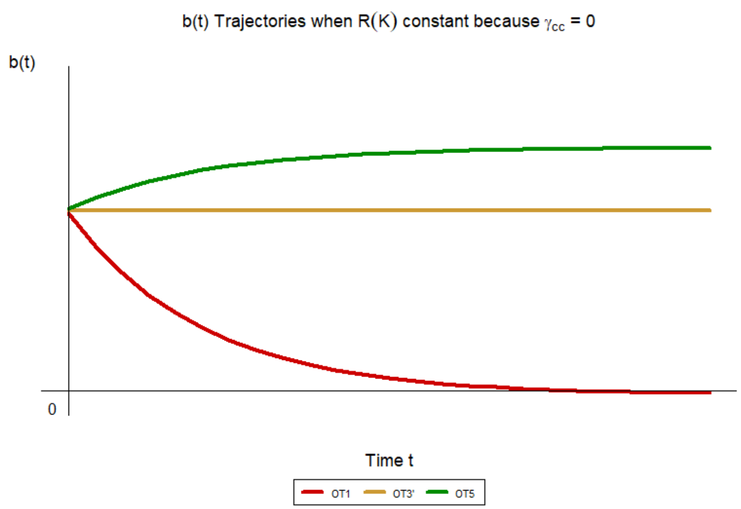




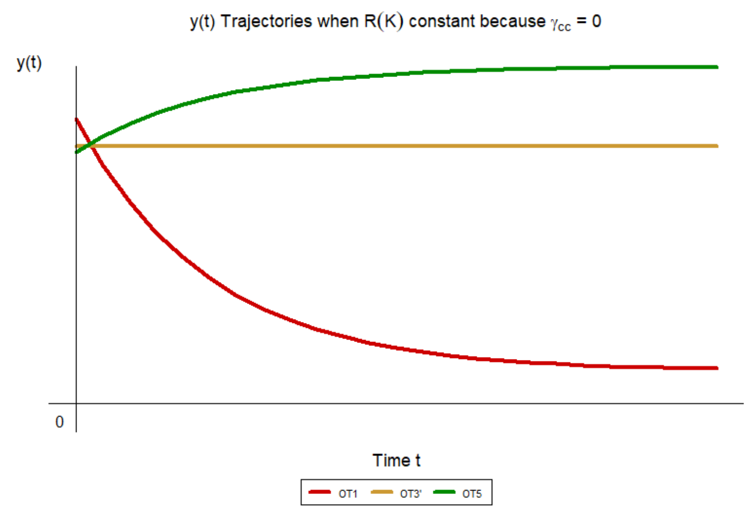
(a) Clean soil stock $K(t)$



(b) Synthetic compound use $c(t)$



(c) Microbes $b(t)$



(d) Yield $y(t)$

Figure 6: Optimal Trajectories When $R(K)$ is constant because $\gamma_{cc} = 0$
Note: We assume $\mu \neq 0$ and $\overline{c} = \mu\overline{C}$.

4.4.1 Optimal Trajectories 1: Disinvest as fast as possible to $K = 0$

If $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{74}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{75}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When we have $\gamma_{cc} = 0$, our gain function is linear in the control variable I:

$$\begin{aligned} G(K, I) = P_j \cdot \Bigg(\alpha_b \Big(\gamma_c \left(\mu \left(\overline{C} - K \right) - I \right) + \gamma_K K + A_b \Big) \\ + \alpha_c \left(\mu \left(\overline{C} - K \right) - I \right) + A_y \Bigg) - \left(\mu \left(\overline{C} - K \right) - I \right), \end{aligned} \tag{76}$$

which means that the farmer will follow a most rapid approach (MRA) policy. Thus, since $R_j(K) < \rho$, the farmer will always disinvest according to the most rapid approach (MRA) policy until he reaches $K = 0$.

If $R_j(K)$ is constant because $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex), and if $R(K)$ is less than ρ, then lower-bound constraints on net investment I *always* bind (for all t), which means the upper bound constraint on synthetic compound use always binds (i.e., $c^{**} > \bar{c}$).

$R_j(K) < \rho$ occurs when the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is positive.

When $\gamma_{cc} = 0$, $R_j(K) < \rho$ implies:

$$P_j \alpha_c > -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + 1 \tag{77}$$

When $R_j(K)$ is constant because $\gamma_{cc} = 0$ and is less than ρ, the farmer's optimal solution is to disinvest as fast as possible until $K = 0$. The optimal synthetic compound use $c(t)$ is to always apply the maximum amount possible $\bar{c}$.

The optimal trajectories are therefore:

$$K(t) = K(0) \cdot e^{-\mu \cdot t} \tag{78}$$

$$I(t) = -\mu \cdot K(0) \cdot e^{-\mu \cdot t} \tag{79}$$

$$c(t) = \underbrace{\mu \overline{C}}_{\overline{c}} \; \forall t \tag{80}$$

$$C(t) = \overline{C} - K(0) \cdot e^{-\mu \cdot t} \tag{81}$$

$$b(t) = \gamma_c \underbrace{\mu \overline{C}}_{=\overline{c}} + \frac{1}{2}\gamma_{cc} \left(\underbrace{\mu \overline{C}}_{=\overline{c}} \right)^2 + \gamma_K K_{0j} \cdot e^{-\mu \cdot t} + A_b$$

$$y(t) = \alpha_b \left(\gamma_c \underbrace{\mu \overline{C}}_{=\overline{c}} + \frac{1}{2}\gamma_{cc} \left(\underbrace{\mu \overline{C}}_{=\overline{c}} \right)^2 + A_b \right) + \alpha_c \underbrace{\mu \overline{C}}_{=\overline{c}} + A_y + \alpha_b \gamma_K K_{0j} \cdot e^{-\mu \cdot t}$$

4.4.2 Optimal Trajectories 3': Stay at initial clean soil stock and do not invest or disinvest

We always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock when $R_j(K)$ is constant and equal to ρ.

In this case it is optimal to stay at initial clean soil stock and not to invest or disinvest. In other words, in each period our chemical input use $c(t)$ should exactly offset the stock of chemicals in the soil decays on its own so that the stock of chemicals in the soil stays constant, and therefore the clean soil stock stays constant at its initial value.

Thus, the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

Thus, for OT3', the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

$$K(t) = K_{0j} \, \forall t \tag{82}$$

$$I(t) = 0 \, \forall t \tag{83}$$

$$c(t) = \mu C(0)\,\forall t \tag{84}$$

$$C(t) = \overline{C} - K_{0j} = C(0)\,\forall t \tag{85}$$

$$b(t) = \left(\gamma_c \mu \left(\overline{C} - K_{0j}\right) + \frac{1}{2}\gamma_{cc}\mu^2 \left(\overline{C} - K_{0j}\right)^2 + \gamma_K K_{0j} + A_b\right)\,\forall t \tag{86}$$

$$\tilde{f}(t) = \alpha_b \left(\gamma_c \mu \left(\overline{C} - K_{0j}\right) + \frac{1}{2}\gamma_{cc}\mu^2 \left(\overline{C} - K_{0j}\right)^2 + \gamma_K K_{0j} + A_b\right) + \alpha_c \mu \left(\overline{C} - K_{0j}\right) + A_y\,\forall t \tag{87}$$

4.4.3 Optimal Trajectories 5: Invest as fast as possible until $K = \overline{C}$

There are two main cases in which the farmer will wish to continually invest when $R_j(K)$ is a constant (that does not depend on K).

First, if $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough to satisfy the following condition for net investment to have a non-negative effect on contemporaneous net gain (so that $R_j(K)$ is not useful for analyzing net investment):

$$P_j^{-1} \geq \alpha_b \gamma_c + \alpha_c, \tag{88}$$

then the farmer will wish to continually invest in clean soil stock.

Second, if $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough that $R_j(K) > \rho$:

$$P_j^{-1} > \frac{\alpha_b}{\rho + \mu}\left((\rho + \mu)\gamma_c - \gamma_K\right) \tag{89}$$

but also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c, \tag{90}$$

then the farmer will wish to continually invest in clean soil stock.

Moreover, when we have $\gamma_{cc} = 0$, our gain function is linear in the control variable I:

$$G(K,I) = P_j \cdot \Bigg(\alpha_b \Big(\gamma_c \left(\mu \left(\overline{C} - K \right) - I \right) + \gamma_K K + A_b \Big) + \alpha_c \left(\mu \left(\overline{C} - K \right) - I \right) + A_y \Bigg) - \left(\mu \left(\overline{C} - K \right) - I \right) \tag{91}$$

which means that the farmer will follow a most rapid approach (MRA) policy. Thus, the farmer will always invest according to the most rapid approach (MRA) policy until he reaches $K = \overline{C}$.

When $R_j(K)$ is constant because $\gamma_{cc} = 0$ (i.e., no convex costs of synthetic compounds on soil microbe production), and $R(K)$ is greater than ρ, then upper-bound constraints on net investment I *always* bind (for all t).

In this case, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is negative even when there are no convex costs of synthetic compounds on soil microbes

When $\gamma_{cc} = 0$, $R_j(K) > \rho$ implies:

$$P_j \alpha_c < -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + 1 \tag{92}$$

When $\gamma_{cc} = 0$ and $\mu \neq 0$, such that $R_j(K)$ is constant but $M(K) \neq 0 \, \forall K$, the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. It is optimal not to use any synthetic compounds $c(t)$ at all.

$$c_j^*(t) = 0 \tag{93}$$

$$K^*(t)_j = K(t)_{UC,j} = \overline{C} - \left(\overline{C} - K_{0j} \right) e^{-\mu \cdot t} \tag{94}$$

$$I^*(t)_j = I(t)_{UC,j} = \mu \cdot \left(\overline{C} - K(t)_{UC,j} \right) \tag{95}$$

$$C^*(t)_j = \left(\overline{C} - K_{0j} \right) e^{-\mu \cdot t} \tag{96}$$

$$b^*(t)_j = \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j} \right) e^{-\mu \cdot t} \right) + A_b \right) \tag{97}$$

$$y^*(t)_j = \alpha_b \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j} \right) e^{-\mu \cdot t} \right) + A_b \right) + A_y \tag{98}$$

4.5 Optimal Trajectories for Stage j When $R(K)$ is Constant Because $\mu = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) then $R_j(K)$ is a constant (that does not depend on K).

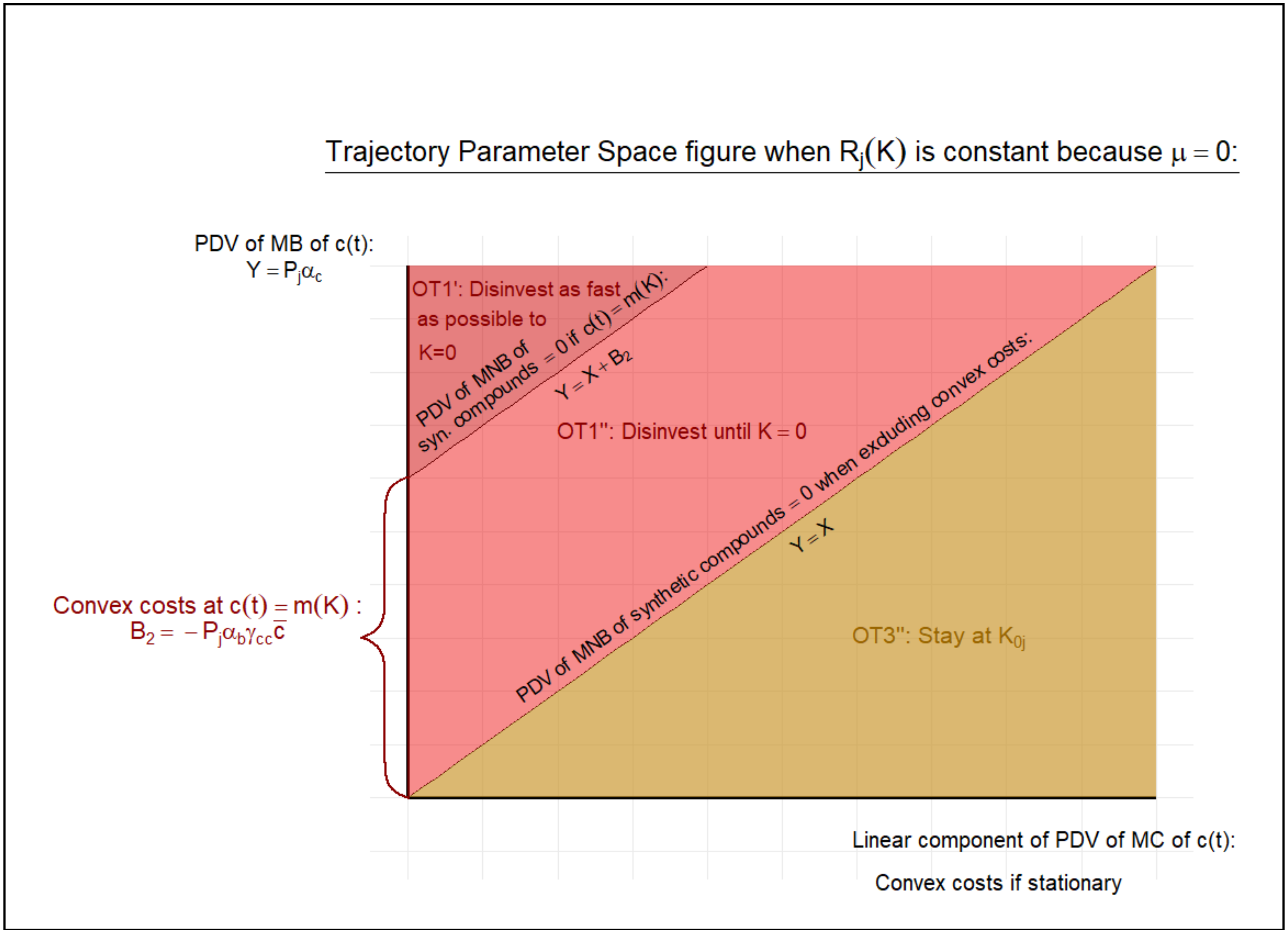
There are three possible types of optimal trajectories that arise when $R(K)$ is constant because $\mu = 0$, depending on the parameters. In order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least syntheic compound use and the least disinvestment in the stock of clean soils, these three types of optimal trajectories are as follows:

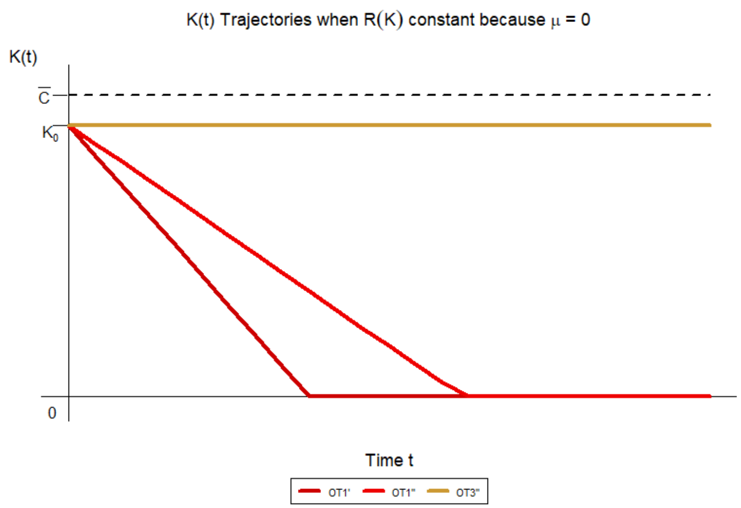
- **Optimal Trajectories 1' [OT1']**: Disinvest as fast as possible by applying $c(t) = \bar{c}$ until $K = 0$ is reached
- **Optimal Trajectories 1" [OT1"]**: Disinvest by applying c_j^{**} at which PDV of MNB equals 0 until $K = 0$ is reached
- **Optimal Trajectories 3" [OT3"]**: Stay at initial clean soil stock K_0 by never applying any synthetic compounds at all

Figure 7 presents the parameter spaces for each of the three types of optimal trajectories when $R(K)$ is constant because $\mu = 0$. Figure 8 plots examples of each of the three types of optimal trajectories that arise when $R(K)$ is constant because $\mu = 0$.

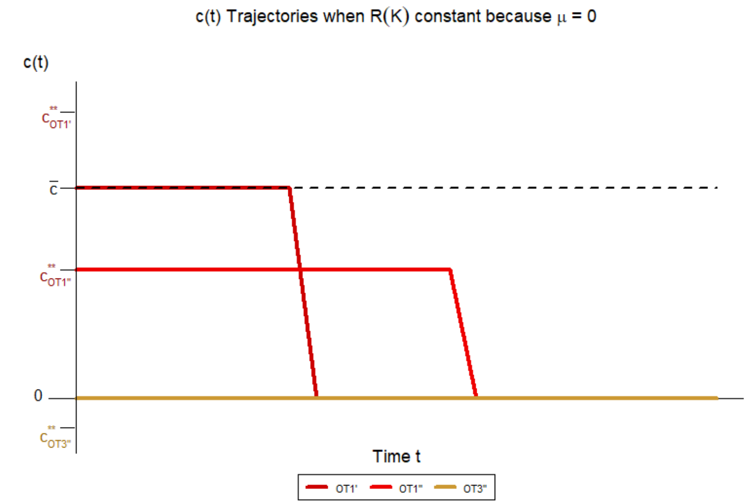
As seen in Figure 8, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the order of the optimal trajectory type by per-period yield reverses, and the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

Figure 7: Parameter Space for Optimal Trajectories When $R(K)$ is Constant Because $\mu = 0$

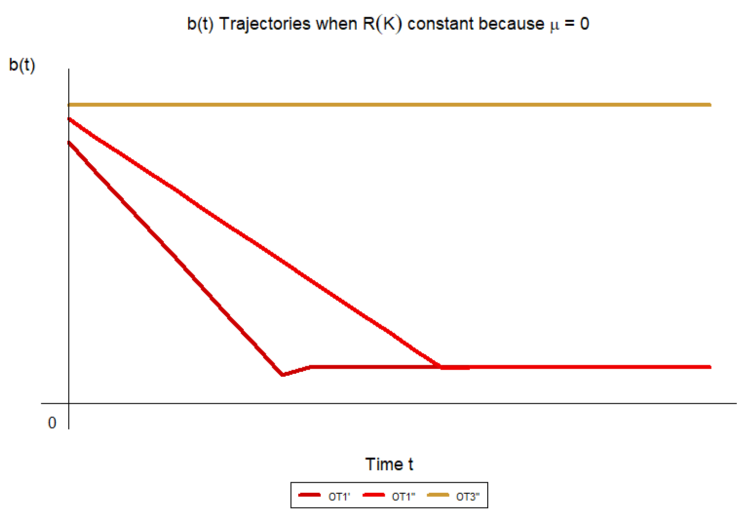




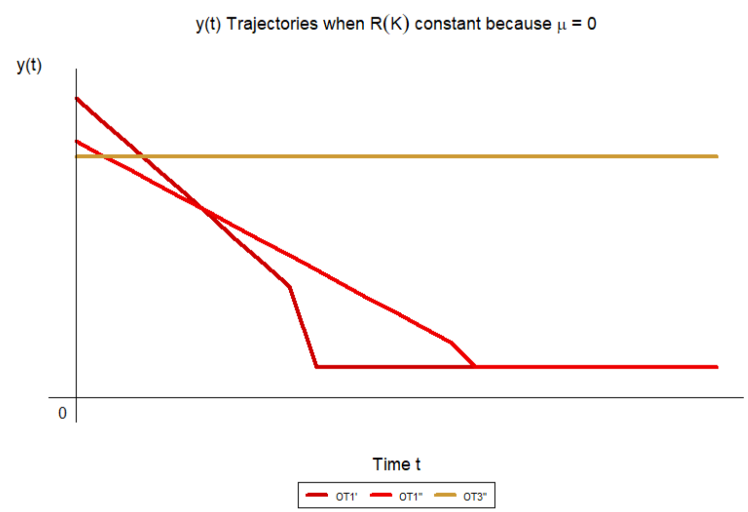
(a) Clean soil stock $K(t)$



(b) Synthetic compound use $c(t)$



(c) Microbes $b(t)$



(d) Yield $y(t)$

Figure 8: Optimal Trajectories When $R(K)$ is constant because $\mu = 0$
Note: We assume $\gamma_{cc} \neq 0$.

4.5.1 Optimal Trajectories 1': Disinvest as fast as possible to $K = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{99}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{100}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

The lower bound to I binds when the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$). If the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$), this means that the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 at $c = \overline{c}$.

The condition $c_j^{**} > \overline{c}$ implies the following when $\mu = 0$:

$$-\frac{P_j \cdot \left(\alpha_c + \alpha_b \left(\gamma_c - \frac{1}{\rho} \cdot \gamma_K \right) \right) - 1}{P_j \alpha_b \gamma_{cc}} > \overline{c} \tag{101}$$

$$\Rightarrow = P_j \cdot \alpha_c > -P_j \alpha_b \gamma_{cc} \overline{c} + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + P_j \alpha_b \left(-\gamma_c \right) + 1 \tag{102}$$

In this case our optimal trajectories are as follows:

$$K(t) = \begin{cases} K(0) - \overline{c} \cdot t & t < T \\ 0 & t \geq T \end{cases} \tag{103}$$

$$I(t) = \begin{cases} -\overline{c} & t < T \\ 0 & t \geq T \end{cases} \tag{104}$$

$$c(t) = \begin{cases} \overline{c} & t < T \\ 0 & t \geq T \end{cases} \tag{105}$$

$$C(t) = \begin{cases} \overline{C} - K(0) + \overline{c} \cdot t & t < T \\ \overline{C} & t \geq T \end{cases} \tag{106}$$

$$b(t) = \begin{cases} \max\{\gamma_c\bar{c} + \frac{1}{2}\gamma_{cc}\bar{c}^2 + \gamma_K\left(K(0) - \bar{c}\cdot t\right) + A_b,\, 0\} & t < T \\ A_b & t \geq T \end{cases} \tag{107}$$

$$y(t) = \begin{cases} \alpha_c\bar{c} + \alpha_b b(t) + A_y & t < T \\ \alpha_b b(t) + A_y & t \geq T \end{cases} \tag{108}$$

$$T = \frac{K(0)}{\bar{c}} \tag{109}$$

4.5.2 Optimal Trajectories 1": Disinvest to $K = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b\gamma_c + \alpha_c \tag{110}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho+\mu}\left((\rho+\mu)\gamma_c - \gamma_K\right) \tag{111}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When $\gamma_{cc} \neq 0$ but $\mu = 0$ the gain function is non-linear in I, and therefore the optimal policy will not be MRA. If the lower corner solution for I does not bind (because $c_j^{**} \leq \bar{c}$), we will have an interior solution.

The condition $c_j^{**} \leq \bar{c}$ implies the following when $\mu = 0$:

$$-\frac{P_j\cdot\left(\alpha_c + \alpha_b\left(\gamma_c - \frac{1}{\rho}\cdot\gamma_K\right)\right) - 1}{P_j\alpha_b\gamma_{cc}} \leq \bar{c} \tag{112}$$

$$\Rightarrow P_j\cdot\alpha_c \leq -P_j\alpha_b\gamma_{cc}\bar{c} + \frac{P_j\alpha_b\gamma_K}{\mu+\rho} + P_j\alpha_b\left(-\gamma_c\right) + 1 \tag{113}$$

Thus, for OT1", the farmer will disinvest by applying the optimal unconstrained synthetic compound level c_j^{**} at which PDV of MNB equals 0 until $K = 0$. The optimal trajectories are as follows:

$$K(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R_j(K)^{-1}\right)\cdot t + K_{0j}, & \forall t \leq T \\ 0, & \forall t > T \end{cases} \tag{114}$$

$$I(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R_j(K)^{-1}\right), & \forall t \leq T \\ 0, & \forall t > T \end{cases} \tag{115}$$

$$c(t) = \begin{cases} \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - R_j(K)^{-1}\right), & \forall t \leq T \\ 0, & \forall > T \end{cases} \tag{116}$$

$$C(t) = \begin{cases} C_{0j} + \frac{\gamma_K}{\gamma_{cc}}\left(\rho^{-1} - R_j(K)^{-1}\right) \cdot t & \forall t \leq T(\\ \overline{C} & \forall t > T \end{cases} \tag{117}$$

$$b(t) = \begin{cases} \gamma_K \cdot \left(\left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}}\right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R_j(K)^{-1}\right)\right) \cdot \left(\rho^{-1} - R_j(K)^{-1}\right)\right. & \\ \left. + K_{0j}\right) + A_b, & \forall t \leq T \\ & \\ A_b, & \forall t > T \end{cases} \tag{118}$$

$$y(t) = \begin{cases} \alpha_b \gamma_K \left(\frac{1}{\gamma_{cc}} \cdot \left(\frac{\alpha_c}{\alpha_b} + \gamma_c - \gamma_K \cdot \left(t + \frac{1}{2} \cdot \left(\rho^{-1} - R_j(K)^{-1}\right)\right)\right)\right. & \\ \left. \cdot \left(\rho^{-1} - R_j(K)^{-1}\right) + K_{0j} + \frac{A_b + \frac{A_y}{\alpha_b}}{\gamma_K}\right), & \forall t \leq T \\ & \\ \alpha_b A_b + A_y, & \forall t > T \end{cases} \tag{119}$$

$$T = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(R_j(K)^{-1} - \rho^{-1}\right)} \geq 0 \tag{120}$$

$$R_j(K) = \frac{\gamma_K}{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}. \tag{121}$$

4.5.3 Optimal Trajectories 3": Stay at initial clean soil stock and do not invest or disinvest

We always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock when $\mu = 0$ and $R_j(K)$ is constant and greater than or equal to ρ.

If $R_j(K)$ is constant and equal to ρ, it is optimal to stay at initial clean soil stock and not to invest or disinvest.

When $\mu = 0$, the condition $R_j(K) \geq \rho$ implies the following:

$$P_j \alpha_c \leq \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + P_j \alpha_b \left(-\gamma_c\right) + 1 \tag{122}$$

When $\mu = 0$ the upper bound constraint on investment is equal to zero, and will always bind when $R_j(K)$ is greater than ρ. In this case, the farmer is constrained by the upper bound on I to stay at their initial capital stock K_{0j} indefinitely.

Thus, for OT3", the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own. Since the stock of chemicals in the soil does not decay on its own when $\mu = 0$, this means the optimal synthetic compound use $c(t)$ is constant at zero.

The optimal trajectories are therefore the following:

$$K(t) = K_{0j} \, \forall t \tag{123}$$

$$I(t) = 0 \, \forall t \tag{124}$$

$$c(t) = 0 \, \forall t \tag{125}$$

$$C(t) = \overline{C} - K_{0j} = C(0) \, \forall t \tag{126}$$

$$b(t) = \gamma_K K_{0j} + A_b \, \forall t. \tag{127}$$

$$y(t) = \alpha_b \left(\gamma_K K_{0j} + A_b\right) + A_y \, \forall t. \tag{128}$$

5 Possible Cases for Combinations of Stage 1 and Stage 2

Given that $P_{con} < P_{org}$, that $\tilde{K}_j \leq \hat{K}_j$, and that both $\hat{K}_j$ and $\tilde{K}_j$ are decreasing in prices P_j, we have the following possible types of cases:

- Case A: **For both Stage 1 and Stage 2, stationary solutions exist and $\hat{K}_j \in [0, \overline{C}]$. In this case our expression for $\hat{K}_j$ tells us that $\hat{K}_{con} > \hat{K}_{org}$.**
  - **This could happen when both conventional and organic crop prices are moderate enough (not too high and not too low). Our conclusion that Stage 1 SS > Stage 2 SS comes from the assumption that P_con < P_org**
  - Sub-cases of Case A include what we will call Cases A1, A2, and A3. These subcases are described below:
    - **Case A1**: Both Conventional Farmer Stationary Solution $\hat{K}_{con}$ and Organic Farmer Stationary Solution $\hat{K}_{org}$ are above K_{org}, and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$
    - **Case A2**: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer Stationary Solution $\hat{K}_{org}$ exists (so is below $\hat{K}_{con}$ and therefore below K_{org} as well), and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$
    - **Case A3**: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is above K_{org}, but Organic Farmer Stationary Solution $\hat{K}_{org}$ is below K_{org}, and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$

- Case B: **A non-negative Stage 1 stationary solution exists but Stage 2 organic farmer disinvests to $K = 0$)**
  - **This could happen when conventional prices are moderate (not too low and not too high) and organic prices are too high**
  - Sub-cases of Case B include what we will call Cases B4-B5. These subcases are described below:
    - **Case B4**: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer disinvests to $K = 0$ because $\hat{K}_{org} < 0$
    - **Case B5**: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is above K_{org} and Organic Farmer disinvests because $\hat{K}_{org} < 0$

- Case C: **Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$**
  - **Assuming that conventional prices are always less than organic prices, this could happen under very high conventional and organic prices.**

– Sub-cases of Case C include what we will call Cases C6-C7. These subcases are described below:

   * **Case C6**: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $\hat{K}_{con} < 0$ (and therefore $\hat{K}_{org} < 0$ as well)
   * **Case C7**: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $R_j(K)$ is constant and less than ρ

- <u>Case D</u>**: Both the Stage 1 conventional farmer and the Stage 2 organic farmer invest until they reach** $K = \overline{C}$

  – **This could happen if we had low conventional prices and low organic prices**

  – α_c is relatively large compared to $(-\gamma_{cc})$, $(-\gamma_c)$, and μ, but also γ_K is sufficiently large relatively to α_c, and α_b is not too small relative to α_c.

  – Sub-cases of Case D include what we will call Cases D8-D9. These subcases are described below:

   * **Case D8**: Both Stage 1 conventional farmer and the Stage 2 organic farmer invest until they reach $K = \overline{C}$ because $\hat{K}_j > \overline{C}$
   * **Case D9**: Both Stage 1 conventional farmer and the Stage 2 organic farmer invest until they reach $K = \overline{C}$ because $R_j(K)$ is constant and greater than ρ

- <u>Case E</u>**: Stage 1 conventional farmer invests until he reaches** $K = \overline{C}$**; a Stage 2 stationary solution exists and** $\hat{K}_{org} \in [0, \overline{C}]$

  – **This could happen if we had low conventional prices and moderate organic prices**

  – Sub-cases of Case E include what we will call Cases E10-E11. These subcases are described below:

   * **Case E10**: Stage 1 conventional farmer invests until he reaches $K = \overline{C}$ because $\hat{K}_{con} > \overline{C}$; and Organic Farmer Stationary Solution $\hat{K}_{org}$ is above K_{org} and $\hat{K}_{org} \in [0, \overline{C}]$
   * **Case E11**: Stage 1 conventional farmer invests until he reaches $K = \overline{C}$ because $\hat{K}_{con} > \overline{C}$; and Organic Farmer Stationary Solution $\hat{K}_{org}$ is below K_{org} and $\hat{K}_{org} \in [0, \overline{C}]$

- **<u>Case F</u>: $R_j(K)$ is constant and $R_{org}(K) \leq \rho \leq R_{con}(K)$**
  - $R_j(K)$ **is constant if either** $\gamma_{cc} = 0$ **(i.e., the negative effects of chemical input use** $c(t)$ **on beneficial soil microbes** $b(t)$ **are linear rather than convex) or** $\mu = 0$ **(i.e., synthetic compounds in the soil do not decay on their own)**
  - In this case, as derived above, $\frac{\partial R_j(K)}{\partial P_j} \leq 0$.
  - Thus, if either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then the constant $R_j(K)$ is lower when prices are higher.
  - Sub-cases of Case F include what we will call Cases F12-F14. These subcases are described below:
    * **Case F12**: $R_j(K)$ is constant and $R_{org}(K) < \rho < R_{con}(K)$
    * **Case F13**: $R_j(K)$ is constant and $R_{org}(K) = \rho < R_{con}(K)$
    * **Case F14**: $R_j(K)$ is constant and $R_{org}(K) < \rho = R_{con}(K)$

5.1 Continuous vs. Discrete Transitions

In summary, we have the following cases:

- Case A1: Both Conventional Farmer Stationary Solution $\hat{K}_{con}$ and Organic Farmer Stationary Solution $\hat{K}_{org}$ are above K_{org}, and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$
- Case A2: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer Stationary Solution $\hat{K}_{org}$ exists (so is below $\hat{K}_{con}$ and therefore below K_{org} as well), and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$
- Case A3: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is above K_{org}, but Organic Farmer Stationary Solution $\hat{K}_{org}$ is below K_{org}, and $\hat{K}_j \in [0, \overline{C}]$ for $j \in \{con, org\}$
- Case B4: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer disinvests to $K = 0$ because $\hat{K}_{org} < 0$
- Case B5: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is above K_{org} and Organic Farmer disinvests because $\hat{K}_{org} < 0$
- Case C6: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $\hat{K}_{con} < 0$ (and therefore $\hat{K}_{org} < 0$ as well)
- Case C7: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $R_j(K)$ is constant and less than ρ
- Case D8: Both Stage 1 conventional farmer and the Stage 2 organic farmer invest until they reach $K = \overline{C}$ because $\hat{K}_j > \overline{C}$
- Case D9: Both Stage 1 conventional farmer and the Stage 2 organic farmer invest until they reach $K = \overline{C}$ because $R_j(K)$ is constant and greater than ρ
- Case E10: Stage 1 conventional farmer invests until he reaches $K = \overline{C}$ because $\hat{K}_{con} > \overline{C}$; and Organic Farmer Stationary Solution $\hat{K}_{org}$ is above K_{org} and $\hat{K}_{org} \in [0, \overline{C}]$
- Case E11: Stage 1 conventional farmer invests until he reaches $K = \overline{C}$ because $\hat{K}_{con} > \overline{C}$; and Organic Farmer Stationary Solution $\hat{K}_{org}$ is below K_{org} and $\hat{K}_{org} \in [0, \overline{C}]$
- Case F12: $R_j(K)$ is constant and $R_{org}(K) < \rho < R_{con}(K)$
- Case F13: $R_j(K)$ is constant and $R_{org}(K) = \rho < R_{con}(K)$
- Case F14: $R_j(K)$ is constant and $R_{org}(K) < \rho = R_{con}(K)$

6 Accidental Organic Transitions

The transition from conventional to organic management is 'accidental' and continuous even in the absence of an organic price premium for either:

1. OT5: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

2. OT4 if $\hat{K}_{con} \geq$K$_{org}$: Invest until $\hat{K}_{con}$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0

since in these cases the optimal solution for a conventional farmer is to invest in the stock of clean soils until $K(t)$ exceeds K_{org}.

This will happen in agricultural systems where soil microbes are sufficiently important for determining crop yields.

If $\hat{K}_{con} \in [K_{org}, \overline{C}]$, then we have Optimal Trajectories 4: Approach $\hat{K}_{con}$ at moderate speed, then the time T_{org} at which a fully informed conventional farmer makes a continuous transition to organic farming is given by:

$$T_{org} = \ln\left(\frac{K_{0,con} - \hat{K}_{con}}{K_{org} - \hat{K}_{con}}\right)^{\frac{1}{\mu}} \tag{129}$$

If $\hat{K}_{con} > \overline{C}$, or $R_{con}(K)$ is constant and greater than ρ, then we have Optimal Trajectories 5: Invest as fast as possible until $K = \overline{C}$, then the time T_{org} at which a fully informed conventional farmer makes a continuous transition to organic farming is given by:

$$T_{org} = \ln\left(\frac{\overline{C} - K_{0,con}}{\overline{C} - K_{org}}\right)^{\frac{1}{\mu}} \tag{130}$$

The transition from conventional stage 1 to organic stage 2 is continuous when the conventional farmer stationary solution $\hat{K}_{con}$ is either above K_{org} or above $\overline{C}$, since then a conventional farmer will tend to invest in the stock of clean soils until he reaches the organic threshold. Thus, of the above 14 cases, the transition from conventional stage 1 to organic stage 2 is continuous for Case A1, Case A3, Case B5, Case D8, Case D9, Case E10, Case E11, Case F12, and Case F13.

7 Premium-Induced Organic Transitions

There is no 'accidental' transition from conventional to organic for:

1. OT1: Disinvest as fast as possible until $K = 0$ by always applying $c(t) = \overline{c}$
2. OT2: Disinvest to $\hat{K}_j < K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
3. OT3: Stay at initial clean soil stock $K_0 = \hat{K}_j$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
4. OT4 if $\hat{K}_{con}$ ¡ K_{org}: Invest until $\hat{K}_{con}$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0

Accidental, continuous transitions cannot occur for OT2 because if $\hat{K}_{con}$ is above K_{org}, then since initial K is above $\hat{K}_{con}$, which is above K_{org}, this means that the 'conventional' farmer already starts out organic.

There is no accidental, continuous transition from conventional stage 1 to organic stage 2 when the conventional farmer stationary solution $\hat{K}_{con}$ is below K_{org}, since then a conventional farmer will tend towards the conventional farmer stationary solution $\hat{K}_{con}$, and therefore stay below K_{org} rather than become organic.

Similarly, there is no accidental, continuous transition from conventional stage 1 to organic stage 2 when $R_{con}(K)$ is constant and less than ρ, since then the conventional farmer will continually disinvest until $K = 0$, and therefore stay below K_{org} rather than become organic.

Nevertheless, when $\hat{K}_{con} < K_{org}$ or when $R_{con}(K)$ is constant and always less than ρ, the organic price premium may still cause some farmers to "jump" to the organic threshold.

If there is no 'accidental' transition, an organic price premium may still induce some farmers to switch to organic management. Given $P_{org} > P_{con}$, it may still be possible for fully informed conventional farmer to prefer organic farming, even when $\hat{K}_j < K_{org}$ or when $R_{con}(K)$ is constant and always less than ρ, and make a "jump" transition to the organic certification threshold. For this to occur we must have:

$$\Delta(\epsilon) \equiv V_{org}(K_{org}) - V_{con}(K_{0,con}) > 0 \tag{131}$$

where $K_{0,con} = K_{org} - \epsilon$ for some $\epsilon > 0$.

We denote our full set of model parameters $\Omega(X)$.

We find the conditions on P_{con}, P_{org}, K_{org}, $\Omega(X)$, and ϵ that satisfy Equation (131).

We solve for the values of the organic price premium $\frac{P_{org}-P_{con}}{P_{con}}$ that satisfy condition (131) for a conventional farmer to want to adopt organic. Similarly, we solve for the values of ϵ, which measures how close ϵ the conventional farmer is to satisfying organic requirement K_{org} at $t = 0$, satisfy condition (131) for a conventional farmer to want to adopt organic.

Because our optimal trajectories change form depending on where we are in the parameter space, Equation (131) also changes form depending on parameter space. Thus, the conditions on the organic price premium $\frac{P_{org}-P_{con}}{P_{con}}$ defining $\{\frac{P_{org}-P_{con}}{P_{con}}) : \Delta(\epsilon) > 0\}$ and also on ϵ defining $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$ do not have a general form. As a result, we must find separate conditions from Equation (131) for each part of parameter space.

7.1 Discrete Analysis for OT1 (Case C6, Case B4; Case C7 when $\gamma_{cc} = 0$)

Discrete Analysis for Case C6: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $\hat{K}_{con} < 0$ (and therefore $\hat{K}_{org} < 0$ as well).

Recall that in Case C6 the optimal solution for each stage $j \in \{con, org\}$ is to disinvest as fast as possible until $K = 0$.

A conventional farmer facing C6 conditions will adopt OT1 solutions.

Case B4: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer disinvests to $K = 0$ because $\hat{K}_{org} < 0$.

Case B4 ends up being exactly the same as Case C6 because these two cases only differ in $\hat{K}_{org}$, but not in their stage 2 trajectories (in both cases we will have $K(t)_{org} = K_{org}$ and $I(t)_{org} = 0$ for all t).

A conventional farmer facing B4 or C6 conditions will adopt OT1 solutions.

Case C7: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $R_j(K)$ is constant and less than ρ

Similarly, Case C7 when $\gamma_{cc} = 0$ ends up being the same as Case C6 because the conventional farmer adopts OT1 solutions while for the stage 2 trajectories we have $K(t)_{org} = K_{org} \forall t$ and $I(t)_{org} = 0 \forall t$, except that the interpretation for ϵ^* based on $\hat{K}_{con}$ in Figure B.1 no longer applies, since $\hat{K}_{con}$ does not exist.

Similarly, Case C7 when $\gamma_{cc} = 0$ ends up being exactly the same as Case C6 because the conventional farmer adopts OT1 solutions, while for the stage 2 trajectories we have $K(t)_{org} = K_{org}$ and $I(t)_{org} = 0$ for all t.

A conventional farmer facing B4 or C6 conditions, or C7 conditions when $\gamma_{cc} = 0$, will adopt OT1 solutions.

In this case, a farmer who starts off organic will disinvest until they reach K_{org}. They then choose to remain organic if and only if

$$V_{org}(K_{org}) > V_{con}(K_{org} - \epsilon). \tag{132}$$

$V_{org}(K_{org})$ is the present discounted value of the entire stream of net benefits that a farmer will receive from the moment they have switched to organic management, into perpetuity, assuming the organic farmer stays organic indefinitely. $V_{org}(K_{org})$ assumes that once in stage 2, the farmer follows the following constrained trajectories:

$$\bar{K}(t)_{org} = K_{org} \, \forall t \tag{133}$$

$$\bar{I}(t)_{org} = 0 \, \forall t \tag{134}$$

$$\overline{C}(t)_{org} = \mu \left(\overline{C} - K_{org} \right) \, \forall t \tag{135}$$

To further simplify our analysis, let's also assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils, such that they will be certifiied organic if and only if $K = \overline{C}$.

When $K_{org} = \overline{C}$, the value $V_{org}(K_{org})$ of the farmer's optimal program for stage 2 following this constrained capital trajectory can be written as follows:

$$V_{org}(K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b \right) + A_y \right) \tag{136}$$

On the other hand, $V_{con}(K_{org} - \epsilon)$ is the present discounted value of the entire stream of net benefits that a farmer will receive if they continue to produce conventionally indefinitely. When the conventional farmer adopts OT1 solutions and $K_{org} = \overline{C}$, $V_{con}(K_{org} - \epsilon)$ is given by:

$$V_{con}(K_{org} - \epsilon) = \frac{1}{\rho} \cdot P_{con} \alpha_b \cdot \left(\frac{\rho}{(\mu + \rho)} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon \right) + \left(\frac{1}{2} \gamma_{cc} \mu \overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{137}$$

With expressions for $V_{org}(K_{org})$ and $V_{con}(K_{org} - \epsilon)$ we can now write the following expression:

$$\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon) \tag{138}$$

as follows when $K_{org} = \overline{C}$:

$$\Delta^{C6}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}(P_{org}-P_{con})\cdot(\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium}\\ \text{on "level effect"}\\ \text{of other agricultural inputs}}}$$

$$- \underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\underbrace{(\overline{C}-\epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} \underbrace{-\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\overline{C}}_{\substack{\text{PDV of using synthetic compounds}\\ \text{at dynamically optimal rate } \mu\overline{C}}}$$

The sign of $\frac{\partial\Delta(\epsilon)}{\partial\epsilon}$ is given by:

$$\frac{\partial\Delta(\epsilon)}{\partial\epsilon} \geq \underbrace{P_{con}\cdot\alpha_b\cdot\frac{1}{(\mu+\rho)}\cdot\gamma_K}_{\geq 0} \tag{139}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.
Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$. Note that $\Delta(\epsilon^*) = 0$.
The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\begin{aligned}\epsilon^* = &\left(\frac{\mu+\rho}{\gamma_K}\cdot\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-1\right)\cdot\frac{\mu}{\rho}\cdot\overline{C}\\ &-\frac{1}{\gamma_K}\cdot\underbrace{\frac{\mu+\rho}{\rho}\cdot\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)\end{aligned}$$

$$\begin{aligned}\epsilon^* = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K}\cdot\frac{1}{\rho}}_{\leq 0}\Bigg(&\underbrace{(P_{org}-P_{con})\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\\ &-P_{con}\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\mu-\underbrace{\left(\frac{P_{org}}{P_{con}}-\frac{\rho}{\mu+\rho}\right)\gamma_K}_{\geq 0}\right)\overline{C}\Bigg)\end{aligned} \tag{140}$$

This means that when $\epsilon^* \leq 0$ the farmer will face $V_{org}(K_{org}) - V_{con}(K_{org}-\epsilon) > 0\,\forall\epsilon \geq 0$, and will therefore prefer to produce organically for all feasible initial capital stocks (i.e. they always prefer to produce organically).

Given $\frac{\partial\Delta(\epsilon^*)}{\partial\epsilon} \geq 0$, we will have that:

- The lower the threshold ϵ^*, the larger the set $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$
- The higher the threshold ϵ^*, the smaller the $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$

We conduct a comparative statics analysis of $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$ to analyze how $\Delta(\epsilon)$ responds to changes in parameters ($\mu, \rho, \gamma_{cc}, \gamma_c, \gamma_K, \alpha_1, \alpha_c, P_{con},$ and P_{org}). The results are summarized in Table B.1 and the derivations are presented in Appendix B.1.1.

We similarly conduct a comparative static analysis for ϵ^*. The results are summarized in Table B.2 and the derivations are presented in Appendix B.1.2.

We also want to find how large the price premium needs to be in order to induce the fully informed farmer to prefer organic management. We derive this requirement for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ below.

The range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho \cdot \epsilon\right)}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{141}$$

We also conduct a comparative statics analysis for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$. The results are summarized in Table B.3 and full derivations are presented in Appendix B.1.3.

7.2 Discrete Analysis for OT2/OT3/OT4 (Case A2)

Case A2: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer Stationary Solution $\hat{K}_{org}$ exists (so is below $\hat{K}_{con}$ and therefore below K_{org} as well), and $\hat{K}_{S_j} \in [0, \overline{C}]$ for $j \in \{con, org\}$

In A2 the farmer will also adopt the same stage 2 trajectories as in B4 and C6, namely $K(t)_{org} = K_{org} \forall t$ and $I(t)_{org} = 0 \forall t$.

A conventional farmer facing A2 conditions will adopt either an OT2, OT3, or OT4 solution.

The conventional A2 farmer faces:

$$0 < \hat{K}_j < K_{org} \leq \overline{C} \tag{142}$$

and adopts the following trajectories:

$$K^*(t)_{S_j} = \hat{K}_j + \left(K(0)_j - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{143}$$

$$I^*(t)_{S_j} = \mu\left(\hat{K}_j - K(t)\right) \tag{144}$$

$$c^*(t)_{S_j} = \mu\left(\overline{C} - \hat{K}_j\right) \tag{145}$$

For the conventional A2 farmer, assuming $K_{org} = \overline{C}$, $V_{con}(K_{org} - \epsilon)$ is given by:

$$V_{con}^{A2}(K_{org}-\epsilon) = \underbrace{\frac{1}{\rho}\cdot P_{con}\left(\alpha_b \cdot A_b + A_y\right)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at conventional prices}}} + \underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b \cdot \gamma_K \cdot \left(\underbrace{(K_{org}-\epsilon)}_{=K_0} - \hat{K}_{con}\right)}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}}$$
$$+ \underbrace{\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right)\cdot \mu\left(\overline{C} - \hat{K}_{con}\right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu\left(\overline{C} - \hat{K}_{con}\right)}$$

Given $K_{org} = \overline{C}$ the stage-2 A2 farmer faces:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}P_{org}\cdot\left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at organic prices}}} \tag{146}$$

Given $K_{org} = \overline{C}$ the $\Delta^{A2}(\epsilon)$ faced by the conventional A2 farmer is given by:

$$\Delta^{A2}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K\underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org}-P_{con}\right)\cdot\left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of organic price premium}\\ \text{on "level effect"}\\ \text{of other agricultural inputs}}}$$

$$-\underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\left(\underbrace{(\overline{C}-\epsilon)}_{=K_0}-\hat{K}_{con}\right)}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}}$$

$$-\underbrace{\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\left(\overline{C}-\hat{K}_{con}\right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu\left(\overline{C}-\hat{K}_{con}\right)}$$

The sign of $\frac{\partial\Delta(\epsilon)}{\partial\epsilon}$ is given by:

$$\frac{\partial\Delta(\epsilon)}{\partial\epsilon} = \frac{P_{con}\alpha_b\gamma_K}{\mu+\rho}\cdot\epsilon \geq 0 \tag{147}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.
Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$.
The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where, when $K_{org} = \overline{C}$:

$$\epsilon^* = \frac{1}{\gamma_K}\cdot\frac{\mu+\rho}{\rho}\cdot\left(\underbrace{\frac{1}{2}\cdot\frac{\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{\gamma_K}{\mu+\rho}\right)^2}{(-\gamma_{cc})}}_{\geq 0} - \underbrace{\left(\frac{P_{org}}{P_{con}}-1\right)\cdot\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\right) \tag{148}$$

As shown in Appendix B.2, case A2 allows for the possibility that ϵ^* exceeds $\overline{C}$. When this happens there will be no feasible ϵ for which $\Delta(\epsilon) \geq 0$, and there will therefore be no feasible capital stock for which the fully informed farmer facing Case A2 OT2/OT3/OT3 conditions will prefer to produce organically. ϵ^* will be more likely to exceed $\overline{C}$ when the farmer faces small organic price premia.

We conduct a comparative statics analysis of $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$ to analyze how $\Delta(\epsilon)$ responds to changes in parameters ($\mu, \rho, \gamma_{cc}, \gamma_c, \gamma_K, \alpha_1, \alpha_c, P_{con},$ and P_{org}). The results are summarized in Table B.4 and the derivations are presented in Appendix B.2.1.

We similarly conduct a comparative statics analysis for ϵ^*. The results are summarized in Table B.5 and the derivations are presented in Appendix B.2.2.

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing A2, OT2/OT3/OT4 conditions to prefer to produce organically. We derive an inequality describing the necessary conditions in Appendix B.2.3.

Given the assumption that $K_{org} = \overline{C}$, and assuming conventional crop prices are not zero, we can write:

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \frac{\int_0^\infty P_{con} \cdot \alpha_b \cdot \frac{1}{2} \cdot (-\gamma_{cc})\, c(\hat{T})^2 \cdot e^{-\rho \cdot t} dt - \int_0^\infty P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon \cdot e^{-(\mu+\rho)\cdot t} dt}{\int_0^\infty P_{con} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b\right) + A_y\right) \cdot e^{-\rho \cdot t} dt} \tag{149}$$

We're finding that there's a value to (1) not managing conventionally (i.e. reducing one's per-period synthetic compound use), but also to (2) receiving enough capital to satisfy the organic certification requirement (i.e. having an overall larger capital stock).

$$\begin{gathered}\underbrace{\int_0^\infty (P_{org} - P_{con}) \cdot f_{org} \cdot e^{-\rho \cdot t} dt}_{\text{Net gain from organic premium}} \\ + \underbrace{\int_0^\infty P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(\epsilon \cdot e^{-\mu \cdot t}\right) \cdot e^{-\rho \cdot t} dt}_{\text{gain from organic management}} \geq \\ \underbrace{\int_0^\infty P_{con} \cdot \alpha_b \cdot \frac{1}{2} \cdot (-\gamma_{cc})\, c(\hat{T})^2 \cdot e^{-\rho \cdot t} dt}_{\text{loss from conventional management}}\end{gathered} \tag{150}$$

Let $\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*$ denote the threshold value:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* =$$

$$\left(\frac{1}{2} \cdot \frac{1}{(-\gamma_{cc})} \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\mu+\rho)}\right)^2 - \frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \epsilon\right) \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)^{-1} \tag{151}$$

Then we can determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters. The results are summarized in Table B.6 and derivations are presented in Appendix B.2.3.

7.3 Discrete Analysis for OT3' and OT3" (Case F14)

Case F14: $R(K)_{con}$ and $R(K)_{org}$ Constant with $R(K)_{con} = \rho$ and $R(K)_{org} < \rho$

If $R_j(K)$ is constant $\forall j \in \{con, org\}$, it must be that either $\gamma_{cc} = 0$, $\mu = 0$, or both. Because the conventional farmer's optimal trajectory and value function will depend on which of these conditions holds true, we will have to examine case F14 in two parts:

- F14 A, where $\gamma_{cc} = 0$ and $\mu \neq 0$ (conventional OT3')
- F14 B, where $\mu = 0$ (conventional OT3")

We consider each case in turn below.

7.3.1 Case F14 A: $\gamma_{cc} = 0$, $\mu \neq 0$, $R_{con}(K) = \rho\ \forall K$ (conventional OT3'), and $R_{org}(K) < \rho$:

In this case the stage 1 conventional farmer follows the following solution trajectories (OT3'):

$$K(t)_{con} = K_{org} - \epsilon\, \forall t \tag{152}$$

$$I(t)_{con} = 0\, \forall t \tag{153}$$

$$c(t)_{con} = \mu\left(\overline{C} - K(t)_{con}\right) - I(t)_{con} \tag{154}$$

$$= \mu\left(\overline{C} - K_{org} + \epsilon\right)\ \forall t \tag{155}$$

Given the assumption that $K_{org} = \overline{C}$, $K(t)_{con}$ and $c(t)_{con}$ simply to

$$K(t)_{con} = \overline{C} - \epsilon\, \forall t \tag{156}$$

$$c(t)_{con} = \mu \cdot \epsilon\, \forall t \tag{157}$$

where $\epsilon > 0$ and is determined by the equation $K(0) = K_{org} - \epsilon$.

The stage 2 organic farmer will, conditional on having reached the organic threshold and decided to remain organic, adopt the following constrained trajectory:

$$\bar{K}(t)_{org} = \overline{C}\,\forall t \tag{158}$$

$$\bar{I}(t)_{org} = 0\,\forall t \tag{159}$$

$$\overline{C}(t)_{org} = \mu\left(\overline{C} - \bar{K}(t)_{org}\right) - \bar{I}(t)_{org} \tag{160}$$

$$= 0\,\forall t \tag{161}$$

Applying our solutions for $c(t)_{con}$ and $K(t)_{con}$ from above, as well as the assumption that $K_{org} = \overline{C}$, and $\gamma_{cc} = 0$, we can then write $V_{con}(K_{org} - \epsilon)$ as:

$$\begin{aligned} V_{con}(\overline{C} - \epsilon) = & \\ & \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(A_b + \frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at conventional prices}}} + \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(\overline{C} - \epsilon\right)}_{\substack{\text{PDV of value gained from} \\ \text{microbial productivity} \\ \text{under conventional management}}} \\ & + \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal} \\ \text{conventional rate } \mu \cdot \epsilon}} \end{aligned} \tag{162}$$

When $\gamma_{cc} = 0$ and $\mu \neq 0$, $R_{con}(K) = \rho\,\forall K$ implies:

$$\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} = \frac{\gamma_K}{\mu + \rho} \tag{163}$$

On the other hand, the F14 A fully informed farmer will face the following stage 2 value function:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} P_{org} \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at organic prices}}} \tag{164}$$

Given $K_{org} = \overline{C}$, the $\Delta^{F14\,A}(\epsilon)$ faced by the conventional F14 A farmer is given by:

$$\begin{aligned}
\Delta^{F14\,A}(\epsilon) = & \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} (P_{org} - P_{con}) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} \\
& - \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon}_{\substack{\text{PDV of using synthetic compounts} \\ \text{at dynamically optimal} \\ \text{conventional rate } \mu \cdot \epsilon}}
\end{aligned} \tag{165}$$

which we can simplify as follows:

$$\Delta^{F14A}(\epsilon) = \underbrace{\frac{1}{\rho}\left(P_{org}-P_{con}\right)\cdot\alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of organic price premium} \\ \text{from organic stock effect}}} + \underbrace{\frac{1}{\rho}\alpha_b\left(P_{org}-P_{con}\right)\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of organic price premium} \\ \text{from ag. base productivity}}}$$
$$+ \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\epsilon}_{\substack{\text{PDV of additional value gained from} \\ \text{microbial productivity after adopting} \\ \text{organic management practices,} \\ \text{valued at conventional prices}}} - \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{\gamma_K}{\mu+\rho}\cdot\mu\cdot\epsilon}_{\substack{\text{PDV of using synthetic} \\ \text{compounds at dynamically} \\ \text{optimal conventional rate } \mu\cdot\epsilon}} \tag{166}$$

In Appendix B.3.1, we discuss the signs of $\frac{\partial\Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org} = \overline{C}$. The results are summarized in Table B.7.

In Appendix B.3.2, we calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$. The results are summarized in Table B.8.

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing F14A conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below.

Assuming that $P_{con} \neq 0$, and assuming that $\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right) \neq 0$, we can write:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = -\frac{1}{\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}}\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon \leq 0 \tag{167}$$

Given

$$\frac{\partial\Delta^{F14A}(\epsilon)}{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)} = \frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right) \geq 0, \tag{168}$$

$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* \leq 0$ implies that the F14A farmer prefers organic given any non-negative price premium. Still, we may at some point be interested in how the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ responds to changes in our parameter values in this case. In Appendix B.3.3, we determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters. The results are summarized in Table B.9.

7.3.2 <u>Case F14 B: $\mu = 0$,</u> $R_{con}(K) = \rho \; \forall K$ (conventional OT3"), and $R_{org}(K) < \rho$:

In this case the stage 1 conventional farmer follows the following solution trajectories (OT3"):

$$K(t)_{con} = K_{org} - \epsilon \, \forall t \tag{169}$$

$$I(t)_{con} = 0 \, \forall t \tag{170}$$

$$c(t)_{con} = \mu \left(\overline{C} - K(t)_{con}\right) - I(t)_{con} = 0 \, \forall t \tag{171}$$

Given the assumption that $K_{org} = \overline{C}$, $K(t)_{con}$ and $c(t)_{con}$ simply to

$$K(t)_{con} = \overline{C} - \epsilon \, \forall t \tag{172}$$

$$c(t)_{con} = 0 \, \forall t \tag{173}$$

where $\epsilon > 0$ and is determined by the equation $K(0) = K_{org} - \epsilon$.

The stage 2 organic farmer will, conditional on having reached the organic threshold and decided to remain organic, adopt the following constrained trajectory:

$$\bar{K}(t)_{org} = \overline{C} \, \forall t \tag{174}$$

$$\bar{I}(t)_{org} = 0 \, \forall t \tag{175}$$

$$\overline{C}(t)_{org} = \mu \left(\overline{C} - \bar{K}(t)_{org}\right) - \bar{I}(t)_{org} = 0 \, \forall t \tag{176}$$

Applying our solutions for $c(t)_{con}$ and $K(t)_{con}$ from above, as well as the assumption that $K_{org} = \overline{C}$, and $\mu = 0$, and given that $\rho > 0$:, we can then write $V_{con}(K_{org} - \epsilon)$ as:

$$V_{con}(K_{org} - \epsilon) = \underbrace{\frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(A_b + \frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at conventional prices}}} + \underbrace{\frac{1}{\rho} \cdot P_{con}\alpha_b\gamma_K\left(\overline{C} - \epsilon\right)}_{\substack{\text{PDV of value gained from} \\ \text{microbial productivity} \\ \text{under conventional management}}} \tag{177}$$

On the other hand, the F14 B fully informed farmer will face the following stage 2 value function:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho}P_{org} \cdot \alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}P_{org} \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at organic prices}}} \tag{178}$$

Given $K_{org} = \overline{C}$, the conventional F14 B farmer faces:

$$\Delta^{F14\,B}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org} \cdot \alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}}$$
$$- \underbrace{\frac{1}{\rho} \cdot P_{con}\alpha_b\gamma_K\left(\overline{C} - \epsilon\right)}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \tag{179}$$

which we can simplify as follows:

$$\Delta^{F14\,B}(\epsilon) = \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of organic price premium} \\ \text{from organic stock effect}}} + \underbrace{\frac{1}{\rho}\alpha_b \left(P_{org} - P_{con}\right)\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of organic price premium} \\ \text{from ag. base productivity}}}$$

$$+ \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon}_{\substack{\text{PDV of additional value gained from} \\ \text{microbial productivity after adopting} \\ \text{organic management practices,} \\ \text{valued at conventional prices}}}$$

8 Key Model Predictions under "Full" Information

- Some conventional farmers prefer organic management, even if $P_{org} = P_{con}$ (OT5)
- Some will pursue active stewardship (invest in clean soil stock), even if $P_{org} = P_{con}$ (OT4 and OT5)
- Among those still preferring conventional, some will reduce $c(t)$ use compared to farmer misperception model, so as to cultivate, and benefit from, $b(t)$ (OT2 and OT4)
- Others will prefer conventional and will choose the maximum level of $c(t)$ use no matter how large the organic price premium (OT1)
- Farmers who prefer to produce conventionally in absence of organic price premium (OT1, OT2, OT4) can be induced to prefer organic (via a 'jump' transition) if organic price premium high enough

9 Farmer Misperception (Unaware of effect of soil bacteria)

Next we want to know how optimal behavior under full information compares to optimal behavior in a model that doesn't account for soil bacteria. Previous research by Murphy et al. (2020) has shown that farmers in developing countries usually do not have sufficient information about their soil nutrient levels to make profit-maximizing decisions about fertilizer usage; and that there can be potentially large net benefits to providing farmers with soil information .

In order to assess how knowledge about soil microbiomes and the feedback between synthetic compounds, soil health, pest resistance, and crop yields may affect farmers' decisions about transitioning from conventional to organic management, we compare the optimal synthetic fertilizer and pesticide strategy determined by our model with the synthetic fertilizer and pesticide strategy that solves a model in which farmers are not aware that soil bacteria have a mediating effect.

Formally, we now consider a model in which farmers are not aware that soil bacteria have a mediating effect. In this model farmers perceive the following (incorrect) production function:

$$\breve{f}(c;X) = \breve{\theta}(X)c(t) + \breve{\Theta}(X) \tag{180}$$

where the notation $\breve{x}$ refers to the farmer's perception of x; and where the farmer infers the perceived value of $\breve{\theta}(X)$ from their observations in a narrow range of values of $c(t)$ typical in conventional production. Importantly, the farmer believes the marginal product of chemical inputs, $\breve{\theta}(X)$, to be constant and non-negative, conditional on other human and natural inputs X:

$$\frac{\partial \breve{f}(c;X)}{\partial c} = \breve{\theta}(X) \geq 0 \tag{181}$$

The true production function, however, remains the same as in earlier in this paper:

$$f(c, C; X) = \theta(X)c(t) + \Theta(X) \tag{182}$$

Thus, the true $\theta(\cdot)$ is actually a function of not only X, but also $c(t)$ and $C(t)$:

$$\theta(c, C; X) = \alpha_c(X) + \alpha_b(X)\left(\gamma_{cc}(X)c(t) + \gamma_b(X)\frac{\left(\overline{C} - C(t)\right)^2}{c(t)} + \gamma_c(X) + \gamma_K(X)\frac{\left(\overline{C} - C(t)\right)}{c(t)}\right) \tag{183}$$

and the true $\Theta(\cdot)$ is given by:

$$\Theta(X) = \alpha_b(X)A_b(X) + A_y(X). \tag{184}$$

Note that since $\gamma_{cc}(X) \leq 0$, $\gamma_b(X) \geq 0$, and $\gamma_K(X) \geq 0$, the true $\theta(\cdot)$ actually *decreases* as $c(t)$ increases, all else constant. As previously noted, the farmer is not aware of this relationship between $\theta(\cdot)$, c, and C though, and instead erroneously thinks that he or she faces a constant $\breve{\theta}(X(t))$ given by:

$$\breve{\theta}(X(t)) = \theta(c_0, C_0; X(t)) \tag{185}$$

for all values of c and C, where c_0 and C_0 are given (we will need to come back and more rigorously think about how the farmer's beliefs about $\theta(c(t), C(t), X(t))$ are formed. Presumably c_0 and C_0 would have resulted from some prior optimization process during which the farmer could have learned how $\theta(\cdot)$ responds to c and C).

We assume that the farmer's perceived $\breve{\Theta}(\cdot)$ is correct and therefore equal to the true $\Theta(\cdot)$:

$$\breve{\Theta}(X) = \Theta(X) = \alpha_b(X)A_b(X) + A_y(X). \tag{186}$$

In the case of misperception, the conventional farmer chooses their synthetic compound trajectory $c(t)$ to maximize the following erroneous optimization problem:

$$\begin{aligned}
\max_{\{c(t)\}} \int_0^\infty & (P_{con} \cdot \ (\theta(c_0, C_0; X(t)) \cdot c(t) + \Theta(X)) - c(t)) \cdot e^{-\rho t} dt \\
s.t. \quad & \dot{C}(t) = c(t) - \mu(X)C(t) \\
& 0 \leq c(t) \leq \overline{c}(K(t)) \\
& 0 \leq C(t) \leq \overline{C} \\
& C(0) = C_{0,con} \, .
\end{aligned} \tag{187}$$

Let $\breve{c}(t)_{TS^*}$ be the farmer's optimal trajectory for transitioning to organic assuming the farmer faces the production function they erroneously perceive to be true; $\breve{c}(t)_{con^*}$, is the farmer's optimal stage 1 trajectory assuming the farmer faces the production function they erroneously perceive to be true; $\breve{c}(t)_{org^*}$, is the farmer's optimal stage 2 trajectory assuming the farmer faces the production function they erroneously perceive to be true; and $\breve{T}^*$ is the first time at which $K(t) = K_{org}$ if the farmer adopts $\breve{c}(t)_{TS^*}$.

Note that the solution to the farmer's erroneous misperceived optimization problem under farmer misperception can be derived respectively, by setting $\alpha_b(X) = 0$, $\alpha_c(X) = \theta(c_0, C_0, X)$, and $A_y(X) = \Theta(X)$ in our previous model. Doing so, we find the following expression for the stationary rate of return in Stage 1:

$$\breve{R}_{con}(K) = -\mu \tag{188}$$

which is not a function of capital. Since we are assuming that $\mu(X) \geq 0$ (the stock of synthetic chemicals weakly decays on its own), $\breve{R}_{con}(K) = -\mu \leq 0$. Thus, the farmer will continually disinvest in clean soils if $\rho > 0$, since then

$$\breve{R}_{con}(K) = -\mu \leq 0 < \rho; \tag{189}$$

and will forever stay at their initial capital stock if $\rho = 0$, since then

$$\breve{R}_{con}(K) = -\mu = \rho = 0. \tag{190}$$

Note that when $\alpha_b(X) = 0$, $\alpha_c(X) = \theta(c_0, C_0, X)$, and $A_y(X) = \Theta(X)$, then the farmer perceives the following net gain function:

$$\breve{G}_{con}(c(t); x) = P_{con} \cdot (\theta(c_0, C_0; X)c(t) + \Theta(X)) - c(t) \tag{191}$$

or

$$\breve{G}_{con}(c(t); X) = P_{con} \cdot \left(\theta(c_0, C_0; X)\left(\mu(X)\left(\overline{C} - K(t)\right) - I(t)\right) + \Theta(X)\right) - \left(\mu(X)\left(\overline{C} - K(t)\right) - I(t)\right), \tag{192}$$

$$\breve{G}_{con}(K(t), I(t)) = \left(P_{con}\breve{\theta} - 1\right)\left(\mu\left(\overline{C} - K(t)\right) - I(t)\right) + P_{con}\Theta \tag{193}$$

which is linear in $I(t)$. Therefore the farmer's perceived optimal policy will be an MRA policy of disinvestment until $K = 0$.

Thus, the solution to the misinformed farmer's erroneously specified optimization problem will be the same as the fully informed farmer's solution to their optimization problem when the fully informed farmer faces OT1 conditions. In particular, the misinformed farmer will adopt the following trajectories when $\mu \neq 0$:

$$K(t) = K(0) \cdot e^{-\mu \cdot t} \, \forall t \tag{194}$$

$$I(t) = -\mu K(t) \, \forall t \tag{195}$$

$$c(t) = \overline{c} = \mu\overline{C} \, \forall t \tag{196}$$

$$C(t) = \overline{C} - K(0) \cdot e^{-\mu \cdot t} \, \forall t \tag{197}$$

9.1 Local Discrete Analysis of Misperception

Under farmer misperception, the (misperceived) value of adopting organic, $\breve{V}_{org}(K_{org})$, is given by:

$$\breve{V}_{org}(K_{org}) = \int_{t=0}^{\infty} \left((\mu - P_{org}\theta\mu)\,\overline{K}(t)_{org} + (1 - P_{org}\theta)\,\overline{I}(t)_{org} + P_{org}\Theta + P_{org}\theta\mu\overline{C} - \mu\overline{C}\right) \cdot e^{-\rho t} dt \tag{198}$$

and

$$\overline{K}(t)_{org} = K_{org}\, \forall t \tag{199}$$

$$\overline{I}(t)_{org} = 0\, \forall t \tag{200}$$

so that we have

$$\breve{V}_{org}(K_{org}) = \frac{1}{\rho}\left(\left(\mu - P_{org}\theta\mu\right) K_{org} + P_{org}\Theta + P_{org}\theta\mu\overline{C} - \mu\overline{C}\right) \tag{201}$$

Given $K_{org} = \overline{C}$ and $\overline{c} = \mu\overline{C}$ the the misinformed conventional farmer's (misperceived) value function for remaining conventional is given by: ${}_{con}(K_{org} - \epsilon)$

$$\breve{V}_{con}(K_{org} - \epsilon) = \int_{t=0}^{\infty} \left(P_{con} \cdot \breve{f}\left(c(t)_{con}\right) - c(t)_{con}\right) e^{-\rho t} dt \tag{202}$$

where:

$$\breve{f}(b(t), c(t); X) = \breve{\theta}c(t) + \breve{\Theta}. \tag{203}$$

$\breve{V}_{con}(K_{org} - \epsilon) =$

$$\frac{1}{\rho} \cdot \left(\left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \left(\mu\overline{C} - \overline{c}\right) \cdot \left(\frac{\frac{1}{\mu} \cdot \left(\overline{c} - \mu \cdot \overline{C}\right)}{\frac{1}{\mu} \cdot \left(\overline{c} - \mu \cdot \overline{C}\right) + (K_{org} - \epsilon)}\right)^{\frac{\rho}{\mu}} + \left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \overline{c} + P_{con}\breve{\Theta}\right) \tag{204}$$

We then want to write out a closed form expression for $\breve{\Delta}(\epsilon)$, using our solutions for $\breve{V}_{org}(K_{org})$ and $\breve{V}_{con}(K_{org} - \epsilon)$. We do so below:

$$\breve{\Delta}(\epsilon) = \breve{V}_{org}(K_{org}) - \breve{V}_{con}(K_{org} - \epsilon) \tag{205}$$

Given $K_{org} = \overline{C}$, the conventional farmer under misperception faces:

$$\breve{\Delta}(\epsilon) = \frac{1}{\rho} \cdot \breve{\Theta} \cdot \left(P_{org} - P_{con}\right) - \frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \mu\overline{C} \tag{206}$$

$$\breve{\Delta}(\epsilon) = \underbrace{\frac{1}{\rho} \cdot \breve{\Theta} \cdot (P_{org} - P_{con})}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \mu\overline{C}}_{\substack{\text{(misperceived) PDV of using} \\ \text{synthetic compounds at} \\ \text{(misperceived) dynamically} \\ \text{optimal conventional rate } \mu\overline{C}}}$$

For a conventional farmer under misperception, since the farmer is unaware of the effects of microbes, clean soil stock does not matter except through its effect on the crop price (organic or conventional). Thus, for a conventional farmer under misperception $\breve{\Delta}(\epsilon)$ is not a function of initial clean soil stock and therefore not a function of ϵ (except possibly through the effects of initial conditions on $\breve{\theta}$.

When $K_{org} = \overline{C}$, the misinformed farmer prefers organic management ($\breve{\Delta}(\epsilon) > 0$) when they face price premia satisfying $\frac{P_{org} - P_{con}}{P_{con}} > \left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* = \frac{\left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \mu\overline{C}}{P_{con} \cdot \breve{\Theta}} \tag{207}$$

Remember that $\breve{\Theta}(X) = \alpha_b(X)A_b(X) + A_y(X)$, and that $\breve{\theta} = \alpha_c + \alpha_b \left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \frac{K_0}{c_0}\right)$, or $\breve{\theta} = \alpha_c + \alpha_b \left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \frac{\overline{C}-\epsilon}{c_0}\right)$.

So we can write $\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^*$ as:

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* = \frac{\left(P_{con} \cdot \left(\alpha_c + \alpha_b \left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \frac{\overline{C}-\epsilon}{c_0}\right)\right) - 1\right) \cdot \mu\overline{C}}{P_{con} \cdot (\alpha_b A_b + A_y)} \tag{208}$$

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* = \frac{\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \frac{\overline{C}-\epsilon}{c_0} + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C}}{A_b + \frac{A_y}{\alpha_b}} \tag{209}$$

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* = \frac{\left(\gamma_{cc}c_0 + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + \gamma_K \cdot \frac{\mu\overline{C}}{c_0} \cdot \left(\overline{C} - \epsilon\right)}{A_b + \frac{A_y}{\alpha_b}} \tag{210}$$

9.2 Comparing Value of Conventional and Value of Organic with and without Farmer Misperception

Table 1: Conditions Producing Misperception Valuation Errors

	Direction of Error by Misinformed Farmer	
	Misinformed farmer Under-values organic $\Delta(\epsilon) > \check{\Delta}(\epsilon)$	**Misinformed farmer Over-values organic** $\Delta(\epsilon) < \check{\Delta}(\epsilon)$
Conventional OT1 farmer (Case C6)	P_{org} sufficiently large and $\alpha_b \gamma_K \overline{C} \neq 0$; or c_0 sufficiently small and $P_{con} \cdot \alpha_b \gamma_K \overline{C} \cdot \mu K_0 \neq 0$	Small enough γ_K and large enough c_0
Conventional OT2/OT3/OT4 farmer (Case A2)	$\hat{K}_{con}$ sufficiently close to $\overline{C}$	c_0 and α_c large enough and γ_K is also small enough
Conventional OT3' (Case F14A)	Always	N/A

9.2.1 Conventional OT1 farmer (Case C6)

Case C6 (Conventional Farmer Stationary Solution $\hat{K}_{con}$ does not exist and Organic Farmer Stationary Solution $\hat{K}_{org}$ does not exist because $\breve{K}_j < 0$ and $R_j(K) < \rho \quad \forall K \geq 0$ for $j \in \{con, org\}$)

Given $K_{org} = \overline{C}$, the fully informed conventional C6 farmer faces:

$$\Delta^{C6}(\epsilon) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} (P_{org} - P_{con}) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}}$$

$$- \underbrace{\frac{1}{(\mu + \rho)} \cdot P_{con} \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \overline{C} + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu \overline{C}}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal rate } \mu\overline{C}}}$$

Given $K_{org} = \overline{C}$, the conventional farmer under misperception faces:

$$\breve{\Delta}(\epsilon) = \underbrace{\frac{1}{\rho} \cdot \breve{\Theta} \cdot (P_{org} - P_{con})}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1 \right) \cdot \mu \overline{C}}_{\substack{\text{(misperceived) PDV of using} \\ \text{synthetic compounds at} \\ \text{(misperceived) dynamically optimal} \\ \text{conventional rate } \mu\overline{C}}}$$

Given $K_{org} = \overline{C}$ for a conventional C6 farmer, the dynamically optimal conventional rate and the (misperceived) dynamically optimal conventional rate is the same: $\overline{c} = \mu\overline{C}$.

$$
\begin{aligned}
\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon) = \quad & \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} \\
- \quad & \underbrace{\frac{1}{(\mu+\rho)} \cdot P_{con} \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \\
- \quad & \underbrace{\frac{1}{\rho} P_{con} \cdot \left(\left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \overline{C} + \gamma_c \right) + \alpha_c \right) - \breve{\theta} \right) \cdot \mu \overline{C}}_{\substack{\text{difference between PDV and misperceived PDV} \\ \text{of using synthetic compounds at dynamically optimal rate } \mu\overline{C}}}
\end{aligned}
$$

We summarize the factors determining the sign and value of $\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon)$ in the table below:

Main components of: $\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon)$	
PDV of stewarding soil microbiome at organic-level capital stock and at organic prices	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}} \geq 0$
PDV of microbial productivity under conventional management	$\frac{1}{(\mu+\rho)} \cdot P_{con} \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0} \geq 0$
Difference between PDV and misperceived PDV of using synthetic compounds at dynamically optimal rate $\mu\overline{C}$	$\frac{1}{\rho} P_{con} \cdot \Big(\big(\alpha_b \cdot \big(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c\big) + \alpha_c\big) - \breve{\theta} \Big) \cdot \mu\overline{C}$

where $\breve{\theta} = \alpha_c + \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)$. Note that we can simplify the last of the three factors,

$$\frac{1}{\rho}P_{con} \cdot \left(\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c\right) + \alpha_c\right) - \underbrace{\left(\alpha_c + \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right)}_{\geq 0\,\text{(by assumption)}}\right) \cdot \mu\overline{C}, \tag{211}$$

as follows:

$$\frac{1}{\rho}P_{con} \cdot \left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c\right) + \alpha_c - \alpha_c - \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right) \cdot \mu\overline{C} \tag{212}$$

$$\frac{1}{\rho}P_{con} \cdot \alpha_b \cdot \left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c\right) - \left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right) \cdot \mu\overline{C} \tag{213}$$

$$\frac{1}{\rho}P_{con} \cdot \alpha_b \cdot \left(\left(\frac{1}{2}\mu\overline{C} - c_0\right) \cdot \gamma_{cc} - \gamma_K \cdot \frac{K_0}{c_0}\right) \cdot \mu\overline{C}. \tag{214}$$

We see that for small enough c_0 we will have $\frac{1}{\rho}P_{con} \cdot \alpha_b \cdot \left(\left(\frac{1}{2}\mu\overline{C} - c_0\right) \cdot \gamma_{cc} - \gamma_K \cdot \frac{K_0}{c_0}\right) \cdot \mu\overline{C} \leq 0$. On the other hand, when we have that both c_0 is large enough and γ_K is small enough we will have $\frac{1}{\rho}P_{con} \cdot \alpha_b \cdot \left(\left(\frac{1}{2}\mu\overline{C} - c_0\right) \cdot \gamma_{cc} - \gamma_K \cdot \frac{K_0}{c_0}\right) \cdot \mu\overline{C} \geq 0$

We use these results in our second summary table below, in which we describe the direction of the misinformed farmer's valuation error.

Sign of $\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon)$	Condition
$\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon) > 0$ (Misinformed farmer under-estimates value of organic)	P_{org} sufficiently large and $\alpha_b\gamma_K\overline{C} \neq 0$; or c_0 sufficiently small and $P_{con} \cdot \alpha_b\gamma_K\overline{C} \cdot \mu K_0 \neq 0$.
$\Delta^{C6}(\epsilon) - \breve{\Delta}(\epsilon) < 0$ (Misinformed farmer over-estimates value of organic)	Small enough γ_K and large enough c_0.

9.2.2 Conventional OT1/OT2/OT3 farmer (Case A2)

Case A2: Conventional Farmer Stationary Solution $\hat{K}_{S_1}$ is below K_{org} and Organic Farmer Stationary Solution $\hat{K}_{S_2}$ exists (so is below $\hat{K}_{S_1}$ and therefore below K_{org} as well), and $\hat{K}_{S_j} \in [0, \overline{C}]$ for $j \in \{con, org\}$

Given $K_{org} = \overline{C}$ the conventional A2 farmer faces:

$$\Delta^{A2}(\epsilon) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}}$$

$$- \underbrace{\frac{1}{(\mu+\rho)} \cdot P_{con} \alpha_b \cdot \gamma_K \cdot \left(\underbrace{(\overline{C} - \epsilon)}_{=K_0} - \hat{K}_{con} \right)}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}}$$

$$- \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \left(\overline{C} - \hat{K}_{con} \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu \left(\overline{C} - \hat{K}_{con} \right)}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal rate } \mu\left(\overline{C} - \hat{K}_{con}\right)}}$$

Given $K_{org} = \overline{C}$, the conventional farmer under misperception faces:

$$\breve{\Delta}(\epsilon) = \underbrace{\frac{1}{\rho} \cdot \breve{\Theta} \cdot \left(P_{org} - P_{con}\right)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1 \right) \cdot \mu \overline{C}}_{\substack{\text{(misperceived) PDV of using} \\ \text{synthetic compounds at} \\ \text{(misperceived) dynamically optimal} \\ \text{conventional rate } \mu\overline{C}}}$$

$$
\begin{aligned}
\Delta^{A2}(\epsilon)-\breve{\Delta}(\epsilon)= \quad & \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K\underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} \\
& -\underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\left(\underbrace{(\overline{C}-\epsilon)}_{=K_0}-\hat{K}_{con}\right)}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} \\
& -\underbrace{\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\left(\overline{C}-\hat{K}_{con}\right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu\left(\overline{C}-\hat{K}_{con}\right)} \\
& +\underbrace{\frac{1}{\rho}\cdot\left(P_{con}\cdot\breve{\theta}-1\right)\cdot\mu\overline{C}}_{\substack{\text{(misperceived) PDV of using}\\ \text{synthetic compounds at}\\ \text{(misperceived) dynamically optimal}\\ \text{conventional rate } \mu\overline{C}}}
\end{aligned}
$$

We summarize the factors determining the sign and value of $\Delta^{A2}(\epsilon)-\breve{\Delta}(\epsilon)$ in the table below:

Main components of: $\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon)$	
PDV of stewarding soil microbiome at organic-level capital stock and at organic prices	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}} \geq 0$
PDV of microbial productivity under conventional management	$\frac{1}{(\mu+\rho)} \cdot P_{con} \alpha_b \cdot \gamma_K \cdot \left(\underbrace{(\overline{C} - \epsilon)}_{=K_0} - \hat{K}_{con} \right)$
PDV of using synthetic compounds at dynamically optimal rate $\mu \left(\overline{C} - \hat{K}_{con} \right)$	$\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \left(\overline{C} - \hat{K}_{con} \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu \left(\overline{C} - \hat{K}_{con} \right)$
(misperceived) PDV of using synthetic compounds at (misperceived) dynamically optimal conventional rate $\mu \overline{C}$	$\frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1 \right) \cdot \mu \overline{C} \geq 0$

where $\breve{\theta} = \alpha_c + \alpha_b \left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)$. Note that

$$\begin{aligned}\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon) =& \frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \overline{C} - P_{con}\alpha_b \cdot \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\overline{C} - \hat{K}_{con} - \epsilon\right) \\ & - \frac{1}{\rho} \cdot \left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \cdot \mu\left(\overline{C} - \hat{K}_{con}\right) \\ & + \frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1\right) \cdot \mu\overline{C}\end{aligned} \tag{215}$$

simplifies as follows:

$$\begin{aligned}=& \frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \overline{C} + P_{con}\alpha_b \cdot \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \epsilon - P_{con}\alpha_b \cdot \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\overline{C} - \hat{K}_{con}\right) \\ & - \frac{1}{\rho} \cdot \left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \cdot \mu\left(\overline{C} - \hat{K}_{con}\right) \\ & + \frac{1}{\rho} \cdot \left(P_{con} \cdot \left(\alpha_c + \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right) - 1\right) \cdot \mu\overline{C}\end{aligned} \tag{216}$$

$$\begin{aligned}=& \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(\frac{P_{org}}{P_{con}} \cdot \overline{C} - \frac{\rho}{(\mu+\rho)} \cdot \left(\overline{C} - \hat{K}_{con}\right) + \frac{\rho}{(\mu+\rho)} \cdot \epsilon\right) \\ & - \frac{1}{\rho} \cdot \left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \cdot \mu\left(\overline{C} - \hat{K}_{con}\right) \\ & + \frac{1}{\rho} \cdot \left(P_{con} \cdot \left(\alpha_c + \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right) - 1\right) \cdot \mu\overline{C}\end{aligned} \tag{217}$$

$$\begin{aligned}=& \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(\underbrace{\frac{P_{org}}{P_{con}} \cdot \overline{C} - \frac{\rho}{(\mu+\rho)} \cdot \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} + \frac{\rho}{(\mu+\rho)} \cdot \epsilon\right)}_{\geq 0} \\ & - \frac{1}{\rho} \cdot \left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \cdot \mu\left(\overline{C} - \hat{K}_{con}\right) \\ & + \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \cdot \left(\alpha_c + \alpha_b\left(\gamma_{cc}c_0 + \gamma_c + \gamma_K \cdot \frac{K_0}{c_0}\right)\right) - 1\right) \cdot \mu\overline{C}}_{\geq 0}\end{aligned} \tag{218}$$

When $\hat{K}_{con}$ is sufficiently close to $\overline{C}$ we will therefore have $\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon) \geq 0$. On the other hand when c_0 large enough so that the third term goes to zero, and γ_K is small enough so that the first term goes to zero, and also α_c is large enough so that $\left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - \hat{K}_{con}\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \geq 0$, then we will have that $\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon) \leq 0$.

We use these results in our second summary table below, in which we describe the direction of the misinformed farmer's valuation error.

Sign of $\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon)$	Condition
$\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon) > 0$ (Misinformed farmer under-estimates value of organic)	$\hat{K}_{con}$ sufficiently close to $\overline{C}$
$\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon) < 0$ (Misinformed farmer over-estimates value of organic)	c_0 and α_c large enough and γ_K is also small enough

9.2.3 Conventional OT3' farmer (Case F14A)

Next we want to compare how the F14 fully informed farmer's $\Delta(\epsilon)$ compares to the misinformed farmer's $\breve{\Delta}(\epsilon)$.

Since the misinformed farmer is assumed to pursue a lower corner solution, our assumptions about $\bar{c}$ become important. In our C6 discrete analysis we had to impose the assumption that $\bar{c} = \mu\overline{C}$ in order to obtain a closed form solution for the fully informed C6 farmer's $\Delta(\epsilon)$. To allow for comparability between the behavior of C6 $\Delta(\epsilon)$ and C6 $\breve{\Delta}(\epsilon)$, we also imposed the assumption that $\bar{c} = \mu\overline{C}$ when deriving C6 $\breve{\Delta}(\epsilon)$, and then to allow for comparability between our C6 findings and our set of findings for the A2 fully informed and misinformed farmer, we imposed $\bar{c} = \mu\overline{C}$ for our A2 misinformed farmer as well, who is also assumed to pursue a lower corner solution.

To allow for comparability between our F14 and C6 and A2 findings then, in this analysis we will also want to study the F14 model under the assumption that $\bar{c} = \mu\overline{C}$. This will only work for F14 A (where $\gamma_{cc} = 0$, $\mu \neq 0$) however, since $\mu = 0$ in case F14 B.

Given $K_{org} = \overline{C}$, the conventional F14A farmer faces:

$$\Delta^{F14\,A}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K\underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}(P_{org}-P_{con})\cdot(\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium}\\ \text{on "level effect"}\\ \text{of other agricultural inputs}}}$$

$$- \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\underbrace{(\overline{C}-\epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{\gamma_K}{\mu+\rho}\cdot\mu\cdot\epsilon}_{\substack{\text{PDV of using synthetic compounds}\\ \text{at dynamically optimal}\\ \text{conventional rate } \mu\cdot\epsilon}} \tag{219}$$

Given $K_{org} = \overline{C}$ and $\overline{c} = \mu\overline{C}$ the conventional farmer under misperception faces:

$$\breve{\Delta}(\epsilon) = \underbrace{\frac{1}{\rho}\cdot\breve{\Theta}\cdot(P_{org}-P_{con})}_{\substack{\text{PDV of organic price premium}\\ \text{on "level effect"}\\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho}\cdot\left(P_{con}\cdot\breve{\theta}-1\right)\cdot\mu\overline{C}}_{\substack{\text{(misperceived) PDV of using}\\ \text{synthetic compounds at}\\ \text{(misperceived) dynamically optimal}\\ \text{conventional rate } \mu\overline{C}}}$$

$$
\begin{aligned}
\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) = \quad & \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} \\
- \quad & \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \\
- \quad & \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal conventional rate } \mu \cdot \epsilon}} \\
+ \quad & \underbrace{\frac{1}{\rho} \cdot \Big(P_{con} \cdot \breve{\theta} - 1 \Big) \cdot \mu \overline{C}}_{\substack{\text{(misperceived) PDV of using synthetic compounds} \\ \text{at (misperceived) dynamically optimal conventional rate } \mu \overline{C}}}
\end{aligned}
$$

We summarize the factors determining the sign and value of $\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon)$ in the table below:

Main components of: $\Delta^{A2}(\epsilon) - \breve{\Delta}(\epsilon)$	
PDV of stewarding soil microbiome at organic-level capital stock and at organic prices	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}} \geq 0$
PDV of microbial productivity under conventional management	$\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}$
PDV of using synthetic compounds at dynamically optimal conventional rate $\mu \cdot \epsilon$	$\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu+\rho} \cdot \mu \cdot \epsilon$
(misperceived) PDV of using synthetic compounds at (misperceived) dynamically optimal conventional rate $\mu\overline{C}$	$\frac{1}{\rho} \cdot \left(P_{con} \cdot \breve{\theta} - 1 \right) \cdot \mu\overline{C} \geq 0$

This equation can be re-written as follows:

$$
\begin{aligned}
\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) = \\
& \frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}} \\
& - \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0} - \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon \\
& + \frac{1}{\rho} \cdot \Big(P_{con} \cdot \breve{\theta} - 1 \Big) \cdot \mu \overline{C}
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) = \\
& \frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \overline{C} - \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \overline{C} \\
& + \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon - \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \frac{\mu}{\mu + \rho} \cdot \epsilon \\
& + \frac{1}{\rho} \cdot \Big(P_{con} \cdot \breve{\theta} - 1 \Big) \cdot \mu \overline{C}
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) = \\
& \frac{1}{\rho} \cdot \alpha_b \cdot \gamma_K \cdot \overline{C} \cdot (P_{org} - P_{con}) \\
& + \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \\
& + \frac{1}{\rho} \cdot \Big(P_{con} \cdot \breve{\theta} - 1 \Big) \cdot \mu \overline{C}
\end{aligned}
$$

$$
\begin{aligned}
\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) = \\
& \underbrace{\frac{1}{\rho} \cdot \alpha_b \cdot \gamma_K \cdot \overline{C} \cdot (P_{org} - P_{con})}_{\geq 0} + \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon}_{\geq 0} \\
& + \underbrace{\frac{1}{\rho} \cdot \Big(P_{con} \cdot \breve{\theta} - 1 \Big) \cdot \mu \overline{C}}_{\geq 0} \geq 0
\end{aligned}
$$

We can summarize these results as follows:

Sign of $\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon)$	Condition
$\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) > 0$ (Misinformed farmer underestimates value of organic)	Always
$\Delta^{F14\,A}(\epsilon) - \breve{\Delta}(\epsilon) < 0$ (Misinformed farmer overestimates value of organic)	N/A

9.3 Comparing threshold organic price premium for adopting organic with and without Farmer Misperception

To assess when a conventional farmer under misperception chooses incorrectly, we compare the price premium required to make a misinformed farmer prefer organic management to the premium necessary to make a fully informed farmer prefer organic management.

Table 2: Comparing Organic Premia Requirements: Full-Information vs Farmer Misperception

$V_{org} - V_{con}(C_0) \geq 0$ **if** $\frac{P_{org}-P_{con}}{P_{con}}$ **greater than:**	
Case	Condition:
OT1	$\frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C}-\frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\left(\mu\overline{C}+\rho\cdot\epsilon\right)}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}}$
OT2/OT3/OT4	$\frac{\rho\cdot\left(\frac{1}{\rho}\cdot\frac{1}{2}\cdot(-\gamma_{cc})\left(\mu\left(\overline{C}-\hat{K}_{con}\right)\right)^2-\frac{1}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\frac{1}{\alpha_b}\cdot\left(\alpha_b\left(\gamma_K\overline{C}+A_b\right)+A_y\right)}$
Farmer Misperception	$\frac{\left(\gamma_{cc}c_0+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C}+\gamma_K\cdot\frac{\mu\overline{C}}{c_0}\cdot\left(\overline{C}-\epsilon\right)}{A_b+\frac{A_y}{\alpha_b}}$

We will therefore have:

$$\left(\frac{P_{org}\breve{-}P_{con}}{P_{con}}\right)^* \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* \tag{220}$$

such that the misinformed farmer chooses incorrectly and requires a higher price premium than would the fully informed farmer in order to prefer organic management, when, for example, the misinformed farmer faces:

- small enough c_0 (such that the naive farmer incorrectly perceives $\breve{\theta}$ to be very large), or

- small enough μ (given $\gamma_{cc} \neq 0$ and $\alpha_b \neq 0$) such that that synthetic compounds are very persistent, or
- small enough $\alpha_b A_b + A_y$, which measures the importance of factors of production other than synthetic compounds and "clean" soil (given $\alpha_b \gamma_K \overline{C} \neq 0$)

9.3.1 Conventional OT1 farmer (Case C6)

The fully informed conventional OT1 farmer prefers organic management when they face price premia satisfying $\frac{P_{org}-P_{con}}{P_{con}} > \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = \underbrace{\frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho \cdot \epsilon\right)}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}}}_{\phi_{informed}} \tag{221}$$

When the fully informed farmer faces a C6 environment, we will have

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* > \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* \tag{222}$$

or

$$\frac{\left(\gamma_{cc}c_0 + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + \gamma_K \cdot \frac{\mu\overline{C}}{c_0} \cdot \left(\overline{C} - \epsilon\right)}{A_b + \frac{A_y}{\alpha_b}} > \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \frac{\mu\overline{C}+\rho\cdot\epsilon}{\mu+\rho}}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{223}$$

such that the misinformed farmer chooses incorrectly and requires a higher organic price premium than the fully informed farmer before adopting organic when, for example:

- γ_K is large enough, assuming that $\alpha_b = 0$ (such that clean soil is sufficiently important for maintaining benificial soil bacteria)
- The misinformed farmer's initial chemical input use, c_0, is sufficiently small, relative to the upper limit on chemical input use, $\mu\overline{C}$ (such that the indirect harms that their current chemical usage imposes through its effects on the soil microbiome is sufficiently small compared to the indirect harms that the incorrectly identified, "optimal" levels of chemical use under conventional management would impose through its effects on the soil microbiome)
- When γ_c and γ_{cc} are sufficiently low, and P_{con} is sufficiently high, such that $\left(\gamma_{cc}c_0 + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \geq 0$, then $\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^* \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ will also be satisfied for large enough $\overline{c} = \mu\overline{C}$, since

when $\overline{c} = \mu\overline{C}$ is large enough the convex indirect costs and stock effects that the misinformed farmer misses will decrease the value of implementing the optimal conventional management plan relative to the value of organic management.

On the other hand, we will have

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^{*} < \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^{*} \tag{224}$$

or

$$\frac{\left(\gamma_{cc}c_0 + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + \gamma_K \cdot \frac{\mu\overline{C}}{c_0} \cdot \left(\overline{C} - \epsilon\right)}{A_b + \frac{A_y}{\alpha_b}} \leq \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \frac{\mu\overline{C} + \rho\cdot\epsilon}{\mu+\rho}}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{225}$$

such that the misinformed farmer accepts a lower organic price premium than the fully informed farmer before prefering organic when, for example:

- c_0 is sufficiently high (i.e. sufficiently close to $\overline{c} = \mu\overline{C}$) and γ_K is sufficiently low.

9.3.2 Conventional OT2/OT3/OT4 farmer (Case A2)

Given $K_{org} = \overline{C}$, for the fully informed conventional A2 farmer we have $\Delta(\epsilon) \geq 0$ when:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right) \geq \frac{\rho \cdot \left(\frac{1}{\rho} \cdot \frac{1}{2} \cdot (-\gamma_{cc}) \left(\mu\left(\overline{C} - \hat{K}_{con}\right)\right)^2 - \frac{1}{\mu+\rho} \cdot \gamma_K \cdot \epsilon\right)}{\frac{1}{\alpha_b} \cdot \left(\alpha_b\left(\gamma_K\overline{C} + A_b\right) + A_y\right)} \tag{226}$$

Note that since the lower bound on $c(t)$ does not bind in Case A2, the value of $\overline{c}$ does not affect the form that $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ takes.

On the other hand, in *Comparing threshold organic price premium for adopting organic with and without Farmer Misperception for Case C6*, we saw that when $\overline{c} = \mu\overline{C}$ and $K_{org} = \overline{C}$ the misinformed farmer will face $\breve{\Delta}(\epsilon) \geq 0$ when:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right) \geq \frac{\left(P_{con} \cdot \breve{\theta} - 1\right)}{P_{con} \cdot \breve{\Theta}} \cdot \mu\overline{C} \tag{227}$$

We can therefore find conditions under which the misinformed farmer requires a higher organic price premium before prefering to produce organically than does the fully informed A2 farmer by examining the following inequality:

$$\left(\frac{P_{org} \breve{-} P_{con}}{P_{con}}\right)^{*} \geq \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^{*} \tag{228}$$

$$\frac{\left(P_{con} \cdot \breve{\theta} - 1\right)}{P_{con} \cdot \breve{\Theta}} \cdot \mu\overline{C} \geq \tag{229}$$

$$\frac{\rho \cdot \left(\frac{1}{\rho} \cdot \frac{1}{2} \cdot (-\gamma_{cc}) \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2 - \frac{1}{\mu+\rho} \cdot \gamma_K \cdot \epsilon\right)}{\frac{1}{\alpha_b} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b\right) + A_y\right)}$$

$$\underbrace{\frac{P_{con} \cdot \breve{\theta} - 1}{P_{con} \cdot \breve{\Theta}}}_{\geq 0} \cdot \mu\overline{C} \geq \tag{230}$$

$$\frac{\alpha_b \cdot \left(\underbrace{\frac{1}{2} \cdot (-\gamma_{cc}) \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\geq 0} - \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon}_{\geq 0}\right)}{\underbrace{\alpha_b \gamma_K \overline{C} + \alpha_b A_b + A_y}_{\geq 0}}$$

$$\underbrace{\left(\frac{P_{con}\left(\alpha_c + \alpha_b \left(\gamma_{cc} c_0 + \gamma_c + \gamma_K \frac{K_0}{c_0}\right)\right) - 1}{P_{con} \cdot (\alpha_b A_b + A_y)}\right) \cdot \mu\overline{C}}_{\geq 0} \geq \tag{231}$$

$$\frac{\alpha_b \cdot \left(\underbrace{\frac{1}{2} \cdot (-\gamma_{cc}) \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\geq 0} - \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon}_{\geq 0}\right)}{\underbrace{\alpha_b \gamma_K \overline{C} + \alpha_b A_b + A_y}_{\geq 0}}$$

This condition will be satisfied (such that $\breve{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}^{*} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}$, and such that the misinformed farmer therefore requires a higher organic premium in order to prefer organic management than does the fully informed farmer) when either of the following sets of conditions are satisfied:

1. $\alpha_b \gamma_K \overline{C} \neq 0$ and $(\alpha_b A_b + A_y)$ is sufficiently small;

2. c_0 is sufficiently small, given $\alpha_b \gamma_K \neq 0$

$\alpha_b \gamma_K \overline{C} \neq 0$ ensures that as $(\alpha_b A_b + A_y)$ tends towards zero the LHS expression in our inequality above will tend towards positive infinity, while the expression on the RHS of our inequality will tend towards a constant, leading the inequality above to be satisfied. On the other hand, if $\alpha_b \gamma_K \neq 0$, then as c_0 tends towards zero, $\breve{\theta}$, and therefore the LHS expression in our inequality, will tend towards positive infinity, while leaving the RHS expression unchanged. This will also result in our inequality being satisfied.

10 Comparison of Local Discrete Analyses Across Model Specifications

The results are summarized in the following summary tables:

Table 3: Summary of $\Delta(\epsilon)$ and $\breve{\Delta}(\epsilon)$: Terms we add

	PDV of stewarding soil microbiome at organic-level capital stock and at organic prices	PDV of organic price premium on "level effect" of other agricultural inputs
OT1 $\Delta^{C6}(\epsilon)$	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}$	$\frac{1}{\rho} \left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)$
OT2/OT3/OT4 $\Delta^{A2}(\epsilon)$	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}$	$\frac{1}{\rho} \left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)$
OT3' $\Delta^{F14\,A}(\epsilon)$	$\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}$	$\frac{1}{\rho} \left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)$
Misperception $\breve{\Delta}(\epsilon)$		$\frac{1}{\rho} \cdot \breve{\Theta} \cdot \left(P_{org} - P_{con}\right)$

Table 4: Summary of $\Delta(\epsilon)$ and $\breve{\Delta}(\epsilon)$: Terms we subtract

	PDV of microbial productivity under conventional management	PDV of using synthetic compounds at dynamically optimal rate
OT1 $\Delta^{C6}(\epsilon)$	$\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\underbrace{(\overline{C}-\epsilon)}_{=K_0}$	$\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\overline{C}$
OT2/OT3/OT4 $\Delta^{A2}(\epsilon)$	$\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\left(\underbrace{(\overline{C}-\epsilon)}_{=K_0}-\hat{K}_{con}\right)$	$\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\left(\overline{C}-\hat{K}_{con}\right)$
OT3' $\Delta^{F14\,A}(\epsilon)$	$\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\underbrace{(\overline{C}-\epsilon)}_{=K_0}$	$\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\gamma_c+\alpha_c\right)-1\right)\cdot\mu\cdot\underbrace{\epsilon}_{=\overline{C}-K_0}$
		(misperceived) PDV of using synthetic compounds at (misperceived) dynamically optimal conventional rate
Misperception $\breve{\Delta}(\epsilon)$		$\frac{1}{\rho}\cdot\left(P_{con}\cdot\breve{\theta}-1\right)\cdot\mu\overline{C}$

Table 5: Summary of $\Delta(\epsilon)$ and $\breve{\Delta}(\epsilon)$: Rate of synthetic compound use

	dynamically optimal rate	(misperceived) dynamically optimal conventional rate
OT1 $\Delta^{C6}(\epsilon)$	$\mu\overline{C}$	
OT2/OT3/OT4 $\Delta^{A2}(\epsilon)$	$\mu\left(\overline{C}-\hat{K}_{con}\right)$	
OT3' $\Delta^{F14\,A}(\epsilon)$	$\mu\cdot\underbrace{\epsilon}_{=\overline{C}-K_0}$	
Misperception $\breve{\Delta}(\epsilon)$		$\mu\overline{C}$

11 Summary of Comparison of Full Information vs. Misperception

When farmers account for soil bacteria:

- Some may transition to organic management 'accidentally' as their optimal trajectories gradually take them toward the certification threshold (this can happen even in the absence of an organic price premium)
- Other transitions may be induced by the organic price premium.

When farmers do not account for soil bacteria:

- They never make an 'accidental' transition to organic, and will instead disinvest as fast as possible to $K = 0$.
- If they transition can only be induced by an organic price premium.
- They will require a higher premium to adopt than a fully informed farmer would when a large enough proportion of organic farming's value-added comes from stock effects/soil microbes.

12 Investment Under Uncertainty

In this section we use dynamic programming and investment under uncertainty to derive the optimal organic switching policy (i.e., the conditions under which a conventional farmer will switch to organic).

Let the action variable a be a dummy for switching to organic.

For now, let the conventional price P_{con} be a fixed parameter that is not stochastic.

If the farmer stays conventional, then he gets a current period payoff (which is from the conventional price, which we assume is a fixed parameter that is not stochastic) plus β times the continuation value from waiting instead of switching to organic.

If the farmer switches to organic, let's assume for now that the farmer can't switch back, so the payoff to switching is a lump-sum payoff which is the value function from being organic from that year t onwards (similarly to the local discrete analysis). Thus, there is no continuation value if the farmer switches to organic since we model the farmer as having no more decisions to make after switching.

The optimal organic switching policy under uncertainty will be a threshold value of the organic price premium above which the conventional farmer will switch to organic. This threshold organic price premium will be a function of K (or ϵ) and of parameters (including parameters in the transition density for the organic price P_{org}).

Thus, for our investment under uncertainty model, we are focusing on the switch to organic, not the quantity of synthetic compound use.

The (infinite horizon) value function for a conventional farmer who has the option to switch to organic is given by:

$$V_{con}(P_{org}, K) = \max \left\{ \max_{I} \left(G_{con}(K, I) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid P_{org}, K, I, a_t = 0 \right] \right), V_{org}(P_{org}, K_{org}) \right\}. \tag{232}$$

where P_{org} is a stochastic state variable, and where the discount factor $\beta \in [0, 1)$. The value function for a conventional farmer who has the option to switch to organic is the maximum of the PDV payoff from 2 possible options: (1) stay conventional, or (2) switch to organic. $V_{org}(K_{org})$ is the value function from organic production, and is therefore the PDV of the entire stream of net benefits from having switched to organic production (similar to the $V_{org}(K_{org})$ we use in the local discrete analysis). We assume for now that there are no costs to switching, aside from any foregone initial per-period net benefits (profits) from lower synthetic compound use.

We can write the following expression for $G_{con}(K, I)$:

$$G_{con}(K, I) = P_{con} \cdot f(c_t, b_t) - c_t \tag{233}$$

If the farmer remains conventional, they choose c_t according to the dynamically optimal chemical-use policy$\{c_t^*\}$ that solves their value function when managing conventionally is their only option. $\{c_t^*\}$ can be determined using optimal control theory or dynamic programming, and expressed in terms of model parameters.

We assume $K_{org} = \overline{C}$.

Right now $V_{org}(P_{org}, K_{org})$ is easy to work with when $K_{org} = \bar{C}$ since we can factor out P_{org} and express $V_{org}(P_{org}, K_{org})$ as $P_{org} \cdot f(K_{org})$ for some function $f(K_{org})$ of K_{org} (which makes it easier to solve for P^*_{org} and the threshold organic premium).

12.1 Case C6

Let's assume we are in Case C6. Since for Case C6, $\hat{K}_{con} < 0$, we will try writing everything, including $G_{con}(K, I)$ and the subsequent $G^*_{con}(K)$, without using $\hat{K}_{con}$.

In case C6 we have:

$$V_{org}(P_{org}, K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b} - \frac{1}{2}\gamma_{cc}\mu K_{org} \right) \mu \left(\overline{C} - K_{org}\right) + \gamma_K K_{org} + A_b \right) + A_y \right) \tag{234}$$

Since both $\hat{K}_{org}$ and $\hat{K}_{con}$ are negative, and since $\hat{K}_{org}$ is a function of P_{org}, we do not write any of the functions (including $V_{org}(P_{org}, K_{org})$, $G_{con}(K, I)$, and $G^*_{con}(K)$, as functions of either $\hat{K}_{org}$ or $\hat{K}_{con}$. Later, for other cases in which $\hat{K}_{con} \in [0, \overline{C}]$, we might want to write $G_{con}(K, I)$ and $G^*_{con}(K)$ as a function of $\hat{K}_{con}$. Since $\hat{K}_{org}$ is a function of P_{org}, however, we might not want to use $\hat{K}_{org}$ any of the functions (including $V_{org}(P_{org}, K_{org})$) even if $\hat{K}_{org} > 0$.

When $K_{org} = \overline{C}$ our expression for $V_{org}(P_{org}, K_{org})$ becomes:

$$V_{org}(P_{org}, K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b\right) + A_y\right) \tag{235}$$

Next we consider

$$G_{con}(K, I) = P_{con} \cdot f(c_t, b_t) - c_t \tag{236}$$

Let's assume for simplicity that for all periods t for which the farmer chooses to continue producing conventionally, they will employ the pesticide policy that solves the conventional management problem conditional on conventional management being thier only option. Then, when the farmer finds themselves in a C6 parameters space, and when $\mu \neq 0$ and $\gamma_{cc} \neq 0$, their management plan comes from the following lower corner (OT1) solution:

Given that $\overline{c} = \mu\overline{C}$, we have:

$$K^*(t) = K(0) \cdot e^{-\mu \cdot t}, \forall t \geq 0 \tag{237}$$

$$I^*(t) = -\mu K(t) \, \forall \, t \tag{238}$$

and:

$$c^*(t) = \underbrace{\mu\overline{C}}_{\overline{c}}, \forall t \geq 0. \tag{239}$$

Given $I^*(t) = -\mu K^*(t)$ from equation (238), and given the following discrete-time version of the equation of motion:

$$I(t) = K' - K, \tag{240}$$

we can derive the discretized value of next period's capital stock, K', as follows:

$$K' - K = -\mu K \tag{241}$$

$$K' = (1-\mu)K \tag{242}$$

Similarly, we can derive the discretized value of this period's capital stock K as a function of next period's capital stock, K', as follows:

$$K' = (1-\mu)K \tag{243}$$

$$K = \frac{K'}{(1-\mu)} \tag{244}$$

$$b_{t,con} = \left(\gamma_c \cdot c_t + \frac{1}{2}\gamma_{cc} \cdot c_t^2 + \gamma_K \cdot K_t + A_b\right) \tag{245}$$

and:

$$f(b_t, c_t) = (\alpha_b \cdot b_t + \alpha_c \cdot c_t + A_y) \tag{246}$$

We can therefore express f as a function of c_t and K_t as follows:

$$f(c_t) = \left(\alpha_b \cdot \left(\gamma_c \cdot c_t + \frac{1}{2}\gamma_{cc} \cdot c_t^2 + \gamma_K \cdot K_t + A_b\right) + \alpha_c \cdot c_t + A_y\right). \tag{247}$$

We can then express G_{con} as follows:

$$G_{con}(K, I) = P_{con} \cdot \left((\alpha_b \cdot \gamma_c + \alpha_c) \cdot c_t + \alpha_b \cdot \frac{1}{2}\gamma_{cc} \cdot c_t^2 + \alpha_b \cdot \left(\gamma_K \cdot K_t + A_b + \frac{A_y}{\alpha_b}\right)\right) - c_t \tag{248}$$

$$G_{con}(K, I) = P_{con} \cdot \alpha_b \cdot \left(\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot c_t + \frac{1}{2}\gamma_{cc} \cdot c_t^2 + \left(\gamma_K \cdot K_t + A_b + \frac{A_y}{\alpha_b}\right)\right) \tag{249}$$

$$G_{con}(K, I) = P_{con} \cdot \alpha_b \cdot \left(\left(\gamma_{cc} \cdot c + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot c - \frac{1}{2}\gamma_{cc} \cdot c^2 + \left(\gamma_K \cdot K + A_b + \frac{A_y}{\alpha_b}\right)\right) \tag{250}$$

Substituting in the optimal $c^*(t)$ from equation (239) for $c(t)$, we get:

$$G_{con}(K,I^*) = P_{con}\cdot\alpha_b\cdot\left(\left(\left(\gamma_{cc}\cdot c^* + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot c^* - \frac{1}{2}\gamma_{cc}\cdot c^{*2} + \left(\gamma_K\cdot K + A_b + \frac{A_y}{\alpha_b}\right)\right)\right) \tag{251}$$

$$G_{con}(K,I^*) = P_{con}\cdot\alpha_b\cdot\left(\left(\left(\gamma_{cc}\cdot\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} - \frac{1}{2}\gamma_{cc}\cdot\left(\mu\overline{C}\right)^2 + \left(\gamma_K\cdot K + A_b + \frac{A_y}{\alpha_b}\right)\right)\right) \tag{252}$$

$$G_{con}(K,I^*) = P_{con}\cdot\alpha_b\cdot\left(\left(\gamma_{cc}\cdot\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + \gamma_K\cdot K + A_b + \frac{A_y}{\alpha_b}\right) \tag{253}$$

When net investment $I(t)$ (and synthetic compound use $c(t)$) is chosen optimally, $G_{con}(K,I^*)$ is no longer a function of I (or c), so we can define the optimized net gain $G^*_{con}(K)$ as:

$$G^*_{con}(K) \equiv G_{con}(K,I^*) \tag{254}$$

$$G^*_{con}(K) = P_{con}\cdot\alpha_b\cdot\left(\left(\gamma_{cc}\cdot\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + \gamma_K\cdot K + A_b + \frac{A_y}{\alpha_b}\right) \tag{255}$$

Note that since the optimal trajectories for OT1 are not piecewise despite being MRA (since we approach $K = 0$ asymptotically), the optimized net gain $G^*_{con}(K)$ is not piecewise.

Since both $\hat{K}_{org}$ and $\hat{K}_{con}$ are negative, and since $\hat{K}_{org}$ is a function of P_{org}, we do not write any of the functions (including $V_{org}(P_{org}, K_{org})$, $G_{con}(K,I)$, and $G^*_{con}(K)$, as functions of either $\hat{K}_{org}$ or $\hat{K}_{con}$. Later, for other cases in which $\hat{K}_{con}\in[0,\overline{C}]$, we might want to write $G_{con}(K,I)$ and $G^*_{con}(K)$ as a function of $\hat{K}_{con}$. Since $\hat{K}_{org}$ is a function of P_{org}, however, we might not want to use $\hat{K}_{org}$ any of the functions (including $V_{org}(P_{org}, K_{org})$) even if $\hat{K}_{org} > 0$.

Taking the derivative of $G^*_{con}(K)$ with respect to K, we get:

$$\frac{dG^*_{con}(K)}{dK} = P_{con}\cdot\alpha_b\cdot\gamma_K \geq 0 \tag{256}$$

Evaluating optimized net gain $G^*_{con}(K)$ at next period's state K', we get:

$$G^*_{con}(K') = P_{con}\cdot\alpha_b\cdot\left(\left(\gamma_{cc}\cdot\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + \gamma_K\cdot K' + A_b + \frac{A_y}{\alpha_b}\right) \tag{257}$$

Substituting in equation (243) into $G^*_{con}(K')$ to write $G^*_{con}(K')$ in terms of K, instead of K', we get:

$$G^*_{con}(K') = P_{con} \cdot \alpha_b \cdot \left(\left(\gamma_{cc} \cdot \mu \overline{C} + \gamma_c + \frac{\alpha_c - P^{-1}_{con}}{\alpha_b} \right) \cdot \mu \overline{C} + \gamma_K \cdot (1-\mu) K + A_b + \frac{A_y}{\alpha_b} \right) \tag{258}$$

$$\begin{aligned} G^*_{con}(K') =& P_{con} \cdot \alpha_b \cdot \left(\left(\gamma_{cc} \cdot \mu \overline{C} + \gamma_c + \frac{\alpha_c - P^{-1}_{con}}{\alpha_b} \right) \cdot \mu \overline{C} + \gamma_K \cdot K + A_b + \frac{A_y}{\alpha_b} \right) \\ & - P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \mu K \end{aligned} \tag{259}$$

$$G^*_{con}(K') = G^*_{con}(K) - \frac{dG^*_{con}(K)}{dK} \cdot \mu \cdot K \tag{260}$$

If the farmer pursues the optimal C6 conventional management plan as long as they remain conventional, then we can simplify our notation for $V_{con}(P_{org}, K)$ and write:

$$V_{con}(P_{org}, K) = \max \left\{ G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid P_{org}, K, a_t = 0 \right], V_{org}(P_{org}, K_{org}) \right\}, \tag{261}$$

Let's assume that P_{org} is distributed iid with pdf $\phi(P_{org})$, cdf $\Phi(P_{org})$, and support $P_{org} \in [\underline{P}_{org}, \overline{P}_{org}]$.

For simplicity we also assume that $a_t = 0$ does not give the farmer information about the distribution of P_{org} because, for example, the distribution of P_{org} is known to the farmer ahead of the starting period and is not affected by their adoption decision in the current period. In this case we can write that

$$V_{con}(P_{org}, K) = \max_{a_t \in \{0,1\}} \left\{ G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K \right], V_{org}(P_{org}, K_{org}) \right\}, \tag{262}$$

Now, the value of continuing to produce conventionally, $G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K \right]$ is not a function of P_{org}. The value of producing organically, $V_{org}(P_{org}, K_{org})$, though, is strictly increasing with respect to P_{org} (assuming $\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \neq 0$.)

Therefore, assuming that $\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \neq 0$, we can find a value of P_{org} for each K at which the value of continuing to produce conventionally is equal to the value of producing organically. We will denote this value as $P^*_{org}(K)$. At $P^*_{org}(K)$ we therefore have that:

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K\right] = V_{org}(P_{org}, K_{org} \mid P_{org} = P^*_{org}(K)) \tag{263}$$

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K\right] = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \tag{264}$$

Next we can write $\mathbb{E}\left[V_{con}(P'_{org}, K') \mid K\right]$ as an integral, using $\underline{P}_{org}$ and $\overline{P}_{org}$ as our lower and upper bounds on P'_{org}, respectively:

$$\begin{aligned} G^*_{con}(K) + \beta \cdot \Bigg[& \int_{\underline{P}_{org}}^{P^*_{org}(K')} \left(G^*_{con}(K') + \beta \cdot \mathbb{E}\left[V_{con}(P''_{org}, K'') \,|\, K'\right]\right) \phi(P'_{org}) dP'_{org} \\ & + \int_{P^*_{org}(K')}^{\overline{P}_{org}} \frac{1}{\rho} P'_{org} \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \cdot \phi(P'_{org}) dP'_{org} \Bigg] \\ & = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \end{aligned} \tag{265}$$

Note that if we evaluate equation (12.1) at K', we can can write :

$$\begin{aligned} G^*_{con}(K) + \beta \cdot \Bigg[& \int_{\underline{P}_{org}}^{P^*_{org}(K')} \underbrace{\left(G^*_{con}(K') + \beta \cdot \mathbb{E}\left[V_{con}(\cdot, K'')\right]\right)}_{\frac{1}{\rho} P^*_{org}(K') \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \phi(P'_{org}) dP'_{org} \\ & + \int_{P^*_{org}(K')}^{\overline{P}_{org}} \frac{1}{\rho} P'_{org} \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \cdot \phi(P'_{org}) dP'_{org} \Bigg] \\ & = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \end{aligned} \tag{266}$$

or:

$$\begin{aligned} P^*_{org}(K) = & \frac{G^*_{con}(K)}{\frac{1}{\rho} \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \frac{\beta}{\frac{1}{\rho} \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \\ & \cdot \Bigg[\int_{\underline{P}_{org}}^{P^*_{org}(K')} \left(\frac{1}{\rho} P^*_{org}(K') \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)\right) \cdot \phi(P'_{org}) dP'_{org} \\ & + \int_{P^*_{org}(K')}^{\overline{P}_{org}} \frac{1}{\rho} P'_{org} \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \cdot \phi(P'_{org}) dP'_{org} \Bigg] \end{aligned} \tag{267}$$

$$
\begin{aligned}
P^*_{org}(K) =& \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} \\
&+ \frac{\beta}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} \cdot \left[\int_{\underline{\mathrm{P}}_{org}}^{P^*_{org}(K')} \left(\frac{1}{\rho}P^*_{org}(K')\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)\right)\cdot\phi(P'_{org})dP'_{org}\right. \\
&\left. + \int_{P^*_{org}(K')}^{\overline{P}_{org}} \frac{1}{\rho}P'_{org}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)\cdot\phi(P'_{org})dP'_{org}\right]
\end{aligned} \tag{268}
$$

$$
P^*_{org}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} + \beta\cdot P^*_{org}(K')\cdot\int_{\underline{\mathrm{P}}_{org}}^{P^*_{org}(K')}\phi(P'_{org})dP'_{org} + \beta\cdot\int_{P^*_{org}(K')}^{\overline{P}_{org}} P'_{org}\cdot\phi(P'_{org})dP'_{org} \tag{269}
$$

$$
P^*_{org}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} + \beta\cdot P^*_{org}(K')\cdot\Phi(P^*_{org}(K')) + \beta\cdot\int_{P^*_{org}(K')}^{\overline{P}_{org}} P'_{org}\cdot\phi(P'_{org})dP'_{org} \tag{270}
$$

$$
P^*_{org}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} + \beta\cdot P^*_{org}(K')\cdot\Pr(P'_{org}\leq P^*_{org}(K')) + \beta\cdot E[P'_{org}|P'_{org}>P^*_{org}(K')] \tag{271}
$$

$$
P^*_{org}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} + \beta\cdot\left(P^*_{org}(K')\cdot\Pr(P'_{org}\leq P^*_{org}(K')) + E[P'_{org}|P'_{org}>P^*_{org}(K')]\right) \tag{272}
$$

where:

$$
\Pr(P'_{org}\leq P^*_{org}(K')) = \Phi(P^*_{org}(K')) = \int_{\underline{\mathrm{P}}_{org}}^{P^*_{org}(K')}\phi(P'_{org})dP'_{org} \tag{273}
$$

$$
E[P'_{org}|P'_{org}>P^*_{org}(K')] = \int_{P^*_{org}(K')}^{\overline{P}_{org}} P'_{org}\cdot\phi(P'_{org})dP'_{org} \tag{274}
$$

Since P_{org} is iid, we have:

$$P^*_{org}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \beta \cdot \left(P^*_{org}(K') \cdot \Pr(P_{org} \leq P^*_{org}(K')) + E[P_{org}|P'_{org} > P^*_{org}(K')]\right) \tag{275}$$

where:

$$\Pr(P_{org} \leq P^*_{org}(K')) = \Pr(P'_{org} \leq P^*_{org}(K')) = \Phi(P^*_{org}(K')) = \int_{\underline{\mathrm{P}}_{org}}^{P^*_{org}(K')} \phi(P_{org})dP_{org} \tag{276}$$

$$E[P_{org}|P_{org} > P^*_{org}(K')] = E[P'_{org}|P'_{org} > P^*_{org}(K')] = \int_{P^*_{org}(K')}^{\overline{P}_{org}} P_{org} \cdot \phi(P_{org})dP_{org} \tag{277}$$

If $K = 0$ we will have $K' = 0$, and thus K will be 0 in all subsequent time periods. In this case $P^*_{org}(K) = P^*_{org}(K') = P^*_{org}(0)$, and we can write:

$$P^*_{org}(0) = \frac{G^*_{con}(0)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \beta \cdot P^*_{org}(0) \cdot \Pr(P_{org} \leq P^*_{org}(0)) + \beta \cdot E[P_{org}|P_{org} > P^*_{org}(0)] \tag{278}$$

When there is no uncertainty about P_{org} and P_{org} is not stochastic, then the threshold $P^*_{orgdet}(K)$ is given by:

$$P^*_{orgdet}(K) = \frac{G^*_{con}(K)}{\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{279}$$

Previously we found that when there is no uncertainty and $K_{org} = \overline{C}$, the range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ for Case C6 is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P^{-1}_{con}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho \cdot \epsilon\right)}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{280}$$

We now confirm that when $K_{org} = \overline{C}$, $P^*_{orgdet}(K)$ yields the same threshold organic premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ above (which we previous derived for Case C6 when there is no uncertainty and when $K_{org} = \overline{C}$ in the full derivations).

From condition , we would then finish writing out our expression for $P^*_{org}(K)$ when there is no uncertainty over the value of P^*_{org} and when $K_{org} = \overline{C}$ as follows:

$$\Sigma_{t=0}^{\infty}\beta^t G^*_{con}(K^{(t)}) = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{281}$$

$$P^*_{org}(K) = \frac{\Sigma_{t=0}^{\infty}\beta^t G^*_{con}(K^{(t)})}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{282}$$

Note though that our denominator comes from a continuous time calculation, while our numerator comes from a discrete time calculation. For consistency, it would be better to have our assertions about discrete vs continous time match across the two. So we would in fact either want to write

$$P^*_{org}(K) = \frac{\Sigma_{t=0}^{\infty}\beta^t G^*_{con}(K^{(t)})}{\Sigma_{t=0}^{\infty}\beta^t \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{283}$$

or

$$P^*_{org}(K) = \frac{\int_0^{\infty} G_{con}(K^*(t), I^*(t)) \cdot e^{-\rho \cdot t} dt}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{284}$$

where $K^*(t)$ and $I^*(t)$ are our solutions to the conventional C6 problem. From our C6 analysis we know that:

$$\begin{aligned} & \int_0^{\infty} G_{con}(K^*(t), I^*(t)) \cdot e^{-\rho \cdot t} dt \\ = \frac{1}{\rho} \cdot P_{con}\alpha_b \cdot & \left(\frac{\rho}{(\mu + \rho)} \cdot \gamma_K \cdot (K_0) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \end{aligned} \tag{285}$$

and so $P^*_{org}(K)$ can be written as

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K_0) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{286}$$

$$P^*_{org}(K) = \frac{P_{con} \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{287}$$

From this equation we can derive an organic price premium needed to induce a preference for organic management:

$$P^*_{org}(K) = \frac{P_{con}\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{288}$$

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{289}$$

$$\frac{P^*_{org}(K)}{P_{con}} - 1 = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 \tag{290}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} - \gamma_K\overline{C} - A_b - \frac{A_y}{\alpha_b}}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{291}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K\overline{C} + \frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{292}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\overline{C} - \frac{\rho}{(\mu+\rho)} \cdot K\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{293}$$

If we define K in terms of its distance from $\overline{C}$, ϵ, such that $K = \overline{C} - \epsilon$ we then have:

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\overline{C} - \frac{\rho}{\mu+\rho} \cdot \left(\overline{C} - \epsilon\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{294}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\frac{\mu+\rho}{\mu+\rho} \cdot \overline{C} - \frac{\rho}{\mu+\rho} \cdot \left(\overline{C} - \epsilon\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{295}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \overline{C} + \frac{\rho}{\mu+\rho} \cdot \epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{296}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{\mu+\rho} \cdot \gamma_K \cdot \left(\mu \cdot \overline{C} + \rho \cdot \epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{297}$$

Note that this organic price premium is the same as the $\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic}$ that we derived in our C6 local discrete analysis when $K_{org} = \overline{C}$.

When there is uncertainty, we find that since $\left(P^*_{org}(K') \cdot \Pr(P_{org} \leq P^*_{org}(K')) + E[P_{org}|P'_{org} > P^*_{org}(K')]\right) \geq 0$, the threshold organic premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{uncertainty}$ is higher, and is given by the following when $K_{org} = \overline{C}$:

From condition (12.1) we have that at $P^*_{org}(K)$ the following condition holds when $K_{org} = \overline{C}$:

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K\right] = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \tag{298}$$

where the RHS comes from the continouous time value of $V_{org}(\overline{C})$, conditional on remaining organic and conditional on $K_{org} = \overline{C}$.

Re-arranging terms, from this equation we can write

$$P^*_{org}(K) = \cdot \frac{\int_{t=0}^{\infty} G^*_{con}(K^*(t)) e^{-\rho \cdot t} dt}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \cdot \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(K) \cdot e^{-\rho \cdot t} dt}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{299}$$

where a conventional farmer will adopt organic ($a = 1$) if:

$$a(P_{org}, K; K_{org}) = \mathbf{1}\{V_{org}(P_{org}, K_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left(V_{con}(P'_{org}, K')K\right)\} \tag{300}$$

continuous C6 optimal control problem. Based on our solution to that problem, we know that we can write:

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con} \alpha_b \cdot \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{301}$$

$$+ \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{302}$$

$$P^*_{org}(K) = \frac{P_{con} \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t)))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{303}$$

From this equation we can write an expression for the organic price premium required to induce adoption of organic management when the farmer faces uncertainty in the value of P_{org}:

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{304}$$

$$\frac{P^*_{org}(K)}{P_{con}} - 1 = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{305}$$

$$\frac{P^*_{org}(K)}{P_{con}} - \frac{P_{con}}{P_{con}} = \frac{\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + \frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{306}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K\overline{C} + \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\rho\overline{C} - \rho\epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{307}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho\epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{308}$$

$$\left(\frac{P_{org}(K)-P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic} + \frac{\int_{t=0}^{\infty}\left(V_{org}(P_{org},K_{org})-G^*_{con}(K^*(t))\right)\cdot a(P_{org},K;K_{org})\cdot e^{-\rho\cdot t}dt}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} \tag{309}$$

$$\left(\frac{P_{org}(K)-P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic} + \underbrace{\frac{\underbrace{\int_{t=0}^{\infty}\left(V_{org}(P_{org},K_{org})-G^*_{con}(K^*(t))\right)\cdot a(P_{org},K;K_{org})\cdot e^{-\rho\cdot t}dt}_{\geq 0}}{\underbrace{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}}}_{\geq 0} \tag{310}$$

We therefore have that:

$$\left(\frac{P_{org}(K)-P_{con}}{P_{con}}\right)^*_{uncertainty} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic} \tag{311}$$

To see why

$$\int_{t=0}^{\infty}\left(V_{org}(P_{org},K_{org})-G^*_{con}(K^*(t))\right)\cdot a(P_{org},K;K_{org})\cdot e^{-\rho\cdot t}dt \geq 0 \tag{312}$$

is satisfied, note the following. From our solution to the C6 conventional optimization problem, we had that when the farmer follows their optimal management plan $G_{con}(K^*(t), I^*(t))$ decreases in value over time as the farmer's capital stock falls. For all periods of time at which $G_{con}(K^*(t), I^*(t)) > V_{org}(P_{org}, K_{org})$, and at which we therefore have $V_{org}(P_{org}, K_{org}) - G_{con}(K^*(t), I^*(t)) < 0$, our indicator function equals zero, and $V_{org}(P_{org}, K_{org}) - G_{con}(K^*(t), I^*(t)) < 0$ does not contribute to the value of our integral. On the otherhand for all periods of time $t > T_{V_{org}}$, where $T_{V_{org}}$ is the first moment in time at which $G_{con}(K^*(t), I^*(t)) \leq V_{org}(P_{org}, K_{org})$, we will have $V_{org}(P_{org}, K_{org}) - G_{con}(K^*(t), I^*(t)) \geq 0$ and $a(P_{org}, K; K_{org}) = 0$, such that $V_{org}(P_{org}, K_{org}) - G_{con}(K^*(t), I^*(t)) \geq 0$ contributes positively (or at least non-negatively) to the value of our integral.

Thus, when there is uncertainty, the threshold organic price premium is higher by the following amount when $K_{org} = \overline{C}$:

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} - \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\int_{t=0}^{\infty} \left(V_{org}(P_{org}, K_{org}) - G^*_{con}(K^*(t))\right) \cdot a(P_{org}, K; K_{org}) \cdot e^{-\rho \cdot t} dt}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{313}$$

where a conventional farmer will adopt organic ($a = 1$) if:

$$a(P_{org}, K; K_{org}) = \mathbf{1}\{V_{org}(P_{org}, K_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(P'_{org}, K') \mid K\right]\} \tag{314}$$

13 Application to Organic Standards in US and elsewhere

13.1 Application to USDA Organic Standards

In the United States, the National Organic Program (NOP), which is directed by the U.S. Department of Agriculture (USDA) Agricultural Marketing Service (AMS) and became effective on February 20, 2001, oversees and enforces the integrity of the rigorous USDA organic standards and the accreditation of organic certifiers (USDA Agricultural Marketing Service, 2000b; Organic Produce Network, 2022).Organic is one of the most heavily regulated and closely monitored food systems in the U.S. Any product labeled as organic must be USDA certified (Organic Produce Network, 2022). The National Organic Program (NOP) establishes national standards for the production and handling of organically produced products, including a National List of substances approved for and prohibited from use in organic production and handling; as well as requirements for labeling products as organic and containing organic ingredients. Under the National Organic Program (NOP), certifying agents certify production and handling operations in compliance with the requirements of this regulation and initiate compliance actions to enforce program requirements (USDA Agricultural Marketing Service, 2000b).

The organic production and handling requirements of the National Organic Program (NOP) include the requirement that production practices implemented must maintain or improve the natural resources of the operation, including soil and water quality, as well as the requirement that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop. The on-side inspection must verify that prohibited substances have not been and are not being applied to the operation through means which, at the discretion of the certifying agent, may include the collection and testing of soil; water; waste; seeds; plant tissue; and plant, animal, and processed products samples (USDA Agricultural Marketing Service, 2000a). Thus, becoming certified organic under the USDA National Organic Program may entail being subject to soil testing (USDA Agricultural Marketing Service, 2000a; Baier and Ahramjian, 2012; USDA Agricultural Marketing Service, 2018).

In our model, we model organic certification requirements as a clean soil stock threshold K_{org}(or, equivalently, a threshold for the stock of synthetic compounds in the soil C_{org}). Our choice to make certification contingent on a stock threshold at least partially captures the main features of the US National Organic Program, including the requirements that practices implemented must maintain or improve the natural resources of the operation, including soil and water quality, and that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop. In order to verify that practices implemented must maintain or improve the natural resources of the operation, including soil and water quality and that prohibited substances have not been applied by a period of 3 years, the certifying agent may collect and test the soil; as a consequence, the certification requirement essentially amounts to the requirement that the stock of synthetic compounds in the soil must not exceed a threshold C_{org} (or, equivalently, clean soil stock must meet a clean soil stock threshold K_{org}).

To see this, consider that a farmer's organic certification agent ultimately reserves the right to reject a farmer's application if they find evidence of prohibited substances in the farmer's soils (as per the USDA's certification requirements) that exceed a threshold C_{org} (or, equivalently, a clean soil stock that does not meet a clean soil stock threshold K_{org}). Therefore, the farmer needs to ensure that the stock of synthetic residues remaining in their soils meets this threshold C_{org}. If the stock of synthetic residues remaining in a farmer's soils after the required 3-year period of not using synthetic compounds has not fallen below the threshold C_{org}, the farmer's application will be rejected, even if the farmer claims to have not used prohibited substances for the last three years. Therefore, our use of a capital stock threshold is justified, because satisfying this synthetic residue stock requirement is a meaningful/necessary part of becoming eligible for certification.

We can additionally impose the requirement that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop as a requirement that the optimal trajectory for synthetic compound input use $c(t) = 0$ for all t. Continuous transitions to organic management that can involve $c(t) = 0$ for at least 3 years include:

1. Optimal Trajectories 5 (OT5): Invest as fast as possible until $K = \overline{C}$
2. Optimal Trajectories 4 (OT4) if $\hat{K}_j = \overline{C}$: Approach $\hat{K}_j = \overline{C}$ at moderate speed; and
3. Optimal Trajectories 4 (OT4, Approach $\hat{K}_j$ at moderate speed) if $\mu = 0$ (i.e., the stock of synthetic compounds in the soil does not decay on its own) and $\hat{K}_j \geq K_{org}$.

For discrete 'jump' transitions, we can additionally impose the requirement that any discrete 'jump' transition must have $c(t)$ for at least 3 years as part of the discrete 'jump'.

For the majority of our analysis, we also assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils, such that they will be certified organic if and only if $K = \overline{C}$. In this case we have that $K_{org} = \overline{C}$.

13.2 Application to Organic Standards Elsewhere

Great Britain's organic standards are currently the same as EU standards. Basically, no synthetic fertilizers or pesticides may be used. Approved fertilizers and pesticides can only be use if other management methods are not working by themselves. Even then their use has to be justified. The certification program establishes nutrient caps (such that farmers cannot apply more than X amount of nitrogen equivalents per Y area of land). Farmers must also demonstrate that they are making efforts to increase the ecological/environmental soundness of their operation (e.g. by minimizing the destruction of important natural habitats, etc) (Soil Association, 2023a). There is a 2- to 3-year transition period before organic premium can be claimed (Soil Association, 2023a). Testing will be performed at the end of the transition period to determine whether farmers need to pursue a longer transition period, and soil samples may be taken to determine if the conversion period need to be extended (Soil Association, 2023b).

In Australia, ACO Certification LTD, Australia's largest organic certifier, tests farmers' soils for pesticide residues (ACO Certification Ltd, 2023a,b)[1].

[1] This was also confirmed via personal communication with ACO's Technical Officer, Ruwi Jayasuriya, January 2024

14 Discussion and Conclusion

When farmers account for soil bacteria:

- Some may transition to organic management 'accidentally' as their optimal trajectories gradually take them toward the certification threshold (this can happen even in the absence of an organic price premium)
- Other transitions may be induced by the organic price premium.

When farmers do not account for soil bacteria:

- They never make an 'accidental' transition to organic, and will instead disinvest as fast as possible to $K = 0$.
- If they transition can only be induced by an organic price premium.
- They will require a higher premium to adopt than a fully informed farmer would when a large enough proportion of organic farming's value-added comes from stock effects/soil microbes.

Not being informed about soil bacteria could change behavior in a way that leads farmers to adopt sub-optimal, and even detrimental management practices.

$\Rightarrow$ Implications for welfare improving policy initiatives

References

ACO Certification Ltd. Certification process, 2023a. URL: `https://www.aco.net.au/Pages/Certification/CertificationProcess.aspx`.

ACO Certification Ltd. Testing and compliance, 2023b. URL: `https://www.aco.net.au/Pages/Testing/Testing.aspx`.

A. H. Baier and L. Ahramjian. Organic certification of farms and businesses producing agricultural products, 2012. May 2012. URL: `https://www.ams.usda.gov/sites/default/files/media/Guide-OrganicCertification.pdf`.

R. Blundell, J. E. Schmidt, A. Igwe, A. L. Cheung, R. L. Vannette, A. C. Gaudin, and C. L. Casteel. Organic management promotes natural pest control through altered plant resistance to insects. *Nature Plants*, 6(5):483–491, 2020.

C. L. Carroll, C. A. Carter, R. E. Goodhue, and C.-Y. C. Lin Lawell. Crop disease and agricultural productivity: Evidence from a dynamic structural model of verticillium wilt management. In W. Schlenker, editor, *Agricultural Productivity and Producer Behavior*, pages 217–249. University of Chicago Press, Chicago, 2019.

C. L. Carroll, C. Carter, R. Goodhue, and C.-Y. C. Lin Lawell. The economics of decision-making for crop disease control. Working paper, Cornell University, 2024a.

C. L. Carroll, C. Carter, R. Goodhue, and C.-Y. C. Lin Lawell. Supply chain externalities and agricultural disease. Working paper, Cornell University, 2024b.

R. G. Chambers, G. Karagiannis, and V. Tzouvelekas. Another look at pesticide productivity and pest damage. *American Journal of Agricultural Economics*, 92(5):1401–1419, 2010.

J. M. Conrad, C.-Y. C. Lin Lawell, and B. B. Shin. On the optimal moratorium in an overfished fishery: The case of Canadian cod. Working paper, Cornell University, 2024.

S. Dabbert and P. Madden. The transition to organic agriculture: A multi-year simulation model of a Pennsylvania farm. *American Journal of Alternative Agriculture*, pages 99–107, 1986.

T. A. Delbridge and R. P. King. Transitioning to organic crop production: A dynamic programming approach. *Journal of Agricultural and Resource Economics*, pages 481–498, 2016.

S. Hussain, T. Siddique, M. Saleem, M. Arshad, and A. Khalid. Chapter 5: Impact of pesticides on soil microbial diversity, enzymes, and biochemical reactions. *Advances in Agronomy*, 102:159–200, 2009.

F. Jacquet, J.-P. Butault, and L. Guichard. An economic analysis of the possibility of reducing pesticides in French field crops. *Ecological Economics*, 70(9):1638–1648, 2011.

E. Jaenicke. Testing for intermediate outputs in dynamic dea models: Accounting for soil capital in rotational crop production and productivity measures. *Journal of Productivity Analysis*, 14(4): 247–266, 2000.

E. Jaenicke and L. Lengnick. A soil-quality index and its relationship to efficiency and productivity growth measures: Two decompositions. *American Journal of Agricultural Economics*, 81(4):881–893, 1999.

A. Kalia and S. Gosal. Effect of pesticide application on soil microorganisms. *Archives of Agronomy and Soil Science*, 57(6):569–596, 2011.

C.-C. Lo. Effect of pesticides on soil microbial community. *Journal of Environmental Science and Health Part B*, 45(5):348–359, 2010.

M. Lori, S. Symnaczik, P. Mäder, G. De Deyn, and A. Gattinger. Organic farming enhances soil microbial abundance and activityâa meta-analysis and meta-regression. *PLOS ONE*, 12(7): e0180442, 2017.

D. M. Murphy, D. Roobroeck, D. R. Lee, and J. Thies. Underground knowledge: Estimating the impacts of soil information transfers through experimental auctions. *American Journal of Agricultural Economics*, 102(5):1468–1493, 2020.

Organic Produce Network. Organic certification requirements, 2022. [Online]. Accessed 19 September 2022. URL: `https://www.organicproducenetwork.com/article-education/6/organic-certification-requirements-and-the-usda-organic-standards`.

O. Sambucci, C.-Y. C. Lin Lawell, and T. J. Lybbert. Pesticide spraying and disease forecasts: A dynamic structural econometric model of grape growers in California. Working paper, Cornell University, 2024.

L. Sears, D. Lim, and C.-Y. C. Lin Lawell. Spatial groundwater management: A dynamic game framework and application to California. *Water Economics and Policy*, 5(1):1850019, 2019.

L. Sears, C.-Y. C. Lin Lawell, and M. T. Walter. Groundwater under open access: A structural model of the dynamic common pool extraction game. Working paper, Cornell University, 2024.

S. E. Sexton, Z. Lei, D. Zilberman, et al. The economics of pesticides and pest control. *International Review of Environmental and Resource Economics*, 1(3):271–326, 2007.

B. B. Shin, J. M. Conrad, and C.-Y. C. Lin Lawell. On the optimality of a fishery moratorium. Working paper, Cornell University, 2024.

Soil Association. Organic standards for Great Britain, 2023a. URL: `https://www.soilassociation.org/our-standards/read-our-organic-standards/organic-standards-for-great-britain`.

Soil Association. Soil Association organic standards for Great Britain farming and growing, 2023b. Version 1.3. 25 May 2023. URL: https://www.soilassociation.org/media/25986/sa-gb-farming-growing.pdf.

A. W. Stevens. The economics of soil health. *Food Policy*, 80:1–9, 2018.

USDA Agricultural Marketing Service. National organic program, 2000a. 7 CFR 205. 21 December 2000.A Rule by the Agriculture Department, and the Agricultural Marketing Service. URL: https://www.ecfr.gov/current/title-7/subtitle-B/chapter-I/subchapter-M/part-205.

USDA Agricultural Marketing Service. National organic program: A rule by the Agricultural Marketing Service on 12/21/2000, 2000b. 65 FR 80637. 21 December 2000. URL: https://www.federalregister.gov/documents/2000/12/21/00-32257/national-organic-program.

USDA Agricultural Marketing Service. Instruction: Sampling procedures for residue testing, 2018. NOP 2610. Effective 8 November 2012. Updated 13 September 2018. URL: https://www.ams.usda.gov/sites/default/files/media/2610.pdf.

A. Varanasi. Is organic food really better for the environment? *State of the Planet*, 2019. URL https://news.climate.columbia.edu/2019/10/22/organic-food-better-environment/.

M. L. Weitzman. *Income, Wealth, and the Maximum Principle*. Harvard University Press, Cambridge, MA, 2003.

S. W. Wilcox, D. R. Just, C.-Y. C. Lin Lawell, M. I. Gómez, and H. Grab. To (rent) bees or not to (rent) bees?: An examination of the farmer's question. Working paper, Cornell University, 2024.

J. Wu. Optimal weed control under static and dynamic decision rules. *Agricultural Economics*, 25 (1):119–130, 2000.

T. Wu, C.-Y. C. Lin Lawell, D. R. Just, J. Zhao, Z. Fei, and Q. Wei. Optimal forest management for interdependent products: A nested dynamic bioeconomic model and application to bamboo. Working paper, Cornell University, 2024.

D. A. Yeh, M. I. Gómez, and C.-Y. C. Lin Lawell. Sustainable pest management, beliefs, and behavior: A dynamic bioeconomic analysis. Working paper, Cornell University, 2024.

Appendix

A Optimal Solution for Each Stage

A.1 Stock of clean soils $\tilde{K}_j$ below which there is no trade-off involved with net investment

The stationary rate of return on capital $R_j(K)$ is undefined when $\frac{\partial G(K,0)}{\partial I} = 0$. The condition that $\frac{\partial G(K,0)}{\partial I} = 0$ simplifies to:

$$\left(P_j \cdot \left(\alpha_b \left(\gamma_c + \gamma_{cc}\mu \left(\overline{C} - K\right)\right) + \alpha_c\right) - 1\right) = 0 \tag{A.1}$$

Let $\tilde{K}_j$ be defined as the stock of clean soils at which $\frac{\partial G(K,0)}{\partial I} = 0$. In other words, at the stock of clean soils $\tilde{K}_j$, the marginal effect of net investment I on the net gain function $G(K, I)$ when net investment is 0 is 0. $\tilde{K}_j$ is given by:

$$\tilde{K}_j = \frac{\frac{\alpha_c - P_j^{-1}}{\alpha_b} + \gamma_c}{\gamma_{cc}\mu} + \overline{C} \tag{A.2}$$

$$\tilde{K}_j = \frac{\frac{\underbrace{\alpha_c}_{\geq 0} - \underbrace{P_j^{-1}}_{\geq 0}}{\underbrace{\alpha_b}_{\geq 0}} + \gamma_c}{\underbrace{\gamma_{cc}\mu}_{\leq 0}} + \underbrace{\overline{C}}_{>0} \tag{A.3}$$

For $K < \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} > 0$ (i.e., net investment has a positive effect on contemporaneous net gain starting from a net investment of $I = 0$), and for $K \leq \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$). Since our analysis using the stationary rate of return on capital $R(K)$ assumes that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain), we cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R_j(K)$ and ρ to describe the optimal solution when $K \leq \tilde{K}_j$.

To see this, we find that for $K < \tilde{K}_j$, $R_j(K) \leq -\mu \leq 0$:

$$R_j(K) = \underbrace{-\mu}_{\leq 0} + \underbrace{\frac{\overbrace{\gamma_K}^{\geq 0}}{\gamma_{cc}\mu\left(\overline{C} - K\right) + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}}_{<0} \leq -\mu \leq 0, \tag{A.4}$$

which would suggest that for $K < \tilde{K}_j$, since $K < \tilde{K}_j$, $R_j(K) \leq -\mu < \rho$, the farmer will always disinvest in clean soil until K is driven down to $K = 0$. But disinvesting would not make sense when

$\frac{\partial G(K,I)}{\partial I} > 0$ (i.e., net investment has a positive effect on contemporaneous net gain), which is the case when $K < \tilde{K}_j$, since net investment increases the future stock K of clean soil and the stock K of clean soil has a positive effect on net gain (i.e., $\frac{\partial G(K,0)}{\partial K} \geq 0$):

$$\frac{\partial G(K,0)}{\partial K} = \underbrace{\mu \frac{\partial G(K,0)}{\partial I}}_{\geq 0} + \underbrace{P_j \cdot \alpha_b \gamma_K}_{\geq 0} \geq 0. \tag{A.5}$$

Thus, since our analysis using the stationary rate of return on capital $R_j(K)$ makes the assumption of the prototype economic control model that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain), we cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R(K)$ and ρ to describe the optimal solution when $K \leq \tilde{K}_j$.

Instead, based on the feature that for $K \leq \tilde{K}_j$, net investment I has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$, $\frac{\partial G(K,0)}{\partial I} \geq 0$, and moreover that net investment increases the future stock K of clean soil and the stock K of clean soil has a positive effect on net gain (i.e., $\frac{\partial G(K,0)}{\partial K} \geq 0$), then we would expect that a farmer with $K \leq \tilde{K}_j$ would invest in the stock of clean soil, not disinvest, since there is no trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well.

Thus, for $K \leq \tilde{K}_j$, the farmer will invest in clean soil.

Although $\tilde{K}_j$ does not actually matter that much for describing investment behavior, it has important economic content and intuition, since $\tilde{K}_j$ (if it exists) is the threshold below which net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$. A farmer with $K \leq \tilde{K}_j$ does not face any trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well. In addition, another important thing about $\tilde{K}_j$ was to note that our standard interpretation of how the relative value of $R(K)$ determines investment behavior becomes invalid for $K < \tilde{K}_j$. Another important aspect of $\tilde{K}_j$ is it determines which parameter space we are in.

We confirm that $\hat{K}_j \geq \tilde{K}_j$:

$$\hat{K}_j = \tilde{K}_j - \frac{\gamma_K}{(\rho + \mu)\, \gamma_{cc} \mu} \tag{A.6}$$

$$\hat{K}_j = \tilde{K}_j - \frac{\underbrace{\gamma_K}_{\geq 0}}{\underbrace{\underbrace{(\rho + \mu)}_{\geq 0} \underbrace{\gamma_{cc}}_{\leq 0} \underbrace{\mu}_{\geq 0}}_{\leq 0}} \geq \tilde{K}_j \tag{A.7}$$

Thus, since for $K \leq \tilde{K}_j$, the farmer will invest in clean soil, this means that for $K_{0j} \leq \tilde{K}_j$, if the stationary solution $\hat{K}_j$ exists, the farmer will continue to invest in clean soil until he reaches the stationary solution $\hat{K}_j$.

The value of P_j determines the value of $\tilde{K}_j$, all other parameters held constant:

$$\tilde{K}_j = \frac{\frac{\alpha_c - P_j^{-1}}{\alpha_b} + \gamma_c}{\gamma_{cc}\mu} + \overline{C} \tag{A.8}$$

$$\tilde{K}_j = \frac{\frac{\underbrace{\alpha_c}_{\geq 0} - \underbrace{P_j^{-1}}_{\geq 0}}{\underbrace{\alpha_b}_{\geq 0}} + \gamma_c}{\underbrace{\gamma_{cc}\mu}_{\leq 0}} + \underbrace{\overline{C}}_{>0}, \tag{A.9}$$

where $\frac{\partial \tilde{K}_j}{\partial P_j}$ is given by:

$$\frac{\partial \tilde{K}_j}{\partial P_j} = \frac{1}{\alpha_b \gamma_{cc} \mu P_j^2} \tag{A.10}$$

$$\frac{\partial \tilde{K}_j}{\partial P_j} = \frac{1}{\underbrace{\alpha_b}_{\geq 0} \underbrace{\gamma_{cc}}_{\leq 0} \underbrace{\mu}_{\geq 0} \underbrace{P_j^2}_{>0}} \leq 0 \tag{A.11}$$

Thus $\tilde{K}_j$ is a decreasing function of prices P_j.

The intuition is as follows. The threshold $\tilde{K}_j$ is such that for all $K \leq \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$). A higher level of net investment $I(t)$ in the stock of clean soil affects the farmer in the following ways. In the current period t, given the clean soil stock $K(t)$ for that period t, a higher level of net investment $I(t)$ means a lower level of chemical input use $c(t)$. The time-t benefits of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would forego) come through the beneficial effects of chemical input use $c(t)$ on crop output $f(\cdot)$. There are two time-t costs of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would no longer incur). First, there is a unit price to chemical input use, which we normalize to 1. Second, chemical input use decreases beneficial soil microbes $b(t)$, and this decrease in beneficial soil microbes $b(t)$ may then have an adverse impact to crop output $f(\cdot)$. In addition to the time-t costs and benefits of a lower level of chemical input use $c(t)$, a higher level of net investment $I(t)$ also means a higher level of future clean soil stock K.

When prices P_j are lower, the threshold $\tilde{K}_j$ below which net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$ is higher because with lower prices P_j, a farmer who net invests would have less revenue to forego from the foregone beneficial effects of chemical input use $c(t)$ on crop revenue $f(\cdot)$, and thus the costs of net investing are lower and more likely to be outweighed by its benefits, which include foregoing the price of chemical input use as well as the adverse effect of chemical input use on soil microbes.

Neither $\tilde{K}_j$ nor $\hat{K}_j$ will exist if either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own).

If either $\gamma_{cc} = 0$ or $\mu = 0$, then the condition that $\frac{\partial G(K,I)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain) simplifies to:

$$P_j^{-1} \geq \alpha_b \gamma_c + \alpha_c \tag{A.12}$$

A.2 Effects of Price P_j on Optimal Solution for each stage $j \in \{con, org\}$

The value of P_j determines the value of $\tilde{K}_j$, all other parameters held constant:

$$\tilde{K}_j = \frac{\frac{\alpha_c - P_j^{-1}}{\alpha_b} + \gamma_c}{\gamma_{cc}\mu} + \overline{C} \tag{A.13}$$

$$\tilde{K}_j = \frac{\frac{\underbrace{\alpha_c}_{\geq 0} - \underbrace{P_j^{-1}}_{\geq 0}}{\underbrace{\alpha_b}_{\geq 0}} + \gamma_c}{\underbrace{\gamma_{cc}\mu}_{\leq 0}} + \underbrace{\overline{C}}_{>0}, \tag{A.14}$$

where $\frac{\partial \tilde{K}_j}{\partial P_j}$ is given by:

$$\frac{\partial \tilde{K}_j}{\partial P_j} = \frac{1}{\alpha_b \gamma_{cc} \mu P_j^2} \tag{A.15}$$

$$\frac{\partial \tilde{K}_j}{\partial P_j} = \frac{1}{\underbrace{\alpha_b}_{\geq 0}\underbrace{\gamma_{cc}}_{\leq 0}\underbrace{\mu}_{\geq 0}\underbrace{P_j^2}_{>0}} \leq 0 \tag{A.16}$$

Thus $\tilde{K}_j$ is a decreasing function of prices P_j.

The intuition is as follows. The threshold $\tilde{K}_j$ is such that for all $K \leq \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$). A higher level of net investment $I(t)$ in the stock of clean soil affects the farmer in the following ways. In the current period t, given the clean soil stock $K(t)$ for that period t, a higher level of net investment $I(t)$ means a lower level of chemical input use $c(t)$. The time-t benefits of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would forego) come through the beneficial effects of chemical input use $c(t)$ on crop output $f(\cdot)$. There are two time-t costs of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would no longer incur). First, there is a unit price to chemical input use, which we normalize to 1. Second, chemical input use decreases beneficial soil microbes $b(t)$, and this decrease in beneficial soil microbes $b(t)$ may then have an adverse impact to crop output $f(\cdot)$. In addition to the time-t costs and benefits of a lower level of chemical input use $c(t)$, a higher level of net investment $I(t)$ also means a higher level of future clean soil stock K.

When prices P_j are lower, the threshold $\tilde{K}_j$ below which net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$ is higher because with lower prices P_j, a farmer who net invests would have less revenue to forego from the foregone beneficial effects of chemical input use $c(t)$ on crop revenue $f(\cdot)$, and thus the costs of net investing are lower and more likely to be outweighed by its benefits, which include foregoing the price of chemical input use as well as the adverse effect of chemical input use on soil microbes.

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then $\frac{\partial \tilde{K}_j}{\partial P_j}$ is given by:

$$\frac{\partial R_j(K)}{\partial P_j} = -\frac{\gamma_K}{\left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}\frac{P_j^{-2}}{\alpha_b} \tag{A.17}$$

$$\frac{\partial R_j(K)}{\partial P_j} = -\frac{\underbrace{\gamma_K}_{\geq 0}}{\underbrace{\left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}_{\geq 0}}\frac{P_j^{-2}}{\underbrace{\alpha_b}_{\geq 0}} \leq 0 \tag{A.18}$$

Thus, if either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then the constant $R_j(K)$ is lower when prices are higher.

A.2.1 Very low P_j

If $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then since $\tilde{K}_j$ is a decreasing function of P_j , for very low P_j we will have $\tilde{K}_j > \overline{C}$, and therefore $K \leq \tilde{K}_j \leq \hat{K}_j$ for all feasible K, and the farmer will continue to invest such that K approaches $\hat{K}_j$ until K reaches its upper bound $\overline{C}$.

The condition for $\tilde{K}_j > \overline{C}$ simplifies to:

$$P_j^{-1} > -\alpha_b \left(\gamma_{cc}\mu - \gamma_c\right) + \alpha_c \tag{A.19}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough to satisfy the following condition for net investment to have a non-negative effect on contemporaneous net gain (so that $R_j(K)$ is not useful for analyzing net investment):

$$P_j^{-1} \geq \alpha_b\gamma_c + \alpha_c, \tag{A.20}$$

then the farmer will again invest in clean soil until $K = \overline{C}$.

A.2.2 Very high P_j

Alternatively, if $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then for very high P_j , we will have $\tilde{K}_j < 0$, and therefore $K \geq \tilde{K}_j$ for all feasible K.

In this case, the farmer's capital stock will approach the stationary solution and reaches it if $\hat{K}_j \in [0, \overline{C}]$. If $\hat{K}_j > \overline{C}$, then the farmer will approach $\hat{K}_j$ from below until they reach the blocked state $K = \overline{C}$, where they will stay indefinitely. If $\hat{K}_j < 0$ (and therefore does not exist since it is less than 0), the farmer will approach $\hat{K}_j$ from above until they reach the blocked state $K = 0$, where they will stay indefinitely.

The condition for $\tilde{K}_j < 0$ simplifies to:

$$P_j^{-1} < -\alpha_b \left(-\gamma_{cc}\mu\overline{C} - \gamma_c\right) + \alpha_c \tag{A.21}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{A.22}$$

as well as the following condition for $R_j(K) < \rho$:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{A.23}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

A.2.3 Intermediate P_j

Finally, if $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then for certain intermediate values of P_j, it will be possible to have $\tilde{K}_j \in [0, \overline{C}]$. For $K_{0j} \leq \tilde{K}_j$, the farmer will continue to invest in clean soil and approach the stationary solution $\hat{K}_j$ until he reaches the stationary solution $\hat{K}_j \leq \overline{C}$. For $K_{0j} > \tilde{K}_j$, the farmer will approach and eventually reach the stationary solution $\hat{K}_j \leq \overline{C}$ by either investing, as in the case in which $K_{0j} < \hat{K}_j$; or by disinvesting, as in the case in which $K_{0j} > \hat{K}_j$.

The condition for $\tilde{K}_j \in [0, \overline{C}]$ is given by:

$$-\alpha_b \left(\gamma_{cc}\mu - \gamma_c \right) + \alpha_c \leq P_j^{-1} \leq -\alpha_b \left(-\gamma_{cc}\mu\overline{C} - \gamma_c \right) + \alpha_c \tag{A.24}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough that $R_j(K) > \rho$:

$$P_j^{-1} > \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{A.25}$$

but also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c, \tag{A.26}$$

then the farmer will always invest until he reaches $K = \overline{C}$ since $R_j(K) > \rho$.

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices satisfy the condition that $R_j(K) = \rho$:

$$P_j^{-1} = \frac{\alpha_b}{\rho+\mu}\left((\rho+\mu)\gamma_c - \gamma_K\right) \tag{A.27}$$

but are also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b\gamma_c + \alpha_c, \tag{A.28}$$

then the farmer will will stay at $K = K_{0j}$ since $R_j(K) = \rho$.

A.3 Comparative Statics for $\hat{K}_j$

$$\hat{K}_j = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{A.29}$$

We evaluate the effect of each parameter on $\hat{K}_j$ by calculating the partials of $\hat{K}_j$:

$$\begin{aligned} \frac{\partial \hat{K}_j}{\partial P_{\mathrm{j}}} &= \frac{\mu+\rho}{P_{\mathrm{j}}^2\alpha_b\left(\mu(\mu+\rho)\gamma_{cc}\right)} \\ &= \frac{1}{P_{\mathrm{j}}^2\alpha_b\mu\gamma_{cc}} \end{aligned} \tag{A.30}$$

$$\begin{aligned} \frac{\partial \hat{K}_j}{\partial \mu} =& \frac{\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} + (\rho+\mu)\,\gamma_{cc}\overline{C}}{(\rho+\mu)\,\gamma_{cc}\mu} \\ &- \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right) - \gamma_K}{\left((\rho+\mu)\,\gamma_{cc}\mu\right)^2}\cdot\left((\rho+\mu)\,\gamma_{cc} + \gamma_{cc}\mu\right) \end{aligned} \tag{A.31}$$

$$\frac{\partial \hat{K}_j}{\partial \mu} = \left(\underbrace{\frac{1}{\rho+\mu} + \frac{1}{\mu}}_{\geq 0}\right)\left(\underbrace{\overline{C} - \hat{K}_j}_{\geq 0}\right) + \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{A.32}$$

We can put a sign on $\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{(\rho+\mu)\gamma_{cc}\mu}$ by remembering that the capital stock must be less than or equal to $\overline{C}$ for all t. This constraint yields:

$$\overline{C} \geq \hat{K}_j \tag{A.33}$$

$$\Rightarrow \overline{C} \geq \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}\right)-\gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{A.34}$$

$$(\rho+\mu)\,\gamma_{cc}\mu\overline{C}+\gamma_K \leq (\rho+\mu)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}\right) \tag{A.35}$$

$$\gamma_{cc}\mu\overline{C}+\frac{\gamma_K}{(\rho+\mu)} \leq \gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.36}$$

$$\frac{\gamma_K}{(\rho+\mu)} \leq \gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.37}$$

$$\underbrace{\frac{\gamma_K}{(\rho+\mu)^2\,\gamma_{cc}\mu}}_{\leq 0} \geq \frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{A.38}$$

so we know that

$$0 \geq \frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu} \tag{A.39}$$

Note also that

$$\frac{\gamma_K}{\underbrace{(\rho+\mu)}_{\geq 0}} \leq \gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.40}$$

$$0 \leq \gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.41}$$

$$\underbrace{-\gamma_c}_{\geq 0} \leq \frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.42}$$

$$0 \leq \frac{\alpha_c-P_j^{-1}}{\alpha_b} \tag{A.43}$$

$$P_j^{-1} \leq \alpha_c \tag{A.44}$$

We see that $\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\gamma_{cc}\mu}$ must be weakly less than a non-positive number, and will therefore be non-positive. Note that our non-negativity constraint on K does not help us more precisely sign $\frac{\partial \hat{K}_j}{\partial \mu}$. So our expression for $\frac{\partial \hat{K}_j}{\partial \mu}$ can be written as

$$\frac{\partial \hat{K}_j}{\partial \mu} = \left(\underbrace{\frac{1}{\rho+\mu} + \frac{1}{\mu}}_{\geq 0}\right)\left(\underbrace{\overline{C} - \hat{K}_j}_{\geq 0}\right) + \underbrace{\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0} \tag{A.45}$$

$$\frac{\partial \hat{K}_j}{\partial \rho} = \frac{\gamma_K}{\gamma_{cc}\mu\,(\rho+\mu)^2} \tag{A.46}$$

$$\begin{aligned} \frac{\partial \hat{K}_j}{\partial \overline{C}} &= \frac{\mu(\mu+\rho)\gamma_{cc}}{\mu(\mu+\rho)\gamma_{cc}} \\ &= 1 \end{aligned} \tag{A.47}$$

$$\begin{aligned} \frac{\partial \hat{K}_j}{\partial \alpha_b} &= -\frac{(\mu+\rho)\left(-\frac{1}{P_j} + \alpha_c\right)}{\alpha_b^2\left(\mu(\mu+\rho)\gamma_{cc}\right)} \\ &= -\frac{\left(-\frac{1}{P_j} + \alpha_c\right)}{\alpha_b^2\mu\gamma_{cc}} \qquad = \frac{\left(\frac{1}{P_j} - \alpha_c\right)}{\mu\alpha_b^2\gamma_{cc}} \end{aligned} \tag{A.48}$$

$$\begin{aligned} \frac{\partial \hat{K}_j}{\partial \alpha_c} &= \frac{\mu+\rho}{\alpha_b\left(\mu(\mu+\rho)\gamma_{cc}\right)} \\ &= \frac{1}{\mu\alpha_b\gamma_{cc}} \end{aligned} \tag{A.49}$$

$$\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} = \frac{1}{\alpha_b}\left(P_j^{-1} - \alpha_c\right) + \frac{\gamma_K}{(\rho+\mu)} - \gamma_c\mu\gamma_{cc}^2 \tag{A.50}$$

$$\begin{aligned}\frac{\partial \hat{K}_j}{\partial \gamma_c} &= \frac{\mu+\rho}{\mu(\mu+\rho)\gamma_{cc}} \\ &= \frac{1}{\mu\gamma_{cc}}\end{aligned} \tag{A.51}$$

We interpret the resulting expressions below.

$$\frac{\partial \hat{K}_j}{\partial P_j} = \frac{1}{\underbrace{P_j^2\alpha_b}_{\geq 0}\,\mu\,\underbrace{\gamma_{cc}}_{\leq 0}} \tag{A.52}$$

So we see that the effect of crop price on the stationary solution depends on the sign on μ. As seen above, in order for $K \to \hat{K}$, we must have $\mu \geq 0$. When $\mu \geq 0$, such that the stock of synthetic chemicals decays on its own, we have $\frac{\partial \hat{K}_j}{\partial P_j} \leq 0$ so that stock of clean soil at the stationary solution is smaller at higher crop prices. Thus, when the farmer faces greater incentives to produce, the organic stationary solution $\hat{K}_j$ at which the stationary rate of return of clean stock capital equals the interest rate ρ is lower.

Note that $\frac{\partial \hat{K}_j}{\partial P_j}$ becomes more negative, and therefore we have a greater decrease in the stock of clean soil at the stationary solution for a unit increase in crop prices, when: the direct effect that soil bacteria have on crop production (α_b) is smaller, the rate at which synthetic compounds decompose (μ) is smaller, synthetic compounds are less derimental to soil bacteria (γ_{cc} is less negative), and when crop prices P_j are lower.

$$\frac{\partial \hat{K}_j}{\partial \mu} = \left(\underbrace{\frac{1}{\rho+\mu}+\frac{1}{\mu}}_{\geq 0}\right)\left(\underbrace{\overline{C}-\hat{K}_j}_{\geq 0}\right) + \underbrace{\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0} \tag{A.53}$$

We will have $\frac{\partial \hat{K}_j}{\partial \mu} \geq 0$ when

$$\left(\underbrace{\frac{1}{\rho+\mu}+\frac{1}{\mu}}_{\geq 0}\right)\left(\underbrace{\overline{C}-\hat{K}_j}_{\geq 0}\right) \geq -\left(\underbrace{\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0}\right) \tag{A.54}$$

This condition is more likely to be satisfied when: γ_c (the linear part of synthetic compounds' effects on soil bacteria) is more negative (but still less than $\frac{\alpha_c - P_j^{-1}}{\alpha_b}$ in magnitude, as required by our upper bound constraint on K) such that our non-negative numerator inside the parenthesis on the RHS is smaller in magnitude; α_c (the productive effect of synthetic compounds on crop yields) is lower in magnitude, P_j is smaller in magnitude (but still satisfies $P_j^{-1} \leq \alpha_c$, as required by our upper bound constraint on K); α_b (soil bacteria's productive effect on crop yields) is greater in magnitude; γ_{cc} (the quadratic part of synthetic compounds' effects on soil bacteria) is more negative; $\overline{C}$ (soil's ability to absorb synthetic compounds without becoming infertile) is more positive; and γ_K (the benefit of the stock of clean soil to soil bacterias' health) is more positive.

$$\frac{\partial \hat{K}_j}{\partial \rho} = \frac{\overbrace{\gamma_K}^{\geq 0}}{\mu \underbrace{\gamma_{cc}(\mu + \rho)^2}_{\leq 0}} \tag{A.55}$$

So we see that the effect that the interest rate ρ has on the stock of clean soil at the stationary solution depends on the sign of μ. As seen above, in order for $K \to \hat{K}_j$, we must have $\mu \geq 0$. When $\mu \geq 0$, such that the stock of synthetic chemicals decays on its own, we have then $\frac{\partial \hat{K}_j}{\partial \rho} \leq 0$, and the stock of clean soil at the stationary solution decreases as the interest rate increases. This is as expected, since as the payoff from the best alternative investment increases, we would expect the farmer to invest a greater amount in the best alternative investment, and therefore a lesser amount in the stock of clean soil.

Note that $\frac{\partial \hat{K}_j}{\partial \rho}$ becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when the benefit of the clean soil stock to the soil microbiome (γ_K) is greater. This is likely because when γ_K is greater the farmer requires less clean soil in order to have a desired positive effect of any given size on the soil bacteria. $\frac{\partial \hat{K}_j}{\partial \rho}$ also becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when the quadratic part of synthetic compound use's effect on soil bacteria (γ_{cc}) is less detrimental (such that γ_{cc} is smaller in magnitude). This is because as γ_{cc} grows smaller in magnitude, not investing in the stock of clean soils (by increasing per-period synthetic compound use) becomes less costly, allowing the farmer to forgo greater amounts of K as the outside option becomes more attractive. $\frac{\partial \hat{K}_j}{\partial \rho}$ becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, as $\mu \geq 0$ becomes smaller. This is as expected, since as soils become less able to clean themselves up on their own it becomes more costly for the farmer to achieve a stock of clean soils of any given size, since the farmer no longer benefits from as much "free capital" as the capital stock's ability to grow on its own shrinks. Finally $\frac{\partial \hat{K}_j}{\partial \rho}$ is more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when ρ is smaller in magnitude. Thus the effect of increasing ρ is higher when ρ is smaller.

$$\frac{\partial \hat{K}_j}{\partial \overline{C}} = 1 \tag{A.56}$$

A one unit increase in the soil's ability to tolerate synthetic compounds results in a one unit increase in the stock of clean soil, or a one unit decrease in the stock of synthetic compounds, at the stationary solution. This result stems from the fact that the amount of clean soil is defined as the difference between the soil's ability to tolerate synthetic compounds and the actual stock of dirty soil. Therefore increasing the soils ability to tolerate synthetics, even while keeping actual stocks of synthetic compounds fixed, will by definition result in an 1 to 1 increase in the stock of clean soil.

$$\frac{\partial \hat{K}_j}{\partial \alpha_b} = \frac{\underbrace{\frac{1}{P_{\mathrm{j}}} - \alpha_c}_{\leq 0\, by\, hypothesis}}{\mu \underbrace{\alpha_b^2 \gamma_{cc}}_{\leq 0}} \tag{A.57}$$

We see that soil bacteria's effect α_b on crop production has an ambiguous effect on the stock $\hat{K}_j$ of clean soil at the stationary solution. Its effect is mediated by crop prices P_{j}, the effect α_c of per-period synthetic compound application has on crop production, and the rate μ at which synthetic compounds decompose from soils on their own. As seen above, in order for $K \to \hat{K}_j$, we must have $\mu \geq 0$. When the stock of synthetic compounds decays on its own ($\mu \geq 0$) we are more likely to have $\frac{\partial \hat{K}_j}{\partial \alpha_b} \geq 0$ such that the stock of clean soils at the stationary solution increases as soil bacteria become more important to production, when crop prices P_{j} are higher, since a greater crop price incentivizes the farmer to produce more, and since as α_b increases they are better able to produce more by increasing their stock of clean soils. The stock of clean soils at the stationary solution also increases as soil bacteria become more important to production when the effect α_c of per-period synthetic compound use on production is higher, since then a farmer does not have to apply as much fertilizer or pesticide in order to produce at any given level of production.

Given a large enough α_c so that $\frac{1}{P_{\mathrm{j}}} - \alpha_c \leq 0$, then $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ is also more positive when the effect α_b of soil bacteria on crop production is smaller in magnitude. Intuitively, this means that when synthetic compounds are sufficiently important to production, then the stock of clean soil at the stationary solution increases, but at a diminishing rate, as soil bacteria's effect on production increases. This diminishing nature of $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ arises because as α_b increases the farmer does not have to increase the stock of clean soil by as much in order to achieve a given level of productivity gain from their soil bacteria. The farmer has an incentive not to increase $\hat{K}_j$ by too much, because this would mean losing out on the productive effects of their synthetic compounds, which are significant, since we have assumed α_c to be large.

Given a large enough α_c so that $\frac{1}{P_{\mathrm{j}}} - \alpha_c \leq 0$, $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ is also more positive when the effect γ_{cc} of chemical inputs on soil microbe production is smaller in magnitude. Thus, when chemical inputs have less of a detrimental effect γ_{cc} on soil microbe production, then the larger the effect α_b that soil microbes have on crop production, the higher the stock of clean soils at the stationary solution. This is because the grower can still benefit from the productive effects of chemical inputs without harming soil microbe production as much, and also benefit from the productive effects of soil microbes.

We hypothesize that the stock of synthetic compounds decays on its own ($\mu \geq 0$), and that the effect of per-period synthetic compound use on production (α_c) is high, such that the stock of clean soils at the stationary solution will increase as soil bacteria become more important to production ($\frac{\partial \hat{K}_j}{\partial \alpha_b} \geq 0$). $\frac{\partial \hat{K}_j}{\partial \alpha_b} \geq 0$ makes sense intuitively since, all else equal, if soil bacteria become more important for production, and if clean soil stocks benefit soil bacteria, then farmers will want to increase their stock of clean soils so as to make better use of their soil bacteria.

$$\frac{\partial \hat{K}_j}{\partial \alpha_c} = \frac{1}{\mu \underbrace{\alpha_b \gamma_{cc}}_{\leq 0}} \tag{A.58}$$

The effect that the effect α_c of per-period synthetic compound use has on crop production has on the stock of clean soil at the stationary solution depends on the rate μ at which synthetic compounds decompose from the soil. As seen above, in order for $K \to \hat{K}_j$, we must have $\mu \geq 0$. When the stock of synthetic compounds decays on its own ($\mu \geq 0$) then $\frac{\partial \hat{K}_j}{\partial \alpha_c} \leq 0$ and the stock of clean soil at the stationary solution decreases as the effect that per-period synthetic compound use has on crop production increases. This makes intuitive sense, since if, all else equal, fertilizers and pesticides yield greater productivity boosts the farmer will choose to use more of these compounds. This results in a smaller stock of clean soils. Note that $\frac{\partial \hat{K}_j}{\partial \alpha_c}$ becomes less negative as α_b increases in value. This makes sense, since if soil bacteria, which depend on the stock of clean soils, are more important to crop production, then we should be less willing to erode our stock of clean soils as synthetic compounds become more productive. Note also that $\frac{\partial \hat{K}_j}{\partial \alpha_c}$ becomes less negative as γ_{cc} becomes more negative. Again, this makes sense, since if per-period application of synthetic compounds becomes more detrimental to soil bacteria, which help crop production, then we should be less willing to apply synthetic compounds, and thus erode our stock of clean soils, as synthetic compounds become more productive.

$$\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} = \frac{1}{\mu \gamma_{cc}^2} \left(\frac{1}{\underbrace{\alpha_b}_{\geq 0}} \left(\frac{1}{P_\mathrm{j}} - \alpha_c \right) + \frac{\overbrace{\gamma_K}^{\geq 0}}{(\mu + \rho)} - \underbrace{\gamma_c}_{\leq 0} \right) \tag{A.59}$$

The effect on the stock of clean soil at the stationary solution of the quadratic part γ_{cc} of the effect that per-period synthetic compound use has on soil bacteria production is ambiguous.

We hypothesize that $\mu \geq 0$, that α_c is relatively large, and therefore that we will have $\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} \leq 0$. $\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} \leq 0$ makes sense intuitively, because if per-period application of synthetic compounds becomes less harmful to soil bacteria, which are themselves beneficial to crop production, then in any given period farmers will face less of an incentive not to use synthetic compounds, and will therefore choose to apply these compounds at greater rates. All else equal this should, in turn, result in greater stocks of synthetic compound at any given period, including the in the period at which the farmer reaches the stationary solution. If the farmer accumulates a greater stock of synthetic compounds at the stationary solution, by definition they accumulate smaller stocks of clean soils.

When the stock of synthetic compounds decays on its own ($\mu \geq 0$) then we are more likely to have $\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} \leq 0$, which means that our stock of clean soil at the stationary solution increases as the negative effect that synthetic compound use has on soil bacteria production becomes more convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes more negative), when crop prices (P_j) are higher, because a farmer is incentivized to produce more, and because a higher stock of clean soil is needed to offset the more convex costs of chemical input use on soil bacteria production, and both chemical input use and soil bacteria are important for crop production.

Our stock of clean soil at the stationary solution is also more likely to increase as the negative effect that synthetic compound use has on soil bacteria production becomes more convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes more negative) when the effect α_c that synthetic compound use has on crop production is higher, because then we can use less of our synthetic compounds while still having the desired effect on production.

Our stock of clean soil at the stationary solution is also more likely to increase as the negative effect that synthetic compound use has on soil bacteria production becomes more convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes more negative) when the effect γ_K that clean soil stocks have on soil bacteria health is lower, because then we need a greater stock of clean soils to achieve a desired level of effect on soil bacteria.

Our stock of clean soil at the stationary solution is also more likely to increase as the negative effect that synthetic compound use has on soil bacteria production becomes more convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes more negative) when the rate at which synthetic compounds decompose on their own (μ) is higher, since the natural decay of synthetic compounds offsets its use in the evolution of the stock of clean soil.

Our stock of clean soil at the stationary solution is also more likely to decrease as the negative effect that synthetic compound use has on soil bacteria production becomes less convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes less negative) when the interest rate (ρ) is higher, since farmers will care less about the future and in any given period farmers will face less of an incentive not to use synthetic compounds, and will therefore choose to apply these compounds at greater rates, leading to a lower stock of clean soil at the stationary solution.

Our stock of clean soil at the stationary solution is also more likely to decrease as the negative effect that synthetic compound use has on soil bacteria production becomes less convex (i.e., as the quadratic part γ_{cc} of the effect that synthetic compound use has on soil bacteria production becomes less negative) when the linear part γ_c of the detrimental effect that per-period synthetic compound use has on soil bacteria production is smaller in magnitude, since in any given period farmers will face less of an incentive not to use synthetic compounds, and will therefore choose to apply these compounds at greater rates, leading to a lower stock of clean soil at the stationary solution.

$$\frac{\partial \hat{K}_j}{\partial \gamma_c} = \frac{1}{\mu \underbrace{\gamma_{cc}}_{\leq 0}} \tag{A.60}$$

The effect on the stock of clean soil at the stationary solution of the linear part γ_c of the effect that per-period synthetic compound use has on soil bacteria depends on the rate at which synthetic compounds decompose from the soil. When the stock of synthetic compounds decays on its own ($\mu \geq 0$) then $\frac{\partial \hat{K}_j}{\partial \gamma_c} \leq 0$ and the stock of clean soil at the stationary solution decreases as per-period synthetic compound use becomes less detrimental to soil bacteria health.

$\frac{\partial \hat{K}_j}{\partial \gamma_c} \leq 0$ makes sense intuitively, because if per-period application of synthetic compounds becomes less harmful to soil bacteria, which are themselves beneficial to crop production, then in any given period farmers will face less of an incentive not to use synthetic compounds, and will therefore choose to apply these compounds at greater rates. All else equal this should, in turn, result in greater stocks of synthetic compound at any given period, including the in the period at which the farmer reaches the stationary solution. If the farmer accumulates a greater stock of synthetic compounds at the stationary solution, by definition they accumulate smaller stocks of clean soils.

A.4 Unconstrained Solution for Stage j when $\hat{K}_j$ Exists: Deriving using Taylor series expansion

We now solve for the farmer's optimal stage j trajectories.

We start by solving for the unconstrained solution for each stage j by using second-order Taylor series approximations of the net gain function $G(K,I)$. Since the net gain function $G(K,I)$ is quadratic, these second-order Taylor series approximations and the solutions derived using them are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K,I)$ is an exact second-order Taylor series expansion of the net gain function $G(K,I)$.

We then solve for the constrained optimal solution for each stage j by solving for an exact solution via direct derivation.

When $\hat{K}_j$ exists, we can write a Taylor Series approximation of $G(K,I)$ around the stationary solution $\left(\hat{K}_j, 0\right)$, and use this approximation to solve for K^* and I^*.

The net gain function $G(K,I)$ is quadratic, so the second-order Taylor series approximation and the solutions derived using it are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K,I)$ is an exact second-order Taylor series expansion of the net gain function $G(K,I)$.

We define the following values:

$$G_1 = \frac{\partial G(\hat{K}_j, 0)}{\partial K} = -\mu\left(P_j \cdot \left(\alpha_b\left(\gamma_{cc}\mu\left(\overline{C} - \hat{K}_j\right) + \gamma_c\right) + \alpha_c\right) - 1\right) + P_j \cdot \alpha_b\left(\gamma_K\right) \tag{A.61}$$

$$G_2 = \frac{\partial G(\hat{K}_j, 0)}{\partial I} = -\left(P_j \cdot \left(\alpha_b\left(\gamma_{cc}\mu\left(\overline{C} - \hat{K}_j\right) + \gamma_c\right) + \alpha_c\right) - 1\right) \tag{A.62}$$

$$G_{11} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial K^2} = P_j \cdot \alpha_b\left(\gamma_{cc}\mu^2\right) \tag{A.63}$$

$$G_{22} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial I^2} = P_j\alpha_b\gamma_{cc} \tag{A.64}$$

$$G_{12} = G_{21} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial K \partial I} = P_j \cdot \alpha_b\mu\gamma_{cc} \tag{A.65}$$

With the first and second order partials of G in now hand, the second order Taylor series approximation of the gain function $G(K,I)$ around a stationary solution $(\hat{K}_j, 0)$ can be written as:

$$G(K,I) \approx G(\hat{K}_j,0)+G_1\cdot(K-\hat{K}_j)+G_2\cdot(I-0)+G_{11}\cdot\frac{(K-\hat{K}_j)^2}{2}+G_{22}\cdot\frac{(I-0)^2}{2}+G_{12}\cdot(K-\hat{K}_j)\cdot(I-0) \tag{A.66}$$

With this Taylor series approximation of the gain function we can, in turn, derive an explicit closed-form solution to the second-order approximation of the optimal control problem for $K(t)$ and $I(t)$. To this end, we begin by noting that the Hamiltonian can now be approximated as:

$$H = G(\hat{K}_j,0)+G_1\cdot(K-\hat{K}_j)+G_2\cdot(I)+G_{11}\cdot\frac{(K-\hat{K}_j)^2}{2}+G_{22}\cdot\frac{(I)^2}{2}+G_{12}\cdot(K-\hat{K}_j)\cdot(I)+pI \tag{A.67}$$

Where p is the shadow price of capital. Letting

$$X = K - \hat{K}_j, \tag{A.68}$$

we can re-write the Hamiltonian as

$$H = G(\hat{K}_j,0) + G_1\cdot X + G_2\cdot I + G_{11}\cdot\frac{X^2}{2} + G_{22}\cdot\frac{I^2}{2} + G_{12}\cdot X\cdot I + pI. \tag{A.69}$$

The first criteria of the maximum principle,

$$\frac{\partial H}{\partial I} = 0 \tag{A.70}$$

implies

$$G_2 + G_{22}\cdot I + G_{12}\cdot X + p = 0 \tag{A.71}$$

So that

$$\widetilde{I}(X,p) = -\frac{(G_2 + G_{12}\cdot X + p)}{G_{22}} \tag{A.72}$$

From this expression for the maximized investment value we get the maximized Hamiltonian:

$$\widetilde{H} = G(\hat{K}_j,0) + G_1\cdot X + G_2\cdot\widetilde{I}(X,p) + G_{11}\cdot\frac{X^2}{2} + G_{22}\cdot\frac{\widetilde{I}(X,p)^2}{2} + G_{12}\cdot X\cdot\widetilde{I}(X,p) + p\widetilde{I}(X,p) \tag{A.73}$$

In preparation for examining the second criteria of the maximum principle, we'll first derive expressions for $\frac{\partial \widetilde{H}}{\partial K}$, $p(t)$, $\dot{p}(t)$, and ρ. Beginning with $\frac{\partial \widetilde{H}}{\partial K}$, note that $\frac{\partial \widetilde{H}}{\partial K} = \frac{\partial \widetilde{H}}{\partial X} \cdot \frac{\partial X}{\partial K} = \frac{\partial \widetilde{H}}{\partial X}$, since $\frac{\partial X}{\partial K} = \frac{\partial (K - \hat{K}_j)}{\partial K} = 1$. Then we have

$$\frac{\partial \widetilde{H}}{\partial K} = G_1 + G_2 \cdot \frac{\partial \widetilde{I}(X,p)}{\partial X} + G_{11} \cdot X + G_{22} \cdot \widetilde{I}(X,p) \frac{\partial \widetilde{I}(X,p)}{\partial X} + G_{12} \cdot \widetilde{I}(X,p) + G_{12} \cdot X \cdot \frac{\partial \widetilde{I}(X,p)}{\partial X} + p \frac{\partial \widetilde{I}(X,p)}{\partial X} \tag{A.74}$$

or

$$\frac{\partial \widetilde{H}}{\partial K} = G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p) + [\frac{G_2 + G_{12} \cdot X + p}{G_{22}} + \widetilde{I}(X,p)] \frac{\partial \widetilde{I}(X,p)}{\partial X} \cdot G_{22} \tag{A.75}$$

But from (A.72) we know that $\frac{G_2 + G_{12} \cdot X + p}{G_{22}} + \widetilde{I}(X,p) = 0$. So we have

$$\frac{\partial \widetilde{H}}{\partial K} = G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p) \tag{A.76}$$

To get our expressions for $p(t)$, and $\dot{p}(t)$ we solve (A.72) for $p(t)$:

$$p(t) = -(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \tag{A.77}$$

and taking the time derivative of (A.77) we get

$$\dot{p}(t) = -G_{22} \cdot \dot{I}(t) - G_{12} \cdot \dot{X}(t) \tag{A.78}$$

And to get an expression for ρ remember that a stationary solution must satisfy

$$R(\hat{K}_j) = \rho \tag{A.79}$$

or

$$\frac{-\frac{\partial G(\hat{K}_j,0)}{\partial K}}{\frac{\partial G(\hat{K}_j,0)}{\partial I}} = \rho \tag{A.80}$$

so

$$\rho = -\frac{G_1}{G_2} \tag{A.81}$$

Now, we continue by examining the second criteria of the maximum principle. Remember this criteria requires that:

$$\dot{p}(t) = -\frac{\partial \widetilde{H}}{\partial K} + \rho p(t) \tag{A.82}$$

Plugging in (A.76), (A.77), and (A.78) gives:

$$-G_{22} \cdot \dot{I}(t) - G_{12} \cdot \dot{X}(t) = \tag{A.83}$$

$$-(G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) - \rho(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \tag{A.84}$$

or

$$G_{22} \cdot \dot{I}(t) + G_{12} \cdot \dot{X}(t) = \tag{A.85}$$

$$(G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) + \rho(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \tag{A.86}$$

Note that $\dot{X} = \frac{d(K-\widehat{K})}{dt} = \frac{d(K)}{dt} = \dot{K} = I$. Evaluating at $I = \widetilde{I}$ gives

$$G_{22} \cdot \dot{\widetilde{I}}(t) + G_{12} \cdot \tilde{I}(t) = \tag{A.87}$$

$$(G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) + \rho(G_2 + G_{22} \cdot \widetilde{I}(t) + G_{12} \cdot X(t)) \tag{A.88}$$

plugging (A.81) into the RHS and simplifying then gives

$$G_{22} \cdot \dot{\widetilde{I}}(t) = G_{11} \cdot X + \rho G_{22} \cdot \tilde{I}(t) + \rho G_{12} \cdot X(t) \tag{A.89}$$

or

$$\dot{\widetilde{I}}(t) - \rho\tilde{I}(t) - \frac{G_{11} \cdot X + \rho G_{12} \cdot X(t)}{G_{22}} = 0 \tag{A.90}$$

But remembering

$$\tilde{I}(t) = \frac{\partial K}{\partial t} = \frac{\partial(K - \widehat{K})}{\partial t} = \frac{\partial X}{\partial t} \tag{A.91}$$

and that therefore also

$$\dot{\tilde{I}}(t) = \frac{\partial \tilde{I}(t)}{\partial t} = \frac{\partial^2 X}{\partial t^2} \tag{A.92}$$

we can rewrite (A.90) as the following second-order linear ODE

$$\frac{\partial^2 X(t)}{\partial t^2} - \rho \frac{\partial X(t)}{\partial t} - \frac{G_{11} + \rho G_{12}}{G_{22}} X(t) = 0 \tag{A.93}$$

We guess a solution to (A.93) of the form:

$$X(t) = X(0)e^{-at} \tag{A.94}$$

then (A.93) becomes

$$X(0)e^{-at}a^2 + \rho X(0)e^{-at}a - \frac{G_{11} + \rho G_{12}}{G_{22}} X(0)e^{-at} = 0 \tag{A.95}$$

Note that if we assume a finite t and a, and that $K_{0j} \neq \widehat{K}$ so that $X(0) \neq 0$, then we can divide both sides of the above equation by $X(0)e^{-at}$ to get

$$a^2 + \rho a - \frac{G_{11} + \rho G_{12}}{G_{22}} = 0 \tag{A.96}$$

Applying the quadratic formula, we get the following solution for a:

$$a = \sqrt{\left(\frac{\rho}{2}\right)^2 + \frac{G_{11} + \rho G_{12}}{G_{22}}} - \frac{\rho}{2} \tag{A.97}$$

(we take the positive root since we want $X(t) = K(t) - \hat{K}_j$ to converge to 0 as $K \to \hat{K}_j$.)

$$a = \sqrt{\left(\frac{\rho}{2}\right)^2 + \mu^2 + \rho\mu} - \frac{\rho}{2} \tag{A.98}$$

$$a = \sqrt{\left(\frac{\rho}{2} + \mu\right)^2} - \frac{\rho}{2} \tag{A.99}$$

$$a = \left(\frac{\rho}{2} + \mu\right) - \frac{\rho}{2} \tag{A.100}$$

$$a = \mu \tag{A.101}$$

The rate of approach to the stationary solution equals the rate at which synthetic compounds decompose from the farmer's soils. Since the speed of approach $a \geq 0$ in order for $X(t) = K(t) - \hat{K}_j$ to converge to 0 as $K \rightarrow \hat{K}_j$, this means that, in order for $K \rightarrow \hat{K}_j$, we must have $\mu \geq 0$. Moreover, $\frac{\partial a}{\partial \mu} = 1 > 0$, which means that the farmer will reach the stationary solution faster when the rate of decay is greater.

Given our solution for a we can now solve for the optimal policy for capital by rearranging equation (A.94) as follows:

$$X(t) = X(0)e^{-at} \tag{A.102}$$

$$K(t) - \hat{K}_j = (K_{0_{S_j}} - \hat{K}_j)e^{-at} \tag{A.103}$$

$$K_j(t) = \hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-at} \tag{A.104}$$

$$K_j(t) = \hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-\mu t} \tag{A.105}$$

We get our optimal policy for investment by taking the time derivative of the equation above and simplifying as follows:

$$I_j(t) = \frac{dK_j(t)}{dt} = -a(K_{0_{S_j}} - \hat{K}_j)e^{-at} \tag{A.106}$$

$$I_j(t) = -a(\hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-at} - \hat{K}_j) \tag{A.107}$$

$$I_j(t) = -a(K(t)_{org} - \hat{K}_j) \tag{A.108}$$

$$I_j(t) = a(\hat{K}_j - K_j(t)) \tag{A.109}$$

$$I_j(t) = \mu(\hat{K}_j - K_j(t)) \tag{A.110}$$

Since our gain function is quadratic, our Taylor series approximation of the farmer's interior solution is the same as the exact interior solution. Therefore our unconstrained Taylor series approximation will be the same as our unconstrained exact solution.

In the next section we derive the exact solution and also apply the upper and lower bound constraints that the farmer faces on $I(t)$. These are the same constrained trajectories that we would get from applying the upper and lower bound constraints on investment to our Taylor series approximation of the farmer's interior solution, since, again, we assume our gain function to be quadratic, and therefore our Taylor series approximation is exact.

A.5 Solution for Stage j when $\hat{K}_j$ Exists: Directly Deriving Exact Solution and Imposing Lower and Upper Bound Constraints

We also derive the exact solution directly when $\hat{K}_j$ exists, show that the exact unconstrained solution is the same as what we derived using a second-order Taylor series approximation, and then impose the lower and upper bound constraints on $I(t)$.

The net gain function $G(K, I)$ is quadratic, so the second-order Taylor series approximation and the solutions derived using it are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K, I)$ is an exact second-order Taylor series expansion of the net gain function $G(K, I)$.

The upper-bound constraint $M(K)$ on net investment I comes from constraint that chemical input use $c(t)$ is non-negative.

The lower-bound constraint $m(K)$ on net investment I comes from upper bound $\bar{c}$ to chemical input use $c(t)$. The upper bound $\bar{c}$ to chemical input use $c(t)$ may depend on the total stock of synthetic compounds present in the soil $C(t)$, and which may represent, for example, the maximum recommended dose for any given application; the maximum chemical input dose that is not lethal to crops and/or humans; the maximum chemical dose above which consumers will no longer purchase the crop; and/or the maximum chemical input flow at any point in time that does not destroy the farmer's land and soil.

We assume that $\bar{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$ and $\bar{c} > 0$ when $\mu(X) = 0$. Since investment is bounded from below by $\mu(X)(\overline{C} - K(t)) - \bar{c}$, the trajectory for net investment must satisfy:

$$\begin{cases} I(t) \geq -\mu(X)K(t) & \text{if } \mu(X) > 0 \\ I(t) > \mu(X)(\overline{C} - K(t)) & \text{if } \mu(X) = 0 \end{cases} \tag{A.111}$$

Our derivation of the exact solution is as follows:

$$\begin{aligned} H =& P_j \cdot f\left((g(K,I)), \left(\mu(\overline{C} - K) - I\right)\right) - \left(\mu(X)(\overline{C} - K) - I\right) + pI && \text{(A.112)} \\ &+ \lambda_1 \left(I - \left(\mu(\overline{C} - K) - \bar{c}\right)\right) + \lambda_b \left(\mu(\overline{C} - K) - I\right) && \text{(A.113)} \\ & && \text{(A.114)} \end{aligned}$$

where

$$\begin{aligned} \lambda_1 \left(I - \left(\mu(\overline{C} - K) - \bar{c}\right)\right) = 0 \\ \lambda_b \left(\mu(\overline{C} - K) - I\right) = 0 \end{aligned}$$

A.5.1 Interior solution for $I(t)$

When $I(t)$ is interior we will have $\lambda_1 = 0$ and $\lambda_b = 0$. In this case the Maximum Principle will yield:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}(-1)\right) + 1 + p = 0$$

$$-\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} - \frac{\partial f}{\partial c}\right) + 1\right) = p \tag{A.115}$$

and

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}\left(-\mu\right)\right) + \mu\right)$$

so that

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu\right) + \rho p(t) \tag{A.116}$$

and the transversality condition requires that

$$\lim_{t\to\infty} p(t)K^*(t)e^{-\rho t} = 0 \tag{A.117}$$

With

$$f(b,c) = \alpha_b b + \alpha_c c + A_y$$

so that $\frac{\partial f}{\partial b} = \alpha_b$ and $\frac{\partial f}{\partial c} = \alpha_c$, and

$$g(K,I) = \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_c\left(\mu\left(\overline{C} - K\right) - I\right) + \gamma_K K + A_b$$

so that

$$\frac{\partial g(K,I)}{\partial K} = \left(-\mu\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\mu + \gamma_K\right)$$

and

$$\frac{\partial g(K,I)}{\partial I} = \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right)$$

the first two conditions of the maximum principle can be re-expressed as follows:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}\left(-1\right)\right) + 1 + p = 0$$

$$-\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} - \frac{\partial f}{\partial c}\right) + 1\right) = p \tag{A.118}$$

$$\left(-\left(P_j \cdot \left(\alpha_b\left(-\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\right)-\alpha_c\right)+1\right)\right)=p \tag{A.119}$$

and

$$\dot{p}(t)=-\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K}-\mu \cdot \frac{\partial f}{\partial c}\right)+\mu\right)+\rho p(t) \tag{A.120}$$

$$\dot{p}(t)=-\left(P_j \cdot \left(\alpha_b\left(-\mu\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\mu+\gamma_K\right)-\mu \cdot \alpha_c\right)+\mu\right)+\rho p(t) \tag{A.121}$$

We solve this system of equations as follows. Note that taking the time derivative of the first condition of the maximum principle we get

$$\dot{p}=-P_j \cdot \alpha_b\gamma_{cc}\left(\mu\dot{K}+\dot{I}\right) \tag{A.122}$$

We can substitute this new identity for $\dot{p}$, as well as our identity for p from the first condition of the maximum principle, into our expression for $\dot{p}$ from the second condition of the maximum principle, we get the following second order differential equation for $K(t)$:

$$(\mu+\rho)\,\mu\left(K(t)-\hat{K}_j\right)+\rho \cdot \dot{K}(t)-\ddot{K}(t)=0 \tag{A.123}$$

This is a second order differential equation with solution:

$$K(t)=c_1e^{-\mu \cdot t}+c_2e^{(\mu+\rho)\cdot t}+\hat{K}_j \tag{A.124}$$

Our initial condition requires that:

$$K_{0j}=c_1e^{-\mu\cdot 0}+c_2e^{(\mu+\rho)\cdot 0}+\hat{K}_j=K_{0j}$$

$$c_1+c_2+\hat{K}_j=K_{0j}$$

$$c_1 + c_2 = K_{0j} - \hat{K}_j \tag{A.125}$$

Note that we can take the time derivative of our preliminary solution for $K(t)$ to derive the following preliminary equation for $\dot{K}(t)$:

$$\dot{K}(t) = -\mu \cdot c_1 e^{-\mu \cdot t} + (\mu + \rho) \cdot c_2 e^{(\mu+\rho) \cdot t}$$

$$K(t) = c_1 e^{-\mu \cdot t} + c_2 e^{(\mu+\rho) \cdot t} + \hat{K}_j$$

This, combined with this:

$$c_1 + c_2 = K_{0j} - \hat{K}_j \tag{A.126}$$

$$c_2 = K_{0j} - \hat{K}_j - c_1 \tag{A.127}$$

gives us:

$$K(t) = \left(c_1 e^{-\mu \cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho) \cdot t} + \hat{K}_j\right)$$

and

$$I(t) = \dot{K}(t) = \left(-\mu \cdot c_1 e^{-\mu \cdot t} + (\mu + \rho) \cdot \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho) \cdot t}\right)$$

Therefore

$$p(t) = \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc} \left(\mu \left(\overline{C} - K\right) - I\right) - \gamma_c\right) - \alpha_c\right) + 1\right)\right) \tag{A.128}$$

can now be written as:

$$p(t) = \left(-P_j \alpha_b \gamma_{cc} (2\mu + \rho) \cdot \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t} - P_j \cdot \left(\alpha_b \left(-\gamma_{cc}\left(\mu\overline{C} - \mu\hat{K}_j\right) - \gamma_c\right) - \alpha_c\right) - 1\right) \tag{A.129}$$

We can use this expression, together with

$$K(t) = \left(c_1 e^{-\mu\cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t} + \hat{K}_j\right)$$

to determine the range of values for c_1 that satisfy the transversality condition:

$$\lim_{t\to\infty} p(t)K^*(t)e^{-\rho t} = 0 \tag{A.130}$$

$$\lim_{t\to\infty} \left(c_1 a \left(K_{0j} - \hat{K}_j - c_1\right) + a\left(K_{0j} - \hat{K}_j - c_1\right)^2 e^{(2\mu+\rho)\cdot t} + a\hat{K}_j \left(K_{0j} - \hat{K}_j - c_1\right) e^{\mu\cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{\mu\cdot t} \cdot b\right) \tag{A.131}$$

with $a = -P_j \alpha_b \gamma_{cc} (2\mu + \rho)$. Assuming $a \neq 0$, then the transversality condition is satisfied if and only if

$$c_1 = \left(K_{0j} - \hat{K}_j\right) \tag{A.132}$$

So the farmer's optimal trajectory is:

$$K(t) = \left(\left(K_{0j} - \hat{K}_j\right) e^{-\mu\cdot t} + \left(K_{0j} - \hat{K}_j - \left(K_{0j} - \hat{K}_j\right)\right) e^{(\mu+\rho)\cdot t} + \hat{K}_j\right)$$

$$K(t) = \left(K_{0j} - \hat{K}_j\right) e^{-\mu\cdot t} + \hat{K}_j$$

$$K_j(t) = \hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-\mu t} \tag{A.133}$$

$$I_j(t) = \mu(\hat{K}_j - K_j(t)) \tag{A.134}$$

To derive trajectories for $c(t)$ and $C(t)$ under an interior solution for $I(t)$, we can write:

$$c(K, I) = \mu\left(\overline{C} - K(t)\right) - I(t) \tag{A.135}$$

$$c(K, I) = \mu\left(\overline{C} - K(t)\right) - \mu\left(\hat{K}_j - K(t)\right)$$

$$c(K, I) = \mu\left(\overline{C} - \hat{K}_j\right) \forall t \geq 0, \tag{A.136}$$

and

$$C(K, I) = \overline{C} - K(t) \tag{A.137}$$

$$C(K, I) = \overline{C} - \hat{K}_j - \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}. \tag{A.138}$$

We also have the following expressions for soil microbes $b(t)$ and crop output $y(t)$ under an interior solution for $I(t)$:

$$g(K, I; X) = \gamma_c(X)c + \frac{1}{2}\gamma_{cc}(X)c^2 + \gamma_K(X)K + A_b(X)$$

$$g(K, I; X) = \left(\gamma_c\mu\left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K\hat{K}_j + A_b + \gamma_K\left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}\right), \tag{A.139}$$

and

$$\tilde{f}(b, c; X) = \alpha_b(X)b + \alpha_c(X)c + A_y(X) \tag{A.140}$$

$$\tilde{f}(b,c;X) = \alpha_b\left(\gamma_c\mu\left(\overline{C}-\hat{K}_j\right)+\frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C}-\hat{K}_j\right)\right)^2+\gamma_K\hat{K}_j+A_b\right) \\ +\alpha_c\mu\left(\overline{C}-\hat{K}_j\right)+A_y+\alpha_b\gamma_K\left(K_{0j}-\hat{K}_j\right)\cdot e^{-\mu\cdot t} \tag{A.141}$$

A.5.2 Lower corner solution for $I(t)$

On the other hand if $I(t)$ has a lower corner solution (such that the lower bound constraint on $I(t)$ binds, but the upper bound constraint does not) we will have $\lambda_1 \geq 0$ and $\lambda_b = 0$.

In this case, the Hamiltonian is given by:

$$H = P_j\cdot f\left(\left(g(K,I)\right),\left(\mu(\overline{C}-K)-I\right)\right)-\left(\mu(X)(\overline{C}-K)-I\right)+pI \tag{A.142}$$

$$+\lambda_1\left(I-\left(\mu(\overline{C}-K)-\overline{c}\right)\right) \tag{A.143}$$

$$\tag{A.144}$$

The Maximum Principle will then yield:

[#1]:

$$\frac{\partial H}{\partial I} = P_j\cdot\left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I}+\frac{\partial f}{\partial c}(-1)\right)+1+p+\lambda_1 = 0 \tag{A.145}$$

$$\Rightarrow P_j\cdot\left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I}+\frac{\partial f}{\partial c}(-1)\right)+1+p = -\lambda_1 \leq 0$$

and

$$\frac{\partial H}{\partial \lambda_1} = \left(I+(\mu-\kappa)K\right) = 0 \tag{A.146}$$

[#2]:

Given:

$$-\frac{\partial H}{\partial K} = -\left(P_j\cdot\left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K}+\frac{\partial f}{\partial c}(-\mu)\right)+\mu+\lambda_1\mu\right) \tag{A.147}$$

[#2] yields:

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu + \lambda_1 \mu\right) + \rho p(t) \tag{A.148}$$

and [#3]: the transversality condition requires that

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0. \tag{A.149}$$

With

$$f(b, c) = \alpha_b b + \alpha_c c + A_y \tag{A.150}$$

so that $\frac{\partial f}{\partial b} = \alpha_b$ and $\frac{\partial f}{\partial c} = \alpha_c$, and

$$g(K, I) = \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_c\left(\mu\left(\overline{C} - K\right) - I\right) + \gamma_K K + A_b \tag{A.151}$$

so that

$$\frac{\partial g(K, I)}{\partial K} = \left(-\mu\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c \mu + \gamma_K\right) \tag{A.152}$$

and

$$\frac{\partial g(K, I)}{\partial I} = \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right) \tag{A.153}$$

the first condition of the maximum principle can be re-expressed as follows:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}(-1)\right) + 1 + p + \lambda_1 = 0 \tag{A.154}$$

$$\Rightarrow -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} - \frac{\partial f}{\partial c}\right) + 1\right) - \lambda_1 = p \tag{A.155}$$

$$\Rightarrow \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right) - \alpha_c\right) + 1\right) - \lambda_1\right) = p \tag{A.156}$$

and

$$\frac{\partial H}{\partial \lambda_1} \Rightarrow -\left(\mu(\overline{C} - K) - \overline{c}\right) = 0. \tag{A.157}$$

We can use (A.157) to find the farmer's optimal trajectories when the lower bound constraint on $I(t)$ binds as follows:

$$I(t) = \left(\mu(\overline{C} - K) - \overline{c}\right) \tag{A.158}$$

Remember that the upper bound constraint on per-period synthetic compound use, $\overline{c}$, is assumed to satisfy the following conditions:

- $\overline{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$
- and $\overline{c} > 0$ when $\mu(X) = 0$

When $\mu(X) = 0$, then $\hat{K}_j$ does not exist, so the case when $\mu(X) = 0$ does not apply when $\hat{K}_j$ exists.

When $\mu \neq 0$ the farmer's optimal constrained trajectory is:

$$K(t) = K(0) \cdot e^{-\mu \cdot t} \, \forall \, t \tag{A.159}$$

$$I(t) = -\mu K(t) \, \forall \, t \tag{A.160}$$

$$c(t) = \overline{c} = \mu \overline{C} \, \forall \, t \tag{A.161}$$

$$C(t) = \overline{C} - K(0) \cdot e^{-\mu \cdot t} \, \forall \, t \tag{A.162}$$

Given $\tilde{g}(C(t), c(t)) = \gamma_c c + \frac{1}{2}\gamma_{cc}c^2 + \gamma_K \left(\overline{C} - C(t)\right) + A_b$:

$$b(t) = \max\{\gamma_c \overline{c} + \frac{1}{2}\gamma_{cc}\overline{c}^2 + \gamma_K \cdot \left(K(0) \cdot e^{-\mu \cdot t}\right) + A_b, \, 0\} \, \forall \, t \tag{A.163}$$

Given $f(c(t), b(t)) = \alpha_c c(t) + \alpha_b b(t) + A_y$:

$$y(t) = \alpha_c \cdot \overline{c} + \alpha_b \cdot b(t)_{LC} + A_y \, \forall \, t \tag{A.164}$$

The lower bound to I binds when the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$). If the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$), this means that the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 at $c = \overline{c}$.

When $\hat{K}_j$ exists, the optimal unconstrained synthetic compound level c_j^{**} is given by $c^{**} = \hat{c}_j$. If $c^{**} = \hat{c}_j > \overline{c}$, this means $\hat{K}_j < 0$. A negative $\hat{K}_j$ means that even at $K = 0$, the marginal net benefit of synthetic compound use at $c = \overline{c}$ is positive.

So a sufficient condition for the farmer to adopt a lower corner solution when $\mu \neq 0$ and $\hat{K}_j$ exists, assuming that $K_{0_j} \neq \hat{K}_j$, is for the marginal net benefit of synthetic compound use to be positive, even when we are accounting for convex costs, and even when those convex costs are evaluated at $c(t) = \overline{c}$.

When the lower bound constraint on investment binds, the multiplier λ_1 on the lower bound constraint is given by:

$$\lambda_1(t) = \underbrace{P_j \cdot \left(\alpha_c + \alpha_b \left(\gamma_{cc} \cdot c(t) + \gamma_c - \frac{1}{(\mu + \rho)}\gamma_K\right)\right) - 1}_{\text{MNB of c(t)}} \tag{A.165}$$

A.5.3 When lower bound on $I(t)$ binds or not

If $\hat{K}_j$ is positive and greater than $K(t)$, then the farmer is *never at the lower corner solution for* $I(t)$. This makes sense since if $\hat{K}_j > K(t)$, then means we will be investing in the stock of clean soil to increase $K(t)$ and approach $\hat{K}_j$ from below.

The lower bound binds when $\hat{K}_j < 0$. A negative $\hat{K}_j$ means that even at $K = 0$, the marginal net benefit of synthetic compound use at $c = \bar{c}$ is positive. In this case the farmer is *always at the lower corner solution for* $I(t)$, and the farmer's capital trajectory converges to $K(t) = 0$ from above.

So a sufficient condition for the farmer to adopt a lower corner solution when $\mu \neq 0$, assuming that $K_{0_j} \neq \hat{K}_j$, is for the marginal net benefit of synthetic compound use to be positive, even when we are accounting for convex costs, and even when those convex costs are evaluated at $c(t) = \bar{c}$.

A.5.4 Upper corner solution for $I(t)$

On the other hand if $I(t)$ has an upper corner solution (such that the upper bound constraint on $I(t)$ binds, but the lower bound constraint does not) we will have $\lambda_1 = 0$ and $\lambda_b \geq 0$. Then the Maximum Principle will yield:

[#1]:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p - \lambda_b = 0 \tag{A.166}$$

$$\Rightarrow P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p = \lambda_b \geq 0 \tag{A.167}$$

and

$$\frac{\partial H}{\partial \lambda_b} = \left(\mu(\overline{C} - K) - I\right) = 0 \tag{A.168}$$

[#2]:

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}(-\mu)\right) + \mu - \lambda_b \mu\right) \tag{A.169}$$

so that

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu - \lambda_b \mu\right) + \rho p(t) \tag{A.170}$$

and [#3]: the transversality condition requires that

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.171}$$

With

$$f(b, c) = \alpha_b b + \alpha_c c + A_y \tag{A.172}$$

so that $\frac{\partial f}{\partial b} = \alpha_b$ and $\frac{\partial f}{\partial c} = \alpha_c$, and

$$g(K, I) = \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_c\left(\mu\left(\overline{C} - K\right) - I\right) + \gamma_K K + A_b \tag{A.173}$$

so that

$$\frac{\partial g(K, I)}{\partial K} = \left(-\mu\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c \mu + \gamma_K\right) \tag{A.174}$$

and

$$\frac{\partial g(K, I)}{\partial I} = \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right) \tag{A.175}$$

the first condition of the maximum principle can be re-expressed as follows:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p - \lambda_b = 0 \tag{A.176}$$

$$\Rightarrow - \left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} - \frac{\partial f}{\partial c} \right) + 1 \right) + \lambda_b = p \tag{A.177}$$

$$\Rightarrow \left(- \left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc} \left(\mu \left(\overline{C} - K \right) - I \right) - \gamma_c \right) - \alpha_c \right) + 1 \right) + \lambda_b \right) = p \tag{A.178}$$

and

$$\mu \left(\overline{C} - K(t) \right) - I(t) = 0 \tag{A.179}$$

We can use (A.179) to find the farmer's optimal trajectories when the upper bound constraint on $I(t)$ binds as follows:

$$\mu \overline{C} - \mu K(t) - I(t) = 0 \tag{A.180}$$

$$\mu K(t) + \dot{K}(t) = \mu \overline{C} \tag{A.181}$$

$$\left(\mu K(t) + \dot{K}(t) \right) \cdot e^{\mu \cdot t} = \mu \overline{C} \cdot e^{\mu \cdot t} \tag{A.182}$$

$$\int_{s=0}^{t} \left(\mu K(s) + \dot{K}(s) \right) \cdot e^{\mu \cdot s} ds = \int_{s=0}^{t} \mu \overline{C} \cdot e^{\mu \cdot s} ds \tag{A.183}$$

$$K(s) \cdot e^{\mu \cdot s} \big]_0^t = \overline{C} \cdot e^{\mu \cdot s} \big]_0^s \tag{A.184}$$

$$K(t) \cdot e^{\mu \cdot t} - K_{0j} = \overline{C} \cdot e^{\mu \cdot t} - \overline{C} \tag{A.185}$$

$$K(t) \cdot e^{\mu \cdot t} = \overline{C} \cdot e^{\mu \cdot t} + K_{0j} - \overline{C} \tag{A.186}$$

$$K(t) = \left(\overline{C} + \left(K_{0j} - \overline{C}\right) e^{-\mu \cdot t}\right) \tag{A.187}$$

$$K(t) = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.188}$$

$$I(t) = \mu \cdot \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.189}$$

$$I(t) = \mu \cdot \left(\overline{C} - K(t)\right) \tag{A.190}$$

A.5.5 When upper bound for $I(t)$ binds or not

Next we compare the interior solution for $I(t)$:

$$I(K(t))_{Int} = \mu(\hat{K}_j - K(t)_{Int}) \tag{A.191}$$

to the upper corner solution for $I(t)$:

$$I(K(t))_{UC} = \mu \cdot \left(\overline{C} - K(t)_{UC}\right) \tag{A.192}$$

in order to determine the conditions under which the interior solution falls above the upper corner solution, at which point the upper bound constraint on $I(t)$ will bind: we examine the following inequality

$$I(K(t))_{UC} < I(K(t))_{Int} \tag{A.193}$$

$$\mu \cdot \left(\overline{C} - K(t)_{UC}\right) < \mu(\hat{K}_j - K(t)_{Int}) \tag{A.194}$$

$$\overline{C} - K(t)_{UC} < \hat{K}_j - K(t)_{Int} \tag{A.195}$$

$$\overline{C} - K(t) < \hat{K}_j - K(t) \tag{A.196}$$

$$\Rightarrow \overline{C} < \hat{K}_j \tag{A.197}$$

Note that if $\overline{C} < \hat{K}_j$, then we have $I(t)_{UC} < I(t)_{Int}$ for all t . In this case the farmer is stuck at the upper corner solution indefinitely. On the other hand if $\overline{C} \geq \hat{K}_j$, then we have $I(t)_{UC} \geq I(t)_{Int}$ for all t . In this case the farmer's investment trajectory will never be constrained by its upper bound, and we will have $\lambda_b = 0 \, \forall t$.

If $\overline{C} < \hat{K}_j$, then we are stuck at the upper corner solution indefinitely such that we will have $\lambda_b(t) \geq 0$ indefinitely, so we are able to use the transversality condition to get more information about $\lambda_b(t)$ and derive the closed form solution above.

A.5.6 Case: Upper bound on $I(t)$ always binds

When $\hat{K}_j > \overline{C}$, the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. In this case, the farmer's optimal solutions take the form:

$$K_j^*(t) = K(t)_{UC,S_j} = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.198}$$

$$I_j^*(t) = I(t)_{UC,S_j} = \mu \cdot \left(\overline{C} - K(t)_{UC,S_j}\right) \tag{A.199}$$

$$c_j^*(t) = 0 \tag{A.200}$$

$$C_j^*(t) = \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.201}$$

$$b_j^*(t) = \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) \tag{A.202}$$

$$y_j^*(t) = \alpha_b \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) + A_y \tag{A.203}$$

We can derive trajectories for $c(t)$ and $C(t)$ as follows:

$$c(K, I) = \mu \left(\overline{C} - K(t)\right) - I(t) \tag{A.204}$$

$$c(K, I) = \mu \left(\overline{C} - K(t)_{UC,S_j}\right) - \mu \left(\overline{C} - K(t)_{UC,S_j}\right) \tag{A.205}$$

$$c(K, I) = 0, \tag{A.206}$$

and

$$C(K, I) = \overline{C} - K(t) \tag{A.207}$$

$$C(K, I) = \overline{C} - \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) \tag{A.208}$$

$$C(K, I) = \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}. \tag{A.209}$$

We can derive trajectories for soil microbes $b(t)$ and output $y(t)$ as follows:

$$g(K, I; X) = \gamma_c(X)c + \frac{1}{2}\gamma_{cc}(X)c^2 + \gamma_K(X)K + A_b(X) \tag{A.210}$$

$$g(K, I; X) = \left(\gamma_K(X) \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b(X)\right) \tag{A.211}$$

$$b(t) = \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right), \tag{A.212}$$

and

$$\tilde{f}(b, c; X) = \alpha_b(X)b + \alpha_c(X)c + A_y(X) \tag{A.213}$$

$$\tilde{f}(b, c; X) = \alpha_b(X) \left(\gamma_K(X) \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b(X)\right) + A_y(X) \tag{A.214}$$

$$y(t) = \alpha_b \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) + A_y. \tag{A.215}$$

To determine λ_b:

For the intervals of time over which we have an upper corner solution for $I(t)$, we have a continuous $\lambda_b(t)$. In that we can solve for $\lambda_b(t)$ as follows:

From (A.178) we have:

$$p(t) = \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc} \left(\mu \left(\overline{C} - K(t)\right) - I(t)\right) - \gamma_c\right) - \alpha_c\right) + 1\right) + \lambda_b(t)\right) \tag{A.216}$$

or, given (A.188) and (A.190):

$$p(t) = \left(P_j \cdot (\alpha_b \gamma_c + \alpha_c) - 1 + \lambda_b(t)\right) \tag{A.217}$$

Taking the time derivative of both sides of the equation above we get:

$$\dot{p}(t) = \dot{\lambda}_2(t) \tag{A.218}$$

But from the second condition of the maximum principle, [#2], we have:

$$\dot{p}(t) = -\frac{\partial H}{\partial K} + \rho p(t) \tag{A.219}$$

where

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}(-\mu)\right) + \mu - \lambda_b \mu\right),$$

so that

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu - \lambda_b \mu\right) + \rho p(t). \tag{A.220}$$

Given our assumed functional forms for f and g, the equation above can be written as:

$$\dot{p}(t) = -\left(P_j \cdot \left(\alpha_b \left(-\mu \gamma_{cc} \left(\mu \left(\overline{C} - K\right) - I\right) - \gamma_c \mu + \gamma_K\right) - \mu \cdot \alpha_c\right) + \mu - \lambda_b \mu\right) + \rho p(t) \tag{A.221}$$

We can substitute (A.188), (A.190), (A.217) and (A.218) into the above to get:

$$\begin{aligned}\dot{\lambda}_2(t) = &-\left(P_j \cdot \left(\alpha_b \left(-\mu\gamma_{cc}\left(\mu\left(\overline{C} - \left(\overline{C} + \left(K_{0j} - \overline{C}\right)e^{-\mu\cdot t}\right)\right)\right.\right.\right.\right. \\ &\left.\left.\left.\left. -\mu\cdot\left(\overline{C} - K_{0j}\right)e^{-\mu\cdot t}\right) - \gamma_c\mu + \gamma_K\right) - \mu\cdot\alpha_c\right) + \mu - \lambda_b\mu\right) \\ &+\rho\left(P_j\cdot\left(\alpha_b\gamma_c + \alpha_c\right) - 1 + \lambda_b(t)\right)\end{aligned} \tag{A.222}$$

We simplify the equation above and solve the resulting second order ODE for $\lambda_b(t)$:

$$\lambda_b(t) = \left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j - \overline{C}\right)\left(\cdot e^{(\mu+\rho)\cdot t} - 1\right) + \lambda_b(0)\cdot e^{(\mu+\rho)\cdot t}\right) \tag{A.223}$$

Remember that the transversality condition requires that:

$$\lim_{t\to\infty} p(t)K(t)e^{-\rho t} = 0$$

Note that we can make use of the transversality condition to find $\lambda_b(0)$ because as we previously (later) show(ed), whenever the upper bound on investment binds, it will bind for all $t \geq 0$.

So, substituting $p(t) = \left(P_j\cdot(\alpha_b\gamma_c + \alpha_c) - 1 + \lambda_b(t)\right)$, (A.223), and $K(t)_{UC} = \left(\overline{C} + \left(K_{0j} - \overline{C}\right)\cdot e^{-\mu\cdot t}\right)$ into our transversality condition, we get

$$\begin{aligned}\lim_{t\to\infty}\Bigg(&\overline{C}P_j\alpha_b\left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \gamma_{cc}\mu\left(\hat{K}_j - \overline{C}\right)\right)e^{-\rho t} + \overline{C}\left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j - \overline{C}\right) + \lambda_b(0)\right)\cdot e^{\mu\cdot t} \\ &+P_j\alpha_b\left(K_{0j} - \overline{C}\right)\left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \gamma_{cc}\mu\left(\hat{K}_j - \overline{C}\right)\right)e^{-(\rho+\mu)t} \\ &+\left(K_{0j} - \overline{C}\right)\left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j - \overline{C}\right) + \lambda_b(0)\right)\Bigg) = 0\end{aligned}$$

which is equivalent to:

$$\lim_{t\to\infty}\left(\overline{C}\left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)+\lambda_b(0)\right)\cdot e^{\mu\cdot t}\right.$$
$$\left.+\left(K_{0j}-\overline{C}\right)\left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)+\lambda_b(0)\right)\right)=0$$

which is satisfied if

$$\lambda_b(0)=-P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)$$

So we have

$$\lambda_b(t)=\left(P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)\left(\cdot e^{(\mu+\rho)\cdot t}-1\right)-P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)\cdot e^{(\mu+\rho)\cdot t}\right) \tag{A.224}$$

$$\lambda_b(t)=-P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)\ \forall t\geq 0 \tag{A.225}$$

$$p(t)=\left(P_j\cdot(\alpha_b\gamma_c+\alpha_c)-1-P_j\alpha_b\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)\right)$$

$$p(t)=\alpha_b\left(\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}-\gamma_{cc}\mu\left(\hat{K}_j-\overline{C}\right)\right) \tag{A.226}$$

A.6 Optimal Solution for Stage j When $R(K)$ is Constant Because $\mu=0$

If $\mu=0$ (i.e., synthetic compounds in the soil do not decay on their own) then $R_j(K)$ is a constant (that does not depend on K).

There are 3 possible types of optimal trajectories that arise when $R(K)$ is constant because $\mu=0$, depending on the parameters.

A.6.1 Optimal Trajectories 1: Disinvest as fast as possible to $K=0$

If $\mu=0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1}<\alpha_b\gamma_c+\alpha_c \tag{A.227}$$

as well as the following condition for $R_j(K) < \rho$:

$$P_j^{-1} < \frac{\alpha_b}{\rho+\mu}\left((\rho+\mu)\gamma_c - \gamma_K\right) \tag{A.228}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When $\mu = 0$, if we have a lower corner solution for $I(t)$ (i.e., if $c_j^{**} > \bar{c}$), then

The lower bound to I binds when the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \bar{c}$). If the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \bar{c}$), this means that the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 at $c = \bar{c}$.

The condition $c_j^{**} > \bar{c}$ implies the following when $\mu = 0$:

$$-\frac{P_j \cdot \left(\alpha_c + \alpha_b\left(\gamma_c - \frac{1}{\rho}\cdot\gamma_K\right)\right) - 1}{P_j\alpha_b\gamma_{cc}} > \bar{c} \tag{A.229}$$

$$\Rightarrow = P_j \cdot \alpha_c > -P_j\alpha_b\gamma_{cc}\bar{c} + \frac{P_j\alpha_b\gamma_K}{\mu+\rho} + P_j\alpha_b\left(-\gamma_c\right) + 1 \tag{A.230}$$

In this case our optimal trajectories are as follows:

$$K(t) = \begin{cases} K(0) - \bar{c}\cdot t & t < T(\mu)_{K=0} \\ 0 & t \geq T(\mu)_{K=0} \end{cases} \tag{A.231}$$

$$I(t) = \begin{cases} -\bar{c} & t < T(\mu)_{K=0} \\ 0 & t \geq T(\mu)_{K=0} \end{cases} \tag{A.232}$$

$$c(t) = \begin{cases} \bar{c} & t < T(\mu)_{K=0} \\ 0 & t \geq T(\mu)_{K=0} \end{cases} \tag{A.233}$$

$$C(t) = \begin{cases} \overline{C} - K(0) + \bar{c}\cdot t & t < T(\mu)_{K=0} \\ \overline{C} & t \geq T(\mu)_{K=0} \end{cases} \tag{A.234}$$

$$b(t) = \begin{cases} \max\{\gamma_c\bar{c} + \frac{1}{2}\gamma_{cc}\bar{c}^2 + \gamma_K\left(K(0) - \bar{c}\cdot t\right) + A_b,\, 0\} & t < T(\mu)_{K=0} \\ A_b & t \geq T(\mu)_{K=0} \end{cases} \tag{A.235}$$

$$y(t) = \begin{cases} \alpha_c\bar{c} + \alpha_b b(t) + A_y & t < T(\mu)_{K=0} \\ \alpha_b b(t) + A_y & t \geq T(\mu)_{K=0} \end{cases} \tag{A.236}$$

$$T_{K=0} = \frac{K(0)}{\bar{c}} \tag{A.237}$$

A.6.2 Optimal Trajectories 1': Disinvest to $K = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b\gamma_c + \alpha_c \tag{A.238}$$

as well as the following condition for $R_j(K) < \rho$:

$$P_j^{-1} < \frac{\alpha_b}{\rho+\mu}\left((\rho+\mu)\gamma_c - \gamma_K\right) \tag{A.239}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When $\gamma_{cc} \neq 0$ but $\mu = 0$ the gain function is non-linear in I, and therefore the optimal policy will not be MRA. If the lower corner solution for I does not bind (because $c_j^{**} \leq \bar{c}$), we will have an interior soluion.

The condition $c_j^{**} > \bar{c}$ implies the following when $\mu = 0$:

$$-\frac{P_j\cdot\left(\alpha_c + \alpha_b\left(\gamma_c - \frac{1}{\rho}\cdot\gamma_K\right)\right) - 1}{P_j\alpha_b\gamma_{cc}} > \bar{c} \tag{A.240}$$

$$\Rightarrow= P_j\cdot\alpha_c > -P_j\alpha_b\gamma_{cc}\bar{c} + \frac{P_j\alpha_b\gamma_K}{\mu+\rho} + P_j\alpha_b\left(-\gamma_c\right) + 1 \tag{A.241}$$

If neither constraint on $I(t)$ binds and $I(t)^*$ is interior, the conditions of the Maximum Principle yield:

[#1]:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p = 0 \tag{A.242}$$

$$\Rightarrow p(t) = - \left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 \right) \tag{A.243}$$

[#2]:

$$-\frac{\partial H}{\partial K} = - \left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} + \frac{\partial f}{\partial c} (-\mu) \right) + \mu \right) \tag{A.244}$$

or, given $\mu = 0$,

$$-\frac{\partial H}{\partial K} = -P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} \right) \tag{A.245}$$

so that

$$\dot{p}(t) = -P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} \right) + \rho p(t) \tag{A.246}$$

and [#3]: the transversality condition requires that

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.247}$$

With

$$f(b, c) = \alpha_b b + \alpha_c c + A_y \tag{A.248}$$

so that $\frac{\partial f}{\partial b} = \alpha_b$ and $\frac{\partial f}{\partial c} = \alpha_c$, and

$$g(K, I) = \frac{1}{2} \gamma_{cc} (I)^2 - \gamma_c (I) + \gamma_K K + A_b \tag{A.249}$$

so that

$$\frac{\partial g(K,I)}{\partial K} = \gamma_K \tag{A.250}$$

and

$$\frac{\partial g(K,I)}{\partial I} = (\gamma_{cc} I - \gamma_c) \tag{A.251}$$

the first condition of the maximum principle can be re-expressed as follows:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p = 0 \tag{A.252}$$

$$\Rightarrow -\left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} - \frac{\partial f}{\partial c} \right) + 1 \right) = p \tag{A.253}$$

$$\Rightarrow p(t) = -\left(P_j \cdot \left(\alpha_b \left(\gamma_{cc} \cdot I(t) - \gamma_c \right) - \alpha_c \right) + 1 \right) \tag{A.254}$$

$$\Rightarrow p(t) = -P_j \cdot \left(\alpha_b \left(-\gamma_c \right) - \alpha_c \right) - P_j \alpha_b \gamma_{cc} \cdot I(t) - 1 \tag{A.255}$$

$$\Rightarrow P_j \alpha_b \gamma_{cc} \cdot I(t) = -P_j \cdot \left(\alpha_b \left(-\gamma_c \right) - \alpha_c \right) - 1 - p(t) \tag{A.256}$$

$$\Rightarrow I(t) = \frac{-P_j \cdot \left(\alpha_b \left(-\gamma_c \right) - \alpha_c \right) - 1}{P_j \alpha_b \gamma_{cc}} - \frac{1}{P_j \alpha_b \gamma_{cc}} p(t) \tag{A.257}$$

$$\Rightarrow I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_{cc}} - \frac{1}{P_j \alpha_b \gamma_{cc}} p(t) \tag{A.258}$$

The second condition of the maximum principle can be re-expressed as follows:

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} \right) \right) + \rho p(t) \tag{A.259}$$

$$\dot{p}(t) = -\left(P_j \cdot \left(\alpha_b \gamma_K \right) \right) + \rho p(t) \tag{A.260}$$

Using (A.260), we can solve for $p(t)$ as follows:

$$\dot{p}(t) - \rho p(t) = -P_j \alpha_b \gamma_K \tag{A.261}$$

$$p(t) = \frac{P_j \alpha_b \gamma_K}{\rho} + \left(p(0) - \frac{P_j \alpha_b \gamma_K}{\rho} \right) \cdot e^{\rho \cdot t} \tag{A.262}$$

Substituting (A.262) into (A.258) yields:

$$I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} - \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \cdot e^{\rho \cdot t} \tag{A.263}$$

And integrating the above yields:

$$\begin{aligned} K(t) = &\left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} t - \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \cdot e^{\rho \cdot t} \right. \\ &\left. + \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) + K_{0j} \right) \end{aligned} \tag{A.264}$$

where K_{0j} is given.

We then substitute (A.258) and (A.264) into our transversality condition to see if we can learn more about $p(0)$.

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.265}$$

$$\lim_{t\to\infty}\left(\frac{P_j\alpha_b\gamma_K}{\rho}\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}-\frac{1}{\rho}\gamma_K}{\gamma_{cc}}t+\frac{1}{\rho}\left(\frac{p(0)}{P_j\alpha_b\gamma_{cc}}-\frac{\gamma_K}{\rho\gamma_{cc}}\right)+K_{0j}\right)e^{-\rho t}\right.$$
$$-\frac{P_j\alpha_b\gamma_K}{\rho}\frac{1}{\rho}\left(\frac{p(0)}{P_j\alpha_b\gamma_{cc}}-\frac{\gamma_K}{\rho\gamma_{cc}}\right)$$
$$+\left(p(0)-\frac{P_j\alpha_b\gamma_K}{\rho}\right)\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}-\frac{1}{\rho}\gamma_K}{\gamma_{cc}}t+\frac{1}{\rho}\left(\frac{p(0)}{P_j\alpha_b\gamma_{cc}}-\frac{\gamma_K}{\rho\gamma_{cc}}\right)+K_{0j}\right)e^{-2\rho t}$$
$$\left.-\frac{1}{\rho P_j\alpha_b\gamma_{cc}}\left(p(0)-\frac{P_j\alpha_b\gamma_K}{\rho}\right)^2\cdot e^{-\rho\cdot t}\right)=0 \tag{A.266}$$

Applying l'Hospital's rule we see that this limit is equivalent to:

$$\Rightarrow \lim_{t\to\infty}\left(-\frac{P_j\alpha_b\gamma_K}{\rho}\frac{1}{\rho}\left(\frac{p(0)}{P_j\alpha_b\gamma_{cc}}-\frac{\gamma_K}{\rho\gamma_{cc}}\right)\right)=0 \tag{A.267}$$

So we see that the transversality condition will be satisfied if:

$$\Rightarrow p(0)=\frac{P_j\alpha_b\gamma_K}{\rho} \tag{A.268}$$

Let's make this assumption. Then we have that

$$p(t)=\frac{P_j\alpha_b\gamma_K}{\rho} \tag{A.269}$$

$$I(t)=\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}-\frac{1}{\rho}\gamma_K}{\gamma_{cc}} \tag{A.270}$$

$$I(t)=\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}} \tag{A.271}$$

$$I(t)=\frac{\gamma_K}{\gamma_{cc}}\left(R(K)^{-1}-\rho^{-1}\right) \tag{A.272}$$

$$I(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right) \tag{A.273}$$

and

$$K(t) = \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) \tag{A.274}$$

$$K(t) = \frac{\gamma_K}{\gamma_{cc}} \left(R(K)^{-1} - \rho^{-1}\right) \cdot t + K_{0j} \tag{A.275}$$

$$K(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right) \cdot t + K_{0j} \tag{A.276}$$

In this case the optimal trajectories are as follows:

$$p(t) = \frac{P_j \alpha_b \gamma_K}{\rho} \tag{A.277}$$

$$I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} \tag{A.278}$$

$$I(t) = \frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \tag{A.279}$$

$$I(t) = \frac{\gamma_K}{\gamma_{cc}} \left(R(K)^{-1} - \rho^{-1}\right) \tag{A.280}$$

$$I(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right) \tag{A.281}$$

and

$$K(t) = \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) \tag{A.282}$$

$$K(t) = \frac{\gamma_K}{\gamma_{cc}} \left(R(K)^{-1} - \rho^{-1} \right) \cdot t + K_{0j} \tag{A.283}$$

$$K(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \cdot t + K_{0j} \tag{A.284}$$

However, note that if $R(K)$ is constant (as when $\mu = 0$), and $R(K) \leq \rho$, as we've assumed, then the equation for $K(t)$ that we have derived above implies K is weakly decreasing in t, and *strictly* decreasing when $R(K) < \rho$ and $\gamma_K \neq 0$. When this is the case, $K(t)^*$ will eventually fall below zero, and the farmer will have to switch to a constrained optimal solution so as to prevent K from violating our non-negativity condition. We solve for the first moment at which $K(t)^* = 0$, which we will denote $T(\mu)_{K=0}$, below:

$$K(T(\mu)_{K=0}) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \cdot T(\mu)_{K=0} + K_{0j} = 0 \tag{A.285}$$

$$T(\mu)_{K=0} = \frac{-K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right)} \tag{A.286}$$

$$T(\mu)_{K=0} = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(R(K)^{-1} - \rho^{-1} \right)} \geq 0 \tag{A.287}$$

or

$$T(\mu)_{K=0} = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)} \geq 0 \tag{A.288}$$

So $\forall t \leq T(\mu)_{K=0}$ the farmer adopts the unconstrained optimal solution:

$$K(t)^* = \left(\frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \cdot t + K_{0j} \right) \forall t \leq T(\mu)_{K=0} \tag{A.289}$$

and

$$I(t)^* = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right) \forall t \leq T(\mu)_{K=0}. \tag{A.290}$$

Otherwise, when $t > T(\mu)_{K=0}$ the farmer adopts the constrained optimal solution:

$$K(t) = 0, \forall t > T(\mu)_{K=0} \tag{A.291}$$

and

$$I(t) = 0, \forall t > T(\mu)_{K=0}. \tag{A.292}$$

We therefore have the overall solution:

$$K(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right) \cdot t + K_{0j}, & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.293}$$

and

$$I(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1}\right), & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.294}$$

Given this solution, we can solve for $c(t)$, $C(t)$, $b(t)$, and $y(t)$ as follows:

$$c(t) = \underbrace{\mu}_{=0} \left(\overline{C} - K(t)\right) - I(t) \tag{A.295}$$

$$c(t) = -I(t) \tag{A.296}$$

$$c(t) = \begin{cases} -\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}}, & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.297}$$

or:

$$c(t) = \begin{cases} \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - R(K)^{-1}\right), & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall > T(\mu)_{K=0} \end{cases} \tag{A.298}$$

We solve for $C(t)$ as follows:

$$C(t) = \overline{C} - K(t) \tag{A.299}$$

$$C(t) = \begin{cases} \overline{C} - \left(\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}} t + K_{0j}\right), & \forall t \leq T(\mu)_{K=0} \\ \overline{C}, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.300}$$

We can also write:

$$C(t) = \overline{C} - \left(\frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t + K_{0j}\right), \ \forall t \leq T(\mu)_{K=0} \tag{A.301}$$

$$C(t) = \underbrace{\overline{C} - K_{0j}}_{\equiv C_{0j}} + \frac{\gamma_K}{\gamma_{cc}}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t, \ \forall t \leq T(\mu)_{K=0} \tag{A.302}$$

or

$$C(t) = C_{0j} + \frac{\gamma_K}{\gamma_{cc}}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t, \ \forall t \leq T(\mu)_{K=0} \tag{A.303}$$

so that:

$$C(t) = \begin{cases} C_{0j} + \frac{\gamma_K}{\gamma_{cc}}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t & \forall t \leq T(\mu)_{K=0} \\ \overline{C} & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.304}$$

We solve for $b(t)$ as follows:

$$b(t) = \gamma_c c(t) + \frac{1}{2}\gamma_{cc} c(t)^2 + \gamma_K K(t) + A_b \tag{A.305}$$

$$b(t) = \gamma_c \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right) + \frac{1}{2}\gamma_{cc} \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right)^2 \tag{A.306}$$
$$+ \gamma_K \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) + A_b \;\; \forall t \leq T(\mu)_{K=0}$$

On the other hand, when $\forall t > T(\mu)_{K=0}$, we will have

$$b(t) = A_b, \tag{A.307}$$

so that overall the farmer will face:

$$b(t)=\begin{cases}\gamma_c\left(-\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}}\right)+\frac{1}{2}\gamma_{cc}\left(-\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}}\right)^2 \\ +\gamma_K\left(\frac{\gamma_K\left(\frac{\gamma_c+\frac{\alpha_c-P_j^{-1}}{\alpha_b}}{\gamma_K}-\frac{1}{\rho}\right)}{\gamma_{cc}}t+K_{0j}\right)+A_b, & \forall t\le T(\mu)_{K=0} \\ A_b, & \forall t>T(\mu)_{K=0}\end{cases} \tag{A.308}$$

$$\begin{aligned}b(t)&=\gamma_c\cdot\frac{\gamma_K}{\gamma_{cc}}\cdot\left(\rho^{-1}-R(K)^{-1}\right)\\&+\frac{1}{2}\gamma_{cc}\cdot\left(\frac{\gamma_K}{\gamma_{cc}}\cdot\left(\rho^{-1}-R(K)^{-1}\right)\right)^2\\&+\gamma_K\cdot\left(\frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1}-R(K)^{-1}\right)\cdot t+K_{0j}\right)+A_b,\ \forall t\le T(\mu)_{K=0}\end{aligned} \tag{A.309}$$

$$\begin{aligned}b(t)&=\gamma_c\cdot\frac{\gamma_K}{\gamma_{cc}}\cdot\left(\rho^{-1}-R(K)^{-1}\right)\\&+\frac{1}{2}\cdot(-\gamma_{cc})^{-1}\cdot\left(\gamma_K\cdot\left(\rho^{-1}-R(K)^{-1}\right)\right)^2+\frac{\gamma_K^2}{(-\gamma_{cc})}\left(\rho^{-1}-R(K)^{-1}\right)\cdot t\\&+\gamma_K\cdot K_{0j}+A_b,\ \forall t\le T(\mu)_{K=0}\end{aligned} \tag{A.310}$$

$$\begin{aligned}b(t)&=\left(\frac{\gamma_c-\gamma_K\cdot t}{\gamma_{cc}}\right)\cdot\gamma_K\cdot\left(\rho^{-1}-R(K)^{-1}\right)\\&+\frac{1}{2}\cdot(-\gamma_{cc})^{-1}\cdot\left(\gamma_K\cdot\left(\rho^{-1}-R(K)^{-1}\right)\right)^2\\&+\gamma_K\cdot K_{0j}+A_b,\ \forall t\le T(\mu)_{K=0}\end{aligned} \tag{A.311}$$

$$b(t) = \left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}}\right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1}\right)\right) \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1}\right) \tag{A.312}$$
$$+ \gamma_K \cdot K_{0j} + A_b, \, \forall t \leq T(\mu)_{K=0}$$

$$b(t) = \gamma_K \cdot \left(\left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}}\right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1}\right)\right) \cdot \left(\rho^{-1} - R(K)^{-1}\right) \tag{A.313}$$
$$+K_{0j}) + A_b, \, \forall t \leq T(\mu)_{K=0}$$

Overall then, the farmer will face:

$$b(t) = \begin{cases} \gamma_K \cdot \left(\left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}}\right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1}\right)\right) \cdot \left(\rho^{-1} - R(K)^{-1}\right) & \\ +K_{0j}) + A_b, & \forall t \leq T(\mu)_{K=0} \\ & \\ A_b, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.314}$$

We solve for $y(t)$ as follows:

$$y(t) = \alpha_c c(t) + \alpha_b b(t) + A_y \tag{A.315}$$

$$y(t) = \alpha_c \cdot \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right) + \alpha_b \cdot \left(\gamma_c \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right) \right. \tag{A.316}$$

$$+ \frac{1}{2}\gamma_{cc} \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right)^2$$

$$\left. + \gamma_K \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) + A_b \right) + A_y, \, \forall t \leq T(\mu)_{K=0}$$

On the other hand, when $\forall t > T(\mu)_{K=0}$, we will have

$$y(t) = \alpha_b A_b + A_y \tag{A.317}$$

so that overall the farmer will face:

$$
y(t) = \begin{cases} \alpha_c \cdot \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right) + \alpha_b \cdot \left(\gamma_c \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right) \right. \\ + \frac{1}{2} \gamma_{cc} \left(-\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \right)^2 \\ \left. + \gamma_K \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) + A_b \right) + A_y, & \forall t \leq T(\mu)_{K=0} \\ \alpha_b A_b + A_y, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.318}
$$

We can also write that

$$
\begin{aligned} y(t) &= \alpha_c \cdot \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ &+ \alpha_b \cdot \left(\gamma_K \cdot \left(\left(\left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}} \right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right) \cdot \left(\rho^{-1} - R(K)^{-1} \right) + K_{0j} \right) \right. \right. \\ &+ A_b) + A_y, \, \forall t \leq T(\mu)_{K=0} \end{aligned} \tag{A.319}
$$

$$
\begin{aligned} y(t) &= \alpha_c \cdot \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ &+ \alpha_b \cdot \gamma_K \cdot \left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}} \right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right) \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ &+ \alpha_b \cdot \gamma_K \cdot K_{0j} + \alpha_b \cdot A_b + A_y, \, \forall t \leq T(\mu)_{K=0} \end{aligned} \tag{A.320}
$$

$$y(t) = \Bigg(\alpha_c \cdot \frac{\gamma_K}{\gamma_{cc}} \\ + \alpha_b \cdot \gamma_K \cdot \left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}} \right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right) \Bigg) \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ + \alpha_b \cdot \gamma_K \cdot K_{0j} + \alpha_b \cdot A_b + A_y, \forall t \leq T(\mu)_{K=0} \tag{A.321}$$

$$y(t) = \gamma_K \cdot \Bigg(\frac{\alpha_c + \alpha_b \gamma_c - \alpha_b \gamma_K \cdot t}{\gamma_{cc}} \\ - \frac{1}{2} \cdot \alpha_b \left(\gamma_{cc}\right)^{-1} \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1} \right) \Bigg) \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ + \alpha_b \cdot \gamma_K \cdot K_{0j} + \alpha_b \cdot A_b + A_y, \forall t \leq T(\mu)_{K=0} \tag{A.322}$$

or

$$y(t) = \alpha_b \gamma_K \Bigg(\Bigg(\frac{\frac{\alpha_c}{\alpha_b} + \gamma_c - \gamma_K \cdot t}{\gamma_{cc}} \\ - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1} \right) \Bigg) \cdot \left(\rho^{-1} - R(K)^{-1} \right) \\ + K_{0j} + \frac{A_b + \frac{A_y}{\alpha_b}}{\gamma_K} \Bigg), \forall t \leq T(\mu)_{K=0}. \tag{A.323}$$

$$y(t) = \alpha_b \gamma_K \left(\frac{1}{\gamma_{cc}} \cdot \left(\frac{\alpha_c}{\alpha_b} + \gamma_c - \gamma_K \cdot \left(t + \frac{1}{2} \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right) \right) \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right. \\ \left. + K_{0j} + \frac{A_b + \frac{A_y}{\alpha_b}}{\gamma_K} \right), \forall t \leq T(\mu)_{K=0}. \tag{A.324}$$

Overall then the farmer will face

$$
y(t) = \begin{cases} \alpha_b \gamma_K \left(\frac{1}{\gamma_{cc}} \cdot \left(\frac{\alpha_c}{\alpha_b} + \gamma_c - \gamma_K \cdot \left(t + \frac{1}{2} \cdot \left(\rho^{-1} - R(K)^{-1} \right) \right) \right) \right. \\ \left. \cdot \left(\rho^{-1} - R(K)^{-1} \right) + K_{0j} + \frac{A_b + \frac{A_y}{\alpha_b}}{\gamma_K} \right), & \forall t \leq T(\mu)_{K=0} \\ \\ \alpha_b A_b + A_y, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.325}
$$

A.6.3 Optimal Trajectories 3”: Stay at initial clean soil stock and do not invest or disinvest

We always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock when $\mu = 0$ and $R_j(K)$ is constant and greater than or equal to ρ.

If $R_j(K)$ is constant and equal to ρ, it is optimal to stay at initial clean soil stock and not to invest or disinvest.

When $\mu = 0$, the condition $R_j(K) \geq \rho$ implies the following:

$$
P_j \alpha_c \leq \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + P_j \alpha_b \left(-\gamma_c \right) + 1 \tag{A.326}
$$

If $\mu = 0$ and $R_j(K)$ is greater than ρ, the farmer is constrained by upper bound on I to stay at K_{0j} (Optimal Trajectory Type 3’). This is because the upper bound constraint on $I(t)$ was given by:

$$
I(t)_{UC} = \mu \cdot \left(\overline{C} - K(t) \right) \tag{A.327}
$$

or, given $\mu = 0$

$$
I(t)_{UC} = 0 \tag{A.328}
$$

Thus, when $\mu = 0$ the upper bound constraint on investment is equal to zero, and will always bind when $R_j(K)$ is greater than ρ. In this case, the farmer will remain at their initial capital stock indefinitely.

Thus, for OT3”, the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own. Since the stock of chemicals in the soil does not decay on its own when $\mu = 0$, this means the optimal synthetic compound use $c(t)$ is constant at zero.

The optimal trajectories are therefore the following:

$$K(t) = K_{0j} \, \forall t \tag{A.329}$$

$$I(t) = 0 \, \forall t \tag{A.330}$$

$$c(t) = 0 \, \forall t \tag{A.331}$$

$$C(t) = \overline{C} - K_{0j} = C(0) \, \forall t \tag{A.332}$$

$$b(t) = \left(\gamma_c \mu \left(\overline{C} - K_{0j} \right) + \frac{1}{2} \gamma_{cc} \mu^2 \left(\overline{C} - K_{0j} \right)^2 + \gamma_K K_{0j} + A_b \right) \forall t \tag{A.333}$$

Whern $\mu = 0$, the equation above simplifies to:

$$b(t) = \gamma_K K_{0j} + A_b \, \forall t. \tag{A.334}$$

$$\tilde{f}(t) = \alpha_b \left(\gamma_c \mu \left(\overline{C} - K_{0j} \right) + \frac{1}{2} \gamma_{cc} \mu^2 \left(\overline{C} - K_{0j} \right)^2 + \gamma_K K_{0j} + A_b \right) + \alpha_c \mu \left(\overline{C} - K_{0j} \right) + A_y \, \forall t \tag{A.335}$$

When $\mu = 0$, the equation above simplifies to:

$$\tilde{f}(t) = \alpha_b \left(\gamma_K K_{0j} + A_b \right) + A_y \, \forall t.$$

Since $y(t) = \tilde{f}(t)$, we can therefore write that

$$y(t) = \alpha_b \left(\gamma_K K_{0j} + A_b \right) + A_y \, \forall t. \tag{A.336}$$

B Discrete Transitions

B.1 Discrete Analysis for OT1 (Case C6, Case B4; Case C7 when $\gamma_{cc} = 0$)

Discrete Analysis for Case C6: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $\hat{K}_{con} < 0$ (and therefore $\hat{K}_{org} < 0$ as well).

Recall that in Case C6 the optimal solution for each stage $j \in \{con, org\}$ is to disinvest as fast as possible until $K = 0$.

A conventional farmer facing C6 conditions will adopt OT1 solutions.

Case B4: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer disinvests to $K = 0$ because $\hat{K}_{org} < 0$.

Case B4 ends up being exactly the same as Case C6 because these two cases only differ in $\hat{K}_{org}$, but not in their stage 2 trajectories (in both cases we will have $K(t)_{org} = K_{org}$ and $I(t)_{org} = 0$ for all t).

A conventional farmer facing B4 or C6 conditions will adopt OT1 solutions.

Case C7: Both the Stage 1 conventional farmer and the Stage 2 organic farmer disinvest to $K = 0$ because $R_j(K)$ is constant and less than ρ

Similarly, Case C7 when $\gamma_{cc} = 0$ ends up being the same as Case C6 because the conventional farmer adopts OT1 solutions while for the stage 2 trajectories we have $K(t)_{org} = K_{org} \forall t$ and $I(t)_{org} = 0 \forall t$, except that the interpretation for ϵ^* based on $\hat{K}_{con}$ in Figure B.1 no longer applies, since $\hat{K}_{con}$ does not exist.

Similarly, Case C7 when $\gamma_{cc} = 0$ ends up being exactly the same as Case C6 because the conventional farmer adopts OT1 solutions, while for the stage 2 trajectories we have $K(t)_{org} = K_{org}$ and $I(t)_{org} = 0$ for all t.

A conventional farmer facing B4 or C6 conditions, or C7 conditions when $\gamma_{cc} = 0$, will adopt OT1 solutions.

In this case, a farmer who starts off organic will disinvest until they reach K_{org}. They then choose to remain organic if and only if

$$V_{org}(K_{org}) > V_{con}(K_{org} - \epsilon). \tag{B.1}$$

$V_{org}(K_{org})$ is the present discounted value of the entire stream of net benefits that a farmer will receive from the moment they have switched to organic management, into perpetuity, assuming the organic farmer stays organic indefinitely. $V_{org}(K_{org})$ assumes that once in stage 2, the farmer follows the following constrained trajectories:

$$\bar{K}(t)_{org} = K_{org} \, \forall t \tag{B.2}$$

$$\bar{I}(t)_{org} = 0 \, \forall t \tag{B.3}$$

$$\overline{C}(t)_{org} = \mu \left(\overline{C} - K_{org}\right) \, \forall t \tag{B.4}$$

The value $V_{org}(K_{org})$ of the farmer's optimal program for stage 2 following this constrained capital trajectory can be written as follows:

$$\begin{aligned} V_{org}(K_{org}) = \int_{t=0}^{\infty} & \left(P_{org} \cdot \left(\alpha_b \left(\frac{1}{2}\gamma_{cc} \left(\mu \left(\overline{C} - K_{org} \right) \right)^2 + \gamma_c \mu \left(\overline{C} - K_{org} \right) + \gamma_K \left(K_{org} \right) + A_b \right) \right. \right. \\ & \left. \left. + \alpha_c \mu \left(\overline{C} - K_{org} \right) + A_y \right) - \mu \left(\overline{C} - K_{org} \right) \right) e^{-\rho t} dt \end{aligned} \tag{B.5}$$

$$V_{org}(K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b} - \frac{1}{2}\gamma_{cc}\mu K_{org} \right) \mu \left(\overline{C} - K_{org} \right) + \gamma_K K_{org} + A_b \right) + A_y \right) \tag{B.6}$$

On the other hand, $V_{con}(K_{org} - \epsilon)$ is the present discounted value of the entire stream of net benefits that a farmer will receive if they continue to produce conventionally indefinitely. When the conventional farmer adopts OT1 solutions, $V_{con}(K_{org} - \epsilon)$ is given by:

$$V_{con}(K_{org} - \epsilon) = \frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu + \rho)} \cdot \gamma_K \cdot (K_{org} - \epsilon) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{B.7}$$

To further simplify our analysis, let's also assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils, such that they will be certifiied organic if and only if $K = \overline{C}$. In this case we have that $K_{org} = \overline{C}$, and our analytical solution for $V_{con}(K_{org} - \epsilon)$ becomes further to:

$$V_{con}(K_{org} - \epsilon) = \frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu + \rho)} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon \right) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{B.8}$$

With $K_{org} = \overline{C}$ our expression for $V_{org}(K_{org})$ becomes:

$$V_{org}(K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b \right) + A_y \right) \tag{B.9}$$

With expressions for $V_{org}(K_{org})$ and $V_{con}(K_{org} - \epsilon)$ we can now write the following expressions:

$$\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon) \tag{B.10}$$

as follows

$$\begin{aligned}
\Delta(\epsilon) = & \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}}\right) \\
& + \underbrace{P_{con} \cdot \alpha_b \cdot \frac{1}{(\mu + \rho)} \cdot \gamma_K}_{\geq 0} \cdot \epsilon \\
& - \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\mu + \rho)} \cdot \gamma_K\right) \cdot \mu\overline{C}
\end{aligned}$$

Given $K_{org} = \overline{C}$ and $\overline{c} = \mu\overline{C}$ the conventional C6 farmer faces:

$$\begin{aligned}
\Delta^{C6}(\epsilon) = \; & \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} (P_{org} - P_{con}) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} \\
& - \underbrace{P_{con}\alpha_b \cdot \frac{1}{(\mu + \rho)} \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \underbrace{- \frac{1}{\rho} \cdot \left(P_{con}\left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c\right) + \alpha_c\right) - 1\right) \cdot \mu\overline{C}}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal rate } \mu\overline{C}}}
\end{aligned}$$

The sign of $\frac{\partial \Delta(\epsilon)}{\partial \epsilon}$ is given by:

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} \geq \underbrace{P_{con} \cdot \alpha_b \cdot \frac{1}{(\mu + \rho)} \cdot \gamma_K}_{\geq 0} \tag{B.11}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.
Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$. Note that $\Delta(\epsilon^*) = 0$.
The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\epsilon^* = \left(\frac{\mu+\rho}{\gamma_K} \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - 1\right) \cdot \frac{\mu}{\rho} \cdot \overline{C}$$
$$- \frac{1}{\gamma_K} \cdot \underbrace{\frac{\mu+\rho}{\rho} \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}}\right)$$

$$\epsilon^* = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho}}_{\leq 0} \left(\underbrace{(P_{org} - P_{con})\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} - P_{con}\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\mu - \underbrace{\left(\frac{P_{org}}{P_{con}} - \frac{\rho}{\mu+\rho}\right)\gamma_K}_{\geq 0}\right)\overline{C}\right) \tag{B.12}$$

This means that when $\epsilon^* \leq 0$ the farmer will face $V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon) > 0 \, \forall \epsilon \geq 0$, and will therefore prefer to produce organically for all feasible initial capital stocks (i.e. they always prefer to produce organically).

Given $\frac{\partial \Delta(\epsilon^*)}{\partial \epsilon} \geq 0$, we will have that:

- The lower the threshold ϵ^*, the larger the set $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$
- The higher the threshold ϵ^*, the smaller the $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$

Figure B.1: Condition for $\Delta(\epsilon) \geq 0$ for Case C6

$$\underbrace{\frac{1}{\mu+\rho}\cdot P_{con}\alpha_b\gamma_K}_{\substack{\text{PDV of entire stream of}\\ \text{marginal benefit}\\ \text{of additional unit of}\\ \text{clean soil stock } K(t)\\ \text{via its direct effect}\\ \text{on soil microbes}\\ \text{(stock effect)}}} \cdot\epsilon \geq \underbrace{\frac{1}{\rho}P_{con}\alpha_b\left(\frac{1}{2}\gamma_{cc}\left(\mu\overline{C}\right)^2\right)}_{\substack{\text{negative of PDV of entire stream of}\\ \text{nonlinear component of indirect benefits}\\ \text{of clean soil stock } K(t)\\ \text{when evaluated at } K=0\\ \text{via its indirect positive effect}\\ \text{on soil microbes}\\ \text{through its negative effect}\\ \text{on synthetic compound use } c(t)}} \underbrace{-\frac{1}{\rho}P_{con}\alpha_b\left(\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)\right)\mu}_{\substack{\text{PDV of entire stream}\\ \text{of nonlinear component of}\\ \text{indirect marginal benefits of clean soil stock } K(t)\\ \text{when evaluated at } K=\hat{K}_{con}\\ \text{via its indirect positive effect}\\ \text{on soil microbes}\\ \text{through its negative effect}\\ \text{on synthetic compound use } c(t)}} \underbrace{\overline{C}}_{\substack{\text{clean soil stock}\\ \text{at organic threshold}\\ K=\overline{C}}} \underbrace{-\frac{1}{\rho}\left(P_{org}-P_{con}\right)\underbrace{\left(\alpha_b\left(\gamma_K\overline{C}+A_b\right)+A_y\right)}_{\text{crop output at the organic threshold}}}_{\substack{\text{PDV of entire stream of additional crop revenue}\\ \text{at the organic threshold } K=\overline{C}\\ \text{due to organic price premium}}} \qquad \text{(B.13)}$$

B.1.1 Comparative statics for $\Delta(\epsilon)$

First we do a comparative static exercise for $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$. The results are summarized in Table B.1 and derived below.

$$\begin{aligned}\frac{\partial \Delta(\epsilon)}{\partial P_{con}} = \frac{1}{\rho} \cdot \alpha_b \cdot \Bigg(& \underbrace{-\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\leq 0} + \underbrace{\gamma_K \cdot \left(\frac{\rho}{(\mu+\rho)} \cdot \epsilon - \overline{C}\right)}_{\leq 0} \\ & + \underbrace{\left(-\left(\underbrace{\mu\gamma_{cc} \cdot \underbrace{\hat{K}_{con}}_{\leq 0}}_{\geq 0} + \underbrace{\frac{1}{2}\left(-\gamma_{cc}\right)\mu\overline{C}}_{\geq 0} + \underbrace{\frac{P_{con}^{-1}}{\alpha_b}}_{\geq 0}\right)\right) \cdot \mu\overline{C}}_{\leq 0} \Bigg) \leq 0\end{aligned} \tag{B.14}$$

$$\frac{\partial \Delta(\epsilon)}{\partial P_{org}} = \underbrace{\frac{1}{\rho} \cdot \alpha_b \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \geq 0 \tag{B.15}$$

$$\begin{aligned}\frac{\partial \Delta(\epsilon)}{\partial \rho} = -P_{con} \cdot \alpha_b \cdot \frac{1}{\rho^2} \cdot \Bigg(& \underbrace{\left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \cdot \underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}}\right)}_{\geq} \\ & + \underbrace{\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\frac{\rho}{\mu+\rho} \cdot \epsilon + \mu\overline{C} \cdot \left(\frac{1}{\rho} + \frac{1}{(\mu+\rho)}\right)\right)}_{\geq 0} \\ & - \underbrace{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C}}_{\geq 0} \Bigg)\end{aligned}$$

So that for large enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \leq 0$, and for small enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ and large enough α_c we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \geq 0$.

$$\frac{\partial\Delta(\epsilon)}{\partial\mu} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\underbrace{\left(\underbrace{\frac{1}{(\mu+\rho)^2}}_{\geq 0}\cdot\gamma_K\cdot\left(\rho\cdot\epsilon+\mu\cdot\overline{C}\right)+\mu\underbrace{\gamma_{cc}\cdot\hat{K}_{con}}_{\geq 0}\cdot\overline{C}\right)}_{\geq 0}\leq 0$$

$$\frac{\partial\Delta(\epsilon)}{\partial\overline{C}} = \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b}_{\geq 0}\cdot\left(\underbrace{\underbrace{(\gamma_K)}_{\geq 0}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}+\underbrace{(-\gamma_{cc})\,\mu^2\hat{K}_{con}}_{\leq 0}\right) \tag{B.16}$$

So we will have $\frac{\partial\Delta(\epsilon)}{\partial\overline{C}}\geq 0$ for large enough $\frac{P_{org}-P_{con}}{P_{con}}$, and $\frac{\partial\Delta(\epsilon)}{\partial\overline{C}}\leq 0$ for negative enough $\hat{K}_{con}$ (as we might have when α_c is sufficiently large)

$$\begin{aligned}\frac{\partial\Delta(\epsilon)}{\partial\alpha_b} &= \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\left(\gamma_K\cdot\overline{C}+A_b\right)}_{\geq 0}\cdot\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}+\underbrace{P_{con}\cdot\frac{1}{(\mu+\rho)}\cdot\gamma_K}_{\geq 0}\cdot\epsilon\\ &\quad+\underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\left(\frac{1}{2}(-\gamma_{cc})\,\mu\overline{C}+(-\gamma_c)+\frac{1}{(\mu+\rho)}\cdot\gamma_K\right)\cdot\mu\overline{C}}_{\geq 0}\geq 0\end{aligned}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_{cc}} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{1}{2}\cdot\left(\mu\overline{C}\right)^2\leq 0 \tag{B.17}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_c} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\mu\overline{C}\leq 0 \tag{B.18}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_K} = \frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(\overline{C}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)+\underbrace{\frac{1}{(\mu+\rho)}}_{\geq 0}\cdot\left(\rho\epsilon+\mu\overline{C}\right)\right)\geq 0 \tag{B.19}$$

which means that as the marginal product γ_K of an additional unit of clean soil increases, the farmer becomes more likely to prefer organic production.

$$\frac{\partial \Delta (\epsilon)}{\partial A_y} = (-P_{con} \cdot \alpha_b) \cdot \frac{P_{\text{con}} - P_{\text{org}}}{\rho P_{\text{con}} \alpha_b} \geq 0 \tag{B.20}$$

$$\frac{\partial \Delta (\epsilon)}{\partial A_b} = (-P_{con} \cdot \alpha_b) \cdot \frac{P_{\text{con}} - P_{\text{org}}}{\rho P_{\text{con}}} \geq 0 \tag{B.21}$$

$$\frac{\partial \Delta (\epsilon)}{\partial \epsilon} = (-P_{con} \cdot \alpha_b) \cdot \left(-\frac{\gamma_K}{\mu + \rho} \right) \geq 0 \tag{B.22}$$

Table B.1: Comparative Statics for $\Delta(\epsilon)$ When Conventional Farmer Adopts OT1

Parameter	Full Information: C6 (OT1)	
ρ	$+$	Small enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ and large enough α_c.
	$-$	Large enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$
μ	$-$	
P_{org}	$+$	
P_{con}	$-$	
α_b	$+$	
α_c	$-$	
γ_c	$-$	
γ_{cc}	$-$	
γ_K	$+$	
A_y	$+$	
A_b	$+$	
$\overline{C}$	$+$	
	$-$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$. Large enough α_c
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$ when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically when $\Delta(\epsilon) > 0$.

B.1.2 Comparative statics for ϵ^*

Now we do comparative statics of ϵ^* for the parameters: $\mu, \rho,\ \gamma_{cc}, \gamma_c, \gamma_K, \alpha_{1,}\alpha_c, P_{con},$ and P_{org}.

The results are summarized in Table B.2.

$$\frac{\partial \epsilon^*}{\partial P_{org}} = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K}\cdot\frac{1}{\rho}}_{\leq 0}\left(\underbrace{\underbrace{\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}+\underbrace{\gamma_K\overline{C}}_{\geq 0}}_{\geq 0}\right)\leq 0 \tag{B.23}$$

$$\frac{\partial \epsilon^*}{\partial \overline{C}} = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K}\cdot\frac{1}{\rho}}_{\leq 0}\left(\underbrace{\left(P_{org}-P_{con}\right)\gamma_K}_{\geq 0}-\mu^2 P_{con}\gamma_{cc}\hat{K}_{con}\right) \tag{B.24}$$

So for sufficiently large organic price premiums ($(P_{org}-P_{con})$ large) we'll have $\frac{\partial \epsilon^*}{\partial \overline{C}}\leq 0$. On the other hand when synthetic compounds are very effective at increasing yields (such that we have very large $\mu^2 P_{con}\gamma_{cc}\hat{K}_{con}$), we'll have $\frac{\partial \epsilon^*}{\partial \overline{C}}\geq 0$.

$$\frac{\partial \epsilon^*}{\partial \mu} = \left(\frac{1}{\mu+\rho}\right)\epsilon^*+\left(\frac{\mu+\rho}{P_{con}\gamma_K}\right)\cdot\frac{1}{\rho}\left(P_{con}\left(\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\left(\frac{\rho}{(\mu+\rho)^2}\right)\gamma_K\right)\overline{C}\right) \tag{B.25}$$

Since in case C6 we have that $\hat{K}_{con}<0$ (and $\hat{K}_{org}<0$), we have that in case C6:

$$\hat{K}_{con} = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\rho+\mu)\gamma_{cc}\mu}\leq 0 \tag{B.26}$$

Since $\rho>0$, $\mu\geq 0$, and $\gamma_{cc}\leq 0$, we know that $(\rho+\mu)\gamma_{cc}\mu\leq 0$ (and in particular if $\hat{K}_{con}$ is well defined and satisfies $\hat{K}_{con}\leq 0$, it must be the case that $(\rho+\mu)\gamma_{cc}\mu<0$). Then if $\hat{K}_{con}\leq 0$ and $(\rho+\mu)\gamma_{cc}\mu\leq 0$ it must be the case that:

$$(\rho+\mu)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K\geq 0 \tag{B.27}$$

Given $\gamma_K\geq 0$, we know that $\gamma_K\geq\frac{\rho}{\rho+\mu}\cdot\gamma_K\geq 0$ (since $0\leq\frac{\rho}{\rho+\mu}\leq 1$). Thus

$$(\rho+\mu)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\frac{\rho}{(\rho+\mu)}\gamma_K \geq 0 \tag{B.28}$$

Thus:

$$\frac{\partial\epsilon^*}{\partial\mu}=\left(\frac{1}{\mu+\rho}\right)\epsilon^*+\underbrace{\left(\frac{\overline{C}}{\gamma_K}\right)\cdot\frac{1}{\rho}}_{\geq 0}\cdot\left(\underbrace{(\mu+\rho)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\frac{\rho}{(\mu+\rho)}\gamma_K}_{\geq 0}\right)\geq 0 \tag{B.29}$$

where:

$$\begin{aligned}\epsilon^*=&\left(\frac{\mu+\rho}{\gamma_K}\cdot\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-1\right)\cdot\frac{\mu}{\rho}\cdot\overline{C}\\&-\frac{1}{\gamma_K}\cdot\underbrace{\frac{\mu+\rho}{\rho}\cdot\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial\epsilon^*}{\partial\mu}=&\frac{1}{\rho}\cdot\left(\frac{1}{\gamma_K}\cdot\left(\frac{1}{2}\cdot(-\gamma_{cc})\cdot\overline{C}-\frac{1}{(\mu+\rho)^2}\cdot\gamma_K\right)\cdot\mu^2\cdot\overline{C}\right.\\&+\underbrace{\left(-\frac{1}{\gamma_K}\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}\right)}_{\leq 0}\\&\left.+\underbrace{\frac{2\mu+\rho}{\mu+\rho}\cdot\left(\frac{\overline{C}}{\gamma_K}\right)\cdot\left((\mu+\rho)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\frac{\rho}{(\mu+\rho)}\gamma_K\right)}_{\geq 0}\right)\end{aligned} \tag{B.30}$$

We have then that $\frac{\partial\epsilon^*}{\partial\mu}\leq 0$ for large enough organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$, or large enough $A_b+\frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields. We will have $\frac{\partial\epsilon^*}{\partial\mu}\geq 0$, on the other hand, if both the organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$ are sufficiently small and also $(\mu+\rho)$ is sufficiently large.

$$\frac{\partial \epsilon^*}{\partial \rho} = \underbrace{\left(\frac{1}{\rho} - \frac{1}{\mu+\rho}\right)}_{\geq 0} \left(\overline{C} - \epsilon^*\right) \tag{B.31}$$

The sign of $\left(\overline{C} - \epsilon^*\right)$ is ambiguous, and will depend on other parameter values. What we can say is that $\left(\overline{C} - \epsilon^*\right) \geq 0$ is a necessary condition for the feasible set of initial capital stock for which the farmer prefers to produce organically to be non-empty. So if the feasible set of initial capital stocks for which the farmer prefers to produce organically to be non-empty then we will have $\frac{\partial \epsilon^*}{\partial \rho} \geq 0$, such that increasing the interest rate (so that the farmer cares less about the future) increases ϵ^*, and contracts the set of initial capital stock at which the farmer prefers to produce organically.

$$\frac{\partial \epsilon^*}{\partial \gamma_{cc}} = \frac{\mu+\rho}{\gamma_K} \cdot \frac{1}{\rho} \left(\frac{1}{2}\left(\mu\overline{C}\right)^2\right) \geq 0 \tag{B.32}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_c} = \frac{\mu+\rho}{\gamma_K} \cdot \frac{\mu}{\rho}\overline{C} \geq 0 \tag{B.33}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \underbrace{\frac{1}{\gamma_K^2} \cdot \frac{\mu+\rho}{\rho}}_{\geq 0} \cdot \left(\underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}}\right)\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} + \underbrace{\underbrace{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)}_{\geq 0} \cdot \underbrace{\left(-\mu\overline{C}\right)}_{\leq 0}}_{\leq 0} \right), \tag{B.34}$$

where $\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \geq 0$ comes from the fact that in case C6 we have $\hat{K}_{con}$. So we see that we will have:

- $\frac{\partial \epsilon^*}{\partial \gamma_K} \geq 0$ when:

1. We have large enough organic price premia $\frac{P_{org} - P_{con}}{P_{con}}$
2. or large enough $A_b + \frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields.

- We will have $\frac{\partial \epsilon^*}{\partial \gamma_K} \leq 0$ when both:

1. We have small enough organic price premia, $\frac{P_{org} - P_{con}}{P_{con}}$

2. and small enough $A_b + \frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently unimportant for determining yields.

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \frac{1}{\alpha_b^2} \cdot \frac{\mu + \rho}{\rho} \cdot \frac{1}{\gamma_K} \left(\underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot A_y}_{\geq 0} - \underbrace{\left(\alpha_c - P_{con}^{-1}\right)}_{\geq 0} \cdot \underbrace{\mu \overline{C}}_{\geq 0} \right) \tag{B.35}$$

where $\alpha_c - P_{con}^{-1} \geq 0$ comes from $\hat{K}_{con} \leq 0$, which is satisfied in case C6, since:

$$\hat{K}_{con} = \frac{(\rho + \mu) \left(\gamma_{cc} \mu \overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) - \gamma_K}{\underbrace{(\rho + \mu)\, \gamma_{cc} \mu}_{\leq 0}} \leq 0 \tag{B.36}$$

$$\Rightarrow \underbrace{(\rho + \mu)}_{\geq 0} \left(\underbrace{\gamma_{cc} \mu \overline{C}}_{\leq 0} + \underbrace{\gamma_c}_{\leq 0} + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) - \underbrace{\gamma_K}_{\geq 0} \geq 0 \tag{B.37}$$

$$\Rightarrow \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \geq 0 \tag{B.38}$$

$$\Rightarrow \alpha_c - P_{con}^{-1} \geq 0 \tag{B.39}$$

So we have that:

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \underbrace{\frac{1}{\alpha_b^2} \cdot \frac{\mu + \rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \left(\underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot A_y}_{\geq 0} - \underbrace{\left(\alpha_c - P_{con}^{-1}\right)}_{\geq 0} \cdot \underbrace{\mu \overline{C}}_{\geq 0} \right) \tag{B.40}$$

We have $\frac{\partial \epsilon^*}{\partial \alpha_b} \geq 0$ when:

1. We have large enough organic price premia $\frac{P_{org} - P_{con}}{P_{con}}$
2. or large enough A_y, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields.

We will have $\frac{\partial \epsilon^*}{\partial \alpha_b} \leq 0$ when both:

1. We have small enough organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$
2. and small enough A_y, such that factors of production other than synthetic compounds and soil bacteria are sufficiently unimportant for determining yields.

$$\frac{\partial \epsilon^*}{\partial \alpha_c} = \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \frac{1}{\alpha_b} \cdot \mu\overline{C} \geq 0 \tag{B.41}$$

So that increasing the benefit of synthatic compounds (α_c) increases the value of (ϵ^*) and shrinks the set of initial capital stocks for which the farmer prefers to produce organically.

$$\frac{\partial \epsilon^*}{\partial P_{con}} = \frac{1}{P_{con}} \left(\underbrace{\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho} P_{org} \left(A_b + \frac{A_y}{\alpha_b} \right)}_{\geq 0} + \underbrace{\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho} \left(\frac{1}{\alpha_b}\mu + P_{org}\gamma_K \right) \overline{C}}_{\geq 0} \right) \geq 0$$

So that increasing the price at which conventional farmers can sell their crops (P_{con}) increases the value of (ϵ^*) and shrinks the set of initial capital stocks for which the farmer prefers to produce organically.

Table B.2: Comparative Statics for ϵ^* When Conventional Farmer Adopts OT1

Parameter		Full Information: C6 (OT1)
ρ	$-$	ρ and γ_K large enough, $\left(\frac{P_{\text{org}}}{P_{\text{con}}} - 1\right)$ and ϵ small enough.
	$+$	$\left(A_b + \frac{A_y}{\alpha_b}\right)$ sufficiently large
μ	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or $A_b + \frac{A_y}{\alpha_b}$ large enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and $(\mu+\rho)$ large enough.
P_{org}	-	
P_{con}	$+$	
α_b	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and A_y small enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or A_y large enough.
α_c	$+$	
γ_c	$+$	
γ_{cc}	$+$	
γ_K	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and $A_b + \frac{A_y}{\alpha_b}$ small enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or $A_b + \frac{A_y}{\alpha_b}$ large enough.
A_y	$+$	
A_b	$+$	
c_0	0	
C_0	0	
$\overline{C}$	$-$	$(P_{org} - P_{con})$ large enough
	$+$	$(P_{org} - P_{con})$ small enough

Notes: Table reports comparative statics for ϵ^*_+ when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ^*_+ lower than K_{org}.

B.1.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$

We also want to find how large the price premium needs to be in order to induce the fully informed farmer to prefer organic management. We derive this requirement for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ below.

The range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot \mu\overline{C} - \frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\left(\mu\overline{C} + \rho\cdot\epsilon\right)}{\gamma_K\cdot\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{B.42}$$

We now conduct a comparatic statics analysis for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$. The results are summarized in Table B.3.

Given that:

$$\frac{\partial\Delta(\epsilon)}{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)} \geq \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(\gamma_K\cdot\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \tag{B.43}$$

then any change in parameter values that increases the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ will shrink the set of organic price premia for which $\Delta(\epsilon) \geq 0$.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\epsilon} = \frac{-\frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\rho}{\gamma_K\cdot\overline{C} + A_b + \frac{A_y}{\alpha_b}} \leq 0 \tag{B.44}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\rho} = \frac{\mu\cdot\gamma_K}{(\mu+\rho)^2}\cdot\frac{\overline{C}-\epsilon}{\gamma_K\cdot\overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.45}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\mu} = \frac{\underbrace{\left(\gamma_{cc}\mu\hat{K}_{con}\right)}_{\geq 0}\cdot\overline{C} + \underbrace{\frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\left(\frac{\mu}{\mu+\rho}\cdot\overline{C} + \frac{\rho}{\mu+\rho}\cdot\epsilon\right)}_{\geq 0}}{\underbrace{\gamma_K\cdot\overline{C} + A_b + \frac{A_y}{\alpha_b}}_{\geq 0}} \geq 0 \tag{B.46}$$

$$
\begin{aligned}
\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K} = & \underbrace{-\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}}_{\leq 0} \qquad \text{(B.47)}\\
& \cdot\left(\underbrace{\left(\underbrace{\mu\gamma_{cc}\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)}_{\geq 0}+\frac{1}{\mu+\rho}\cdot\gamma_K\right)\cdot\mu\overline{C}^2}_{\geq 0}+\underbrace{\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\frac{\mu\overline{C}+\rho\cdot\epsilon}{\mu+\rho}}_{\geq 0}\right)\leq 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}} = & \underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \qquad \text{(B.48)}\\
& \cdot\left(\underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0}\right.\\
& \left.\cdot\left(\underbrace{\frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\frac{\rho}{\mu}\cdot\frac{\epsilon}{\overline{C}}}_{\geq 0}+\underbrace{(-\gamma_{cc})\,\mu\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)}_{\leq 0}\right)\cdot\underbrace{\gamma_K\mu\overline{C}}_{\geq 0}+\underbrace{\gamma_{cc}\mu^2\cdot\hat{K}_{con}}_{\geq 0}\right)
\end{aligned}
$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}}\leq 0$ for small enough ρ, small enough γ_K and small enough $\left(A_b+\frac{A_y}{\alpha_b}\right)$ (i.e synthetic compounds being relatively important, and the economic agent we caring enough about the future). We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}}\geq 0$ for large enough ρ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y}=\underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}\cdot\left(-\frac{1}{\alpha_b}\right)}_{\leq 0}\cdot\left(\underbrace{\gamma_{cc}\cdot\mu\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)\cdot\mu\overline{C}}_{\geq 0}+\underbrace{\left(\frac{-\rho}{(\mu+\rho)}\right)\cdot\gamma_K\cdot\epsilon}_{\leq 0}\right) \tag{B.49}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y}\geq 0$ for large enough ρ and non-zero ϵ. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y}\leq 0$ for small enough ρ or ϵ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b}=\underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}}_{\geq 0}\cdot\left(\underbrace{\mu\left(-\gamma_{cc}\right)\cdot\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)\cdot\mu\overline{C}}_{\leq 0}+\underbrace{\left(\frac{\rho}{\mu+\rho}\right)\cdot\gamma_K\cdot\epsilon}_{\geq 0}\right) \tag{B.50}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b}\geq 0$ for large enough ρ and non-zero ϵ. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b}\leq 0$ for small enough ρ or ϵ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b}=\underbrace{\frac{A_y}{\left(\alpha_b\cdot\left(\gamma_K\cdot\overline{C}+A_b\right)+A_y\right)^2}}_{\geq 0}\cdot\left(\underbrace{\gamma_{cc}\mu\cdot\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)\cdot\mu\overline{C}}_{\geq 0}+\underbrace{\frac{-\rho}{(\mu+\rho)}\cdot\gamma_K\cdot\epsilon}_{\leq 0}\right) \tag{B.51}$$

$$+\underbrace{\frac{\left(-\left(\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C}\right)}{\left(\alpha_b\left(\gamma_K\cdot\overline{C}+A_b\right)+A_y\right)}}_{\leq 0}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b}\geq 0$ for small enough ρ and and large enough A_y. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b}\leq 0$ for large enough ρ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_{cc}} = \frac{\frac{1}{2}\left(\mu\overline{C}\right)^2}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}} \geq 0 \tag{B.52}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_{c}} = \frac{\mu\overline{C}}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}} \geq 0 \tag{B.53}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_{c}} = \frac{\left(\frac{1}{\alpha_b}\right)\cdot\mu\overline{C}}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}} \geq 0 \tag{B.54}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial P_{con}} = \frac{\left(\frac{P_{con}^{-2}}{\alpha_b}\right)\cdot\mu\overline{C}}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}} \geq 0 \tag{B.55}$$

Table B.3: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ When Conventional Farmer Adopts OT1 (Case C6, B4)

Parameter	Full Information: C6 (OT1)	
ϵ	-	
ρ	+	
μ	+	
γ_K	-	
$\overline{C}$	$-$	For small enough ρ, γ_K, $and\,(A_b + \frac{A_y}{a_b})$
	$+$	For large enough ρ
A_y	$-$	For small enough ρ or ϵ
	$+$	For large enough ρ and non-zero ϵ
A_b	$-$	For small enough ρ or ϵ
	$+$	For large enough ρ and non-zero ϵ
α_b	$-$	For large enough ρ
	$+$	For small enough ρ and large enough A_y
γ_{cc}	+	
γ_c	+	
α_c	+	
P_{con}	+	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically if $\frac{P_{org}-P_{con}}{P_{con}} > \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$.

B.2 Discrete Analysis for OT2/OT3/OT4 (Case A2)

Case A2: Conventional Farmer Stationary Solution $\hat{K}_{con}$ is below K_{org} and Organic Farmer Stationary Solution $\hat{K}_{org}$ exists (so is below $\hat{K}_{con}$ and therefore below K_{org} as well), and $\hat{K}_{S_j} \in [0, \overline{C}]$ for $j \in \{con, org\}$

In A2 the farmer will also adopt the same stage 2 trajectories as in B4 and C6, namely $K(t)_{org} = K_{org} \forall t$ and $I(t)_{org} = 0 \forall t$.

A conventional farmer facing A2 conditions will adopt either an OT2, OT3, or OT4 solution.

The conventional A2 farmer faces:

$$0 < \hat{K}_j < K_{org} \leq \overline{C} \tag{B.56}$$

and adopts the following trajectories:

$$K^*(t)_{S_j} = \hat{K}_j + \left(K(0)_j - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{B.57}$$

$$I^*(t)_{S_j} = \mu \left(\hat{K}_j - K(t)\right) \tag{B.58}$$

$$c^*(t)_{S_j} = \mu \left(\overline{C} - \hat{K}_j\right) \tag{B.59}$$

For the conventional A2 farmer, $V_{con}(K_{org} - \epsilon)$ is given by:

$$\begin{aligned} V_{con}(K_{org} - \epsilon) = & \\ & \frac{1}{\rho} \left(P_{con} \cdot \left(\alpha_b \left(\frac{1}{2} \gamma_{cc} \left(\mu \left(\overline{C} - \hat{K}_{con} \right) \right)^2 + \gamma_c \left(\mu \left(\overline{C} - \hat{K}_{con} \right) \right) + \gamma_K \hat{K}_{con} + A_b \right) + \alpha_c \left(\mu \left(\overline{C} - \hat{K}_{con} \right) \right. \right. \right. \\ & \left. \left. \left. - \left(\mu \left(\overline{C} - \hat{K}_{con} \right) \right) \right) \right) + \frac{P_{con} \alpha_b \gamma_K}{\mu + \rho} \cdot \underbrace{\left(K(0)_{con} - \hat{K}_{con} \right)}_{\geq 0} \end{aligned}$$

We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$.

Given $K_{org} = \overline{C}$ the conventional A2 farmer faces:

$$\begin{aligned} V^{A2}_{con}(K_{org} - \epsilon) = \quad & \underbrace{\frac{1}{\rho} \cdot P_{con} \left(\alpha_b \cdot A_b + A_y \right)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at conventional prices}}} \quad \underbrace{+ P_{con} \alpha_b \cdot \frac{1}{(\mu + \rho)} \cdot \gamma_K \cdot \left(\underbrace{(K_{org} - \epsilon)}_{= K_0} - \hat{K}_{con} \right)}_{\substack{\text{PDV of "cashing in"} \\ \text{on microbial productivity}}} \\ + \quad & \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \left(\overline{C} - \hat{K}_{con} \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu \left(\overline{C} - \hat{K}_{con} \right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu \left(\overline{C} - \hat{K}_{con} \right)} \end{aligned}$$

During stage 2:

$$V_{org}(K_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\left(\frac{1}{2} \gamma_{cc} \mu \overline{C} + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b} - \frac{1}{2} \gamma_{cc} \mu K_{org} \right) \mu \left(\overline{C} - K_{org} \right) + \gamma_K K_{org} + A_b \right) + A_y \right) \tag{B.60}$$

Given $K_{org} = \overline{C}$ the stage-2 A2 farmer faces:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} P_{org} \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at organic prices}}} \tag{B.61}$$

Given $K_{org} = \overline{C}$ the $\Delta^{A2}(\epsilon)$ faced by the conventional A2 farmer is given by:

$$\begin{aligned}\Delta^{A2}(\epsilon) = & \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} (P_{org} - P_{con}) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} \\ & \underbrace{- P_{con} \alpha_b \cdot \frac{1}{(\mu + \rho)} \cdot \gamma_K \cdot \left(\underbrace{(\overline{C} - \epsilon)}_{=K_0} - \hat{K}_{con} \right)}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \\ & - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2} \gamma_{cc} \mu \left(\overline{C} - \hat{K}_{con} \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu \left(\overline{C} - \hat{K}_{con} \right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu \left(\overline{C} - \hat{K}_{con} \right)}\end{aligned}$$

The sign of $\frac{\partial \Delta(\epsilon)}{\partial \epsilon}$ is given by:

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} = \frac{P_{con} \alpha_b \gamma_K}{\mu + \rho} \cdot \epsilon \geq 0 \tag{B.62}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.
Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$.
The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\epsilon^* = \frac{1}{\gamma_K} \cdot \left(-\frac{P_{org}}{P_{con}} \cdot \left(\frac{1}{2} \frac{\mu^2}{\rho} (\mu+\rho) \cdot \gamma_{cc} \left(\overline{C} - K_{org}\right) + \frac{\mu}{\rho} (\mu+\rho) \cdot \left(\gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b} \right) \right) \left(\overline{C} - K_{org}\right) \right.$$
$$- \frac{1}{2} \cdot \frac{1}{\gamma_{cc}} \cdot \frac{1}{\rho} \cdot (\mu+\rho) \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - (\rho+\mu)^{-1} \gamma_K \right)^2 -$$
$$\left. \left(\left(\frac{\mu+\rho}{\rho} \cdot \frac{P_{org}}{P_{con}} - 1 \right) \cdot \gamma_K \cdot K_{org} + \frac{\mu+\rho}{\rho} \cdot \left(\frac{P_{org}}{P_{con}} - 1 \right) \cdot \left(A_b + \frac{A_y}{\alpha_b} \right) \right) \right) + \frac{\mu}{\rho} \overline{C} \tag{B.63}$$

When $K_{org} = \overline{C}$, ϵ^* simplifies to:

$$\epsilon^* = \frac{1}{\gamma_K} \cdot \frac{\mu+\rho}{\rho} \cdot \left(\underbrace{\frac{1}{2} \cdot \frac{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{\mu+\rho} \right)^2}{(-\gamma_{cc})}}_{\geq 0} - \underbrace{\left(\frac{P_{org}}{P_{con}} - 1 \right) \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)}_{\geq 0} \right) \tag{B.64}$$

Case A2 allows for the possibility that ϵ^* exceeds $\overline{C}$. When this happens there will be no feasible ϵ for which $\Delta(\epsilon) \geq 0$, and there will therefore be no feasible capital stock for which the fully informed farmer facing Case A2 OT2/OT3/OT3 conditions will prefer to produce organically. ϵ^* will be more likely to exceed $\overline{C}$ when the farmer faces small organic price premia.

Under Case A2, we have:

$$\hat{K}_j \leq \overline{C} \Rightarrow \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{\gamma_K}{(\rho+\mu)} \geq 0, \tag{B.65}$$

and

$$\hat{K}_j \geq 0 \Rightarrow \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} \leq (-\gamma_{cc})\, \mu \overline{C} + \frac{\gamma_K}{(\rho+\mu)} \tag{B.66}$$

Also, given:

$$\hat{K}_{con} = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu} = \overline{C} + \underbrace{\frac{(\rho+\mu)\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0} \leq \overline{C} \tag{B.67}$$

we know that $\left(\overline{C} - \hat{K}_{con}\right) \to 0$ implies:

$$\underbrace{\frac{(\rho+\mu)\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0}\to 0 \tag{B.68}$$

or

$$\underbrace{\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{\gamma_K}{(\rho+\mu)}}_{\geq 0}\to 0 \tag{B.69}$$

$$\underbrace{\frac{\alpha_c}{\alpha_b}}_{\geq 0}+\underbrace{\left(-\frac{1}{\alpha_b\cdot P_{con}}+\gamma_c-\frac{\gamma_K}{(\rho+\mu)}\right)}_{\leq 0}\to 0 \tag{B.70}$$

$$\alpha_c\to\alpha_b\cdot\underbrace{\left(\frac{1}{\alpha_b\cdot P_{con}}-\gamma_c+\frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0} \tag{B.71}$$

That is, $\left(\overline{C}-\hat{K}_{con}\right)\to 0$, given constant $\overline{C}$, implies small enough α_c, such that $\underbrace{\alpha_c}_{\geq 0}-\alpha_b\cdot\underbrace{\left(\frac{1}{\alpha_b\cdot P_{con}}-\gamma_c+\frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0}$ approaches 0 from above.

B.2.1 Comparative statics for $\Delta(\epsilon)$

Now we discuss the signs of $\frac{\partial\Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org}=\overline{C}$.

The results are summarized in Table B.4.

$$\frac{\partial\Delta(\epsilon)}{\partial P_{\mathrm{org}}}=\underbrace{\frac{1}{\rho}\cdot\left(A_y+\alpha_b\left(A_b+\overline{C}\gamma_K\right)\right)}_{\geq 0}\geq 0$$

$$\frac{\partial \Delta(\epsilon)}{\partial P_{\text{con}}} = \underbrace{\left(-\frac{1}{\rho} \cdot \alpha_b\right)}_{\leq 0} \cdot \left(\underbrace{A_b + \frac{A_y}{\alpha_b}}_{\geq 0} + \mu \cdot \left(\underbrace{\frac{P_{\text{con}}^{-1}}{\alpha_b} + \frac{1}{2} \cdot (-\gamma_{\text{cc}})\, \mu \cdot \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \right) \cdot \underbrace{\left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \right.$$

$$\left. + \underbrace{\gamma_K \left(\overline{C} - \frac{\rho}{\mu+\rho}\epsilon\right)}_{\geq 0} \right) \leq 0 \tag{B.72}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \mu} = -\frac{1}{\rho} P_{\text{con}} \alpha_b \frac{\gamma_K}{(\mu+\rho)} \left(-\frac{\rho \overline{C} + \mu \hat{K}_{con}}{\rho + \mu} + \left(\overline{C} + \frac{\rho}{\mu+\rho} \cdot \epsilon\right)\right) \tag{B.73}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \mu} = -\frac{1}{\rho} \cdot \underbrace{\left(P_{\text{con}} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu+\rho}\right)}_{\geq 0} \cdot \left(\underbrace{\overline{C} - \underbrace{\frac{\rho + \underbrace{\left(\frac{\hat{K}_{con}}{\overline{C}}\right)}_{\leq 1} \cdot \mu}{\rho+\mu}}_{\leq 1} \cdot \overline{C}}_{\geq 0} + \underbrace{\frac{\rho}{\mu+\rho}}_{\geq 0} \cdot \epsilon \right) \leq 0 \tag{B.74}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \rho} = \underbrace{\rho^{-2} P_{\text{con}} \alpha_b}_{\geq 0} \cdot \left(\left(\underbrace{\frac{\rho}{(\mu+\rho)^2} \cdot (-\gamma_K)}_{\leq 0} + \underbrace{\frac{1}{2} \cdot (-\gamma_{\text{cc}})\, \mu \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \right) \underbrace{\cdot \mu \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \right.$$

$$\left. + \underbrace{\left(\frac{\rho}{\mu+\rho}\right)^2 \cdot (-\gamma_K) \cdot \epsilon}_{\leq 0} + \underbrace{\left(\frac{A_y}{\alpha_b} + A_b + \gamma_K \cdot \overline{C}\right) \cdot \left(-\frac{P_{\text{org}} - P_{con}}{P_{con}}\right)}_{\leq 0} \right) \tag{B.75}$$

So we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \geq 0$ for small enough γ_K and small enough organic price premium $\frac{P_{\text{org}} - P_{con}}{P_{con}}$.

On the other hand we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \leq 0$ for large enough $\frac{P_{\text{org}} - P_{con}}{P_{con}}$.

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_c} = -\frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \mu \cdot \underbrace{\left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \leq 0 \tag{B.76}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_{\text{cc}}} = -\frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \cdot \frac{1}{2} \cdot \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2 \leq 0 \tag{B.77}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_K} = \frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \cdot \underbrace{\left(\underbrace{\left(\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}}\right)}_{\geq 0} \overline{C} + \frac{\mu}{\mu + \rho} \cdot \underbrace{\left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} + \frac{\rho}{\mu + \rho} \cdot \underbrace{\epsilon}_{\geq 0} \right)}_{\geq 0} \geq 0 \tag{B.78}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \overline{C}} = \frac{1}{\rho} \cdot \underbrace{(P_{\text{org}} - P_{\text{con}})}_{\geq 0} \cdot \alpha_b \cdot \gamma_K \geq 0 \tag{B.79}$$

$$\begin{aligned}\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} = \frac{1}{\rho} \cdot P_{\text{con}} \cdot \Bigg(& \underbrace{\left(\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}}\right) \cdot \left(A_b + \gamma_K \cdot \overline{C}\right)}_{\geq 0} + \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon}_{\geq 0} \\ & + \underbrace{\frac{1}{2} \gamma_{cc} \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b} \cdot \underbrace{\mu \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0}}_{?} \Bigg)\end{aligned} \tag{B.80}$$

So we will have $\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} \geq 0$ for large enough $\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}}$, and we will have $\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} \leq 0$ for large enough $\hat{c} = \mu\left(\overline{C} - \hat{K}_{con}\right)$, given $\gamma_{cc} \neq 0$. Note that $\hat{c} = \mu\left(\overline{C} - \hat{K}_{con}\right)$ will be larger for larger $\overline{C}$ or μ, or smaller $\hat{K}_{con}$. Given

$$\hat{K}_{con} = \frac{(\mu+\rho)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b}\right) - \gamma_K}{(\mu+\rho)\,\gamma_{cc}\mu} \geq 0$$

$$\Rightarrow (\mu+\rho)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b}\right) - \gamma_K \leq 0$$

we know that $\hat{K}_{con}$ will be smaller for smaller $-\gamma_{cc}$, $-\gamma_c$, or γ_K, or bigger α_c.

$$\frac{\partial \Delta(\epsilon)}{\partial A_y} = \underbrace{\frac{1}{\rho}\left(P_{\text{org}} - P_{\text{con}}\right)}_{\geq 0} \geq 0 \tag{B.81}$$

$$\frac{\partial \Delta(\epsilon)}{\partial A_b} = \underbrace{\frac{1}{\rho} \cdot \alpha_b \cdot \left(P_{\text{org}} - P_{\text{con}}\right)}_{\geq 0} \geq 0 \tag{B.82}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \alpha_c} = -\frac{1}{\rho} \cdot P_{\text{con}} \cdot \underbrace{\mu\left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} \leq 0 \tag{B.83}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} = \frac{1}{\mu+\rho} \cdot P_{con}\alpha_b\gamma_K \geq 0 \tag{B.84}$$

Table B.4: Comparative Statics for $\Delta(\epsilon)$ When Conventional Farmer Adopts OT2 (Case A2) and $K_{org} = \overline{C}$

Parameter i	**Sign**	**Condition**
A_b	$+$	
A_y	$+$	
P_{con}	$-$	
P_{org}	$+$	
$\overline{C}$	$+$	
μ	$-$	
ρ	$-$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$.
	$+$	Small γ_K and $\frac{P_{org}-P_{con}}{P_{con}}$.
γ_{cc}	$-$	
γ_c	$-$	
γ_K	$+$	
α_b	$-$	Large enough $\hat{c} = \mu\left(\overline{C} - \hat{K}_{con}\right)$, given $\gamma_{cc} \neq 0$. (Note that $\uparrow \hat{c} \Rightarrow \uparrow \overline{C}$, μ, α_c, $\downarrow \lvert\gamma_{cc}\rvert$, $\lvert\gamma_c\rvert$, γ_K).
	$+$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$.
α_c	$-$	
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon)$ when $\hat{K}_j \in [0, K_{0,j}]\, \forall j \in \{con, org\}$, assuming $K_{org} = \overline{C}$ (not responsive to assumption that $\overline{c} = \mu\overline{C}$ (looks the same either way)). A conventional farmer is said to prefer producing organically when $\Delta(\epsilon) > 0$.

B.2.2 Comparative statics for ϵ^*

We now calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$.

The results are summarized in Table B.5.

$$\frac{\partial \epsilon^*}{\partial \mu} = \frac{1}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{\frac{1}{2} \cdot \mu \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} + \frac{\gamma_K}{\rho + \mu} \right)}_{\geq 0} \underbrace{\left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} + \underbrace{\left(-\gamma_K \cdot \left(\frac{P_{org}}{P_{con}} + 1 \right) \overline{C} - \left(\frac{P_{org}}{P_{con}} - 1 \right) \cdot \left(A_b + \frac{A_y}{\alpha_b} \right) \right)}_{\leq 0} \right) \tag{B.85}$$

The sign of $\frac{\partial \epsilon^*}{\partial \mu}$ is still ambiguous without futher restrictions to our parameter values. However, we can see that for large enough organic premia we will have $\frac{\partial \epsilon^*}{\partial \mu} \geq 0$. On the other hand, for small enough organic premia we will have

$$\frac{\partial \epsilon^*}{\partial \mu} \to \frac{1}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{\frac{1}{2} \cdot \mu^2 \left(-\gamma_{cc} \right) \left(\overline{C} - \hat{K}_{con} \right)^2}_{\geq 0} - \underbrace{\gamma_K \left(\overline{C} + \frac{\rho}{\rho + \mu} \cdot \overline{C} + \frac{\mu}{\rho + \mu} \cdot \hat{K}_{con} \right)}_{\geq 0} \right) \tag{B.86}$$

Therefore, given small enough organic price premia (such that the value of $\frac{\partial \epsilon^*}{\partial \mu}$ is close enough to the value of the expression we have above), and large enough $\hat{K}_{con}$ (s.t. $\left(\overline{C} - \hat{K}_{con} \right) \to 0$), we will have $\frac{\partial \epsilon^*}{\partial \mu} \leq 0$.

Since $\left(\overline{C} - \hat{K}_{con} \right) \to 0$ implies that $\underbrace{\alpha_c}_{\geq 0} - \alpha_b \cdot \underbrace{\left(\frac{1}{\alpha_b \cdot P_{con}} - \gamma_c + \frac{\gamma_K}{(\rho + \mu)} \right)}_{\geq 0}$ approaches 0 from above, we will have $\frac{\partial \epsilon^*}{\partial \mu} \leq 0$ for small enough organic price premia, and and small enough α_c.

$$\frac{\partial \epsilon^*}{\partial \rho} = -\left(\frac{\mu}{\rho} \right)^2 \cdot \left(\frac{\overline{C} - \hat{K}_{con}}{(\rho + \mu)^2} + K_{org} - \frac{1}{\mu} \overline{C} \right) \tag{B.87}$$

Given $K_{org} = \overline{C}$, this yields:

$$\frac{\partial \epsilon^*}{\partial \rho} = -\left(\frac{\mu}{\rho}\right)^2 \cdot \left(\underbrace{\frac{\overline{C} - \hat{K}_{con}}{(\rho+\mu)^2}}_{\geq 0} + \frac{1}{\mu}\underbrace{(\mu - 1)}_{\leq 0}\overline{C}\right) \tag{B.88}$$

The sign of $\frac{\partial \epsilon^*}{\partial \rho}$ is ambiguous, but we can see that for large enough μ (such that $\frac{1}{\mu}\underbrace{(\mu-1)}_{\leq 0}\overline{C} \to 0$) we will have $\frac{\partial \epsilon^*}{\partial \rho} \leq 0$. On the other hand, for small enough α_c such that $\frac{\overline{C}-\hat{K}_{con}}{(\rho+\mu)^2} \to 0$, we will have $\frac{\partial \epsilon^*}{\partial \rho} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K^2} \cdot \left(\frac{P_{org}}{P_{con}} \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right)\mu\left(\overline{C} - K_{org}\right) + \underbrace{\left(\frac{P_{org}}{P_{con}} - 1\right) \cdot \left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0}\right. \tag{B.89}$$

$$\left. + \left(\underbrace{\frac{1}{2} \cdot \gamma_{cc} \cdot \left(\frac{\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K}{\gamma_{cc}}\right)^2}_{\leq 0} + \underbrace{\frac{\gamma_K}{(\rho+\mu)} \cdot \frac{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)}{\gamma_{cc}}}_{\leq 0}\right)\right)$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_K}$ is still ambiguous, but we can see that as $\left(A_b + \frac{A_y}{\alpha_b}\right)$ increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_K}$ will become non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_K} \geq 0$). On the other hand, as the organic price premia shrinks (such that $\left(\frac{P_{org}}{P_{con}} - 1\right) \to 0$), and the certification criteria becomes stricter (such that $\left(\overline{C} - K_{org}\right) \to 0$), we will eventually get $\frac{\partial \epsilon^*}{\partial \gamma_K} \leq 0$.

$$\frac{\partial \epsilon^*}{\partial \gamma_{cc}} = \underbrace{\frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \left(\underbrace{-\frac{P_{org}}{P_{con}} \cdot \frac{1}{2}\left(\mu\left(\overline{C} - K_{org}\right)\right)^2}_{\leq 0} + \underbrace{\frac{1}{2} \cdot \frac{1}{\gamma_{cc}^2}\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)^2}_{\geq 0}\right) \tag{B.90}$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ is still ambiguous, but we can see that as $\frac{P_{org}}{P_{con}}$ (which reflects the organic premium) increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ will become non-positive ($\frac{\partial \epsilon^*}{\partial \gamma_{cc}} \leq 0$). On the other hand, as the organic certification becomes stricter $\left(K_{org} \to \overline{C}\right)$, $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ eventually becomes non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_{cc}} \geq 0$).

$$\frac{\partial \epsilon^*}{\partial \gamma_c} = \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{-\frac{P_{org}}{P_{con}} \cdot \mu\left(\overline{C} - K_{org}\right)}_{\leq 0} + \underbrace{\frac{1}{(-\gamma_{cc})} \underbrace{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0}}_{\geq 0} \right) \tag{B.91}$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_c}$ is still ambiguous, but we can see that as $\frac{P_{org}}{P_{con}}$ (which reflects the organic premium) increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_c}$ will become non-positive ($\frac{\partial \epsilon^*}{\partial \gamma_c} \leq 0$). On the other hand, as the organic certification becomes stricter $\left(K_{org} \to \overline{C}\right)$, $\frac{\partial \epsilon^*}{\partial \gamma_c}$ eventually becomes non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_c} \geq 0$).

$$\begin{aligned} \frac{\partial \epsilon^*}{\partial P_{con}} = \underbrace{\frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \Bigg(& \underbrace{\frac{P_{org}}{P_{con}^2} \mu\left(\overline{C} - K_{org}\right)}_{\geq 0} \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right) \\ & + \underbrace{\frac{1}{(-\gamma_{cc})} \cdot \left(\frac{P_{con}^{-2}}{\alpha_b}\right)}_{\geq 0} \cdot \underbrace{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)}_{\geq 0} + \underbrace{\frac{P_{org}}{P_{con}^2}\left(\gamma_K \cdot K_{org} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \Bigg) \end{aligned} \tag{B.92}$$

The sign of $\frac{\partial \epsilon^*}{\partial P_{con}}$ is still ambiguous. However, we can see that for relatively large $\overline{C}$ and organic certification criteria is weak enough (such that $\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right) \leq 0$ and also such that $\left(\overline{C} - K_{org}\right)$ is relatively large, so that $\frac{P_{org}}{P_{con}^2}\mu\left(\overline{C} - K_{org}\right) \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right)$ is negative and has a large magnitude), then we will have $\frac{\partial \epsilon^*}{\partial P_{con}} \leq 0$. On the other hand, when the certification criteria is strict enough (so that $\left(\overline{C} - K_{org}\right)$ is close enough to zero), we will have $\frac{\partial \epsilon^*}{\partial P_{con}} \geq 0$.

$$\frac{\partial\epsilon^*}{\partial\overline{C}} = \underbrace{\frac{\mu}{\rho}}_{\geq 0}\left(\underbrace{\frac{(\mu+\rho)}{\gamma_K}\cdot\frac{P_{org}}{P_{con}}}_{\geq 0}\cdot\left(\underbrace{(-\gamma_{cc})\,\mu\left(\overline{C}-K_{org}\right)}_{\geq 0}-\left(\underbrace{\gamma_c+\frac{\alpha_c-P_{org}^{-1}}{\alpha_b}}_{\geq 0}\right)\right)-1\right) \tag{B.93}$$

The sign of $\frac{\partial\epsilon^*}{\partial\overline{C}}$ is still ambiguous. However, we can see that for relatively large $\overline{C}$ (such that $\left(\overline{C}-K_{org}\right)$ is relatively large), we will have $\frac{\partial\epsilon^*}{\partial\overline{C}} \geq 0$. On the other hand, for sufficiently strict certification criteria (such that $\left(\overline{C}-K_{org}\right)\to 0$) we will have $\frac{\partial\epsilon^*}{\partial\overline{C}} \leq 0$.

$$\frac{\partial\epsilon^*}{\partial K_{org}} = \underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\frac{P_{org}}{P_{con}}\cdot\mu}_{\geq 0}\left(\underbrace{\gamma_{cc}\mu\left(\overline{C}-K_{org}\right)}_{\leq 0}+\underbrace{\gamma_c+\frac{\alpha_c-P_{org}^{-1}}{\alpha_b}}_{\geq 0}\right)+\underbrace{\left(1-\frac{\mu+\rho}{\rho}\cdot\frac{P_{org}}{P_{con}}\right)}_{\leq 0} \tag{B.94}$$

We see that the sign of $\frac{\partial\epsilon^*}{\partial K_{org}}$ is ambiguous. We have that for large enough α_c, small enough γ_K , and $K_{org}\neq 0$, we will have $\frac{\partial\epsilon^*}{\partial K_{org}} \geq 0$. On the other hand, for weak enough initial certification criteria (such that $\left(\overline{C}-K_{org}\right)$ is sufficiently large), $|\gamma_{cc}|$ large enough, and $\mu\neq 0$, we will have $\frac{\partial\epsilon^*}{\partial K_{org}} \leq 0$.

$$\frac{\partial\epsilon^*}{\partial\alpha_c} = \underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\frac{1}{\alpha_b}}_{\geq 0}\cdot\left(\underbrace{\frac{P_{org}}{P_{con}}\mu\left(K_{org}-\overline{C}\right)}_{\leq 0}+\underbrace{\frac{1}{(-\gamma_{cc})}\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{1}{(\rho+\mu)}\gamma_K\right)}_{\geq 0}\right) \tag{B.95}$$

The sign of $\frac{\partial\epsilon^*}{\partial\alpha_c}$ is ambiguous, but we see that if $K_{org}\neq\overline{C}$ and the organic premium is high enough, then we will have $\frac{\partial\epsilon^*}{\partial\alpha_c} \leq 0$. On the other hand, if the certification criteria is strict enough (such that $\left(K_{org}-\overline{C}\right)$ is close enough to zero), then we will have $\frac{\partial\epsilon^*}{\partial\alpha_c} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}}_{\geq 0}\cdot\frac{1}{\alpha_b^2}\cdot\left(\underbrace{\frac{P_{org}}{P_{con}}\mu\left(\overline{C}-K_{org}\right)}_{\geq 0}\cdot\underbrace{\left(\alpha_c - P_{org}^{-1}\right)}_{\geq 0}+\underbrace{\left(\frac{P_{org}}{P_{con}}-1\right)\cdot A_y}_{\geq 0}\right. \tag{B.96}$$

$$\left. + \underbrace{\frac{1}{\gamma_{cc}}\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)\left(\alpha_c - P_{con}^{-1}\right)}_{\leq 0}\right) \geq 0$$

The sign of $\frac{\partial \epsilon^*}{\partial \alpha_b}$ is ambiguous. However, we can see that if factors of production other than soil bacteria and synthetic compounds are relatively unimportant for production (so that $A_y \to 0$), and the certification criteria is sufficiently strict (such that $\left(\overline{C}-K_{org}\right) \to 0$), then we will have $\frac{\partial \epsilon^*}{\partial \alpha_b} \leq 0$. On the other hand, if the organic price premium is high enough and also either $A_y \neq 0$ or $K_{org} \neq \overline{C}$, then we will have $\frac{\partial \epsilon^*}{\partial \alpha_b} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial A_b} = \frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\left(1-\frac{P_{org}}{P_{con}}\right) \leq 0 \tag{B.97}$$

$$\frac{\partial \epsilon^*}{\partial A_y} = \frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\left(1-\frac{P_{org}}{P_{con}}\right)\cdot\frac{1}{\alpha_b} \leq 0 \tag{B.98}$$

Table B.5: Comparative Statics for ϵ^* When Conventional Farmer Adopts OT2 (Case A2) and $K_{org} = \overline{C}$

Parameter i	**Sign**	**Condition**
A_b	$-$	
A_y	$-$	
P_{con}	$+$	
P_{org}	$-$	
$\overline{C}$	$-$	
μ	$-$ $+$	Small enough $(P_{org} - P_{con})$ and $\left(\overline{C} - \hat{K}_{con}\right)$. Given constant $\overline{C}$, the latter implies small enough α_c. Large $(P_{org} - P_{con})$.
ρ	$-$ $+$ 0	Large enough μ. Small enough α_c (so that $\hat{K}_j \uparrow$and $\hat{K}_j \to \overline{C}^-$). For: (1) small enough α_c and also large enough μ; or (2) small enough μ.
γ_{cc}	$+$	
γ_c	$+$	
γ_K	$-$ $+$	For $\frac{P_{org}}{P_{con}}$ small enough. For $\frac{P_{org}}{P_{con}}$ large enough, and $(A_b+A_y)\neq 0$.
α_b	$-$ $+$	For small enough A_y or $\frac{P_{org}}{P_{con}}$. For large enough A_y and $\frac{P_{org}}{P_{con}}$.
α_c	$+$	

Notes: Table reports comparative statics for ϵ^* when $\hat{K}_j \in [0, \overline{C}]\, \forall j \in \{con, org\}$, assuming $K_{org} = \overline{C}$ (not responsive to assumption that $\bar{c} = \mu\overline{C}$ (looks the same either way)). A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ^* lower than K_{org}.

B.2.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing A2, OT2/OT3/OT4 conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below.

Given the assumption that $K_{org} = \overline{C}$, and assuming conventional crop prices are not zero, we can write:

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \frac{\left(\frac{1}{2} \cdot (-\gamma_{cc})\, \mu^2 \left(\overline{C} - \hat{K}_{con}\right)^2 - \frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \epsilon\right)}{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{B.99}$$

We'd like to see if we can express this condition in more intuitive terms:

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \frac{\frac{1}{\rho} \cdot \alpha_b \cdot \frac{1}{2} \cdot (-\gamma_{cc})\, c(\hat{T})^2 - \frac{1}{\mu+\rho} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon}{\frac{1}{\rho} \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{B.100}$$

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \frac{P_{con}}{P_{con}} \cdot \frac{\int_0^\infty \frac{1}{2} \cdot \alpha_b \left(-\gamma_{cc}\right) c(\hat{T})^2 \cdot e^{-\rho \cdot t} dt - \int_0^\infty \alpha_b \gamma_K \cdot \epsilon \cdot e^{-(\mu+\rho)\cdot t} dt}{\int_0^\infty \left(\alpha_b \left(\gamma_K \overline{C} + A_b\right) + A_y\right) \cdot e^{-\rho \cdot t} dt} \tag{B.101}$$

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \frac{\int_0^\infty P_{con} \cdot \alpha_b \cdot \frac{1}{2} \cdot (-\gamma_{cc})\, c(\hat{T})^2 \cdot e^{-\rho \cdot t} dt - \int_0^\infty P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon \cdot e^{-(\mu+\rho)\cdot t} dt}{\int_0^\infty P_{con} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b\right) + A_y\right) \cdot e^{-\rho \cdot t} dt} \tag{B.102}$$

$$
\begin{aligned}
&\underbrace{\frac{P_{org}-P_{con}}{P_{con}}}_{\substack{\text{Organic price premium: How much larger organic crop prices are than}\\ \text{conventional prices, as a proportion of conventional crop prices}}} \\
\geq\; &\underbrace{\underbrace{\left(\int_0^{\infty} P_{con}\cdot f_{org}\cdot e^{-\rho\cdot t}dt\right)^{-1}}_{\substack{\text{PDV of entire stream}\\ \text{of net benefits from}\\ \text{managing organically}\\ \text{but selling at}\\ \text{conventional prices}}}\cdot\left(\underbrace{\int_0^{\infty} P_{con}\cdot\alpha_b\cdot\frac{1}{2}\cdot(-\gamma_{cc})\, c(\hat{T})^2\cdot e^{-\rho\cdot t}dt}_{\substack{\text{PDV of value destroyed by managing}\\ \text{conventionally, from the moment}\\ \text{the stationary solution is reached and}\\ \text{onwards, from the perspective of } \hat{T}}}\underbrace{-\underbrace{\int_0^{\infty}\underbrace{P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\Big(\underbrace{\underbrace{\epsilon}_{\substack{\text{Additional amount of capital that}\\ \text{the farmer would have if they}\\ \text{received a capital transfer just large}\\ \text{enough to allow them to satisfy}\\ \text{the organic certification requirement}\\ \text{at s=0.}}}\cdot e^{-\mu\cdot t}}_{\substack{\text{Amount of capital from the}\\ \text{transfer at s=0 that remains at s=t.}}}\Big)}_{\substack{\text{per-period value created in period s=t by a capital transfer at}\\ \text{period s=0, from the perspective of period s=t.}}}\cdot e^{-\rho\cdot t}dt}_{\substack{\text{PDV of entire stream of benefits generated by a one-time transfer of capital,}\\ \text{at time s=0, large enough to allow the farmer to satisfy the organic certification}\\ \text{requirement, from the perspective of period s=0, if prices continued to be convetional}}}}_{\substack{\text{How much greater the loss incurred by using synthetic compounds at stationary state}\\ \text{levels is than the loss incurred by opting not to receive a one-time transfer of}\\ \text{capital large enough to allow the farmer to satisfy the organic certification threshold}}}\right)}_{\substack{\text{PDV of value destroyed by managing conventionally as a proportion of the PDV created by managing (but not selling) organically,}\\ \text{net of relative value of the gain from a one-time transfer of capital just large enough to allow the farmer to satisfy the organic certification requirements}}}
\end{aligned}
\tag{B.103}
$$

We're finding that there's a value to (1) not managing conventionally (i.e. reducing one's per-period synthetic compound use), but also to (2) receiving enough capital to satisfy the organic certification requirement (i.e. having an overall larger capital stock).

$$\underbrace{\int_0^\infty (P_{org} - P_{con}) \cdot f_{org} \cdot e^{-\rho \cdot t} dt}_{\text{Net gain from organic premium}} + \underbrace{\int_0^\infty P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(\epsilon \cdot e^{-\mu \cdot t}\right) \cdot e^{-\rho \cdot t} dt}_{\text{gain from organic management}} \geq \underbrace{\int_0^\infty P_{con} \cdot \alpha_b \cdot \frac{1}{2} \cdot (-\gamma_{cc})\, c(\hat{T})^2 \cdot e^{-\rho \cdot t} dt}_{\text{loss from conventional management}} \tag{B.104}$$

More broadly we may be interested in how the requisite price premia changes in response to changes in our model parameters. To do this, let's first rearrange our expression for $\frac{P_{org}}{P_{con}}$ further.

$$\frac{P_{org} - P_{con}}{P_{con}} \geq \left(\frac{1}{2} \cdot \frac{1}{(-\gamma_{cc})} \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\mu + \rho)} \right)^2 - \frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon \right) \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)^{-1} \tag{B.105}$$

Let $\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*$ denote the threshold value:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = \left(\frac{1}{2} \cdot \frac{1}{(-\gamma_{cc})} \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\mu + \rho)} \right)^2 - \frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon \right) \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)^{-1} \tag{B.106}$$

Then we can determine how $\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters by examining the signs of the partials below. The results are summarized in Table B.6.

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \epsilon} = -\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)^{-1} \leq 0 \tag{B.107}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}}{\partial\rho}=\frac{\underbrace{\gamma_K\cdot\mu}_{\geq 0}\cdot\left(K_0-\hat{K}_{con}\right)}{\underbrace{(\mu+\rho)^2}_{\geq 0}\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}} \tag{B.108}$$

where in case A2 we have that $\hat{K}_{con}\in[0,\overline{C}]$. So we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}}{\partial\rho}\geq 0$ for large enough initial capital stock, K_0, and we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}}{\partial\rho}\leq 0$ for small enough initial capital stock, K_0.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}}{\partial\mu}=\underbrace{\frac{\gamma_K}{\mu+\rho}}_{\geq 0}\cdot\left(\overline{C}-\frac{\mu\cdot\hat{K}_{con}+\rho\cdot K_0}{\mu+\rho}\right)\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \tag{B.109}$$

In case A2 we have that $\hat{K}_{con}\in[0,\overline{C}]$. In all cases we also have that $K_0\in[0,\overline{C}]$. Therefore, it must be the case that

$$\left(\overline{C}-\frac{\mu\cdot\hat{K}_{con}+\rho\cdot K_0}{\mu+\rho}\right)\geq 0. \tag{B.110}$$

So we have that

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}}{\partial\mu}=\underbrace{\frac{\gamma_K}{\mu+\rho}}_{\geq 0}\cdot\underbrace{\left(\overline{C}-\frac{\mu\cdot\hat{K}_{con}+\rho\cdot K_0}{\mu+\rho}\right)}_{\geq 0}\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0}\geq 0 \tag{B.111}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K}=\left(\underbrace{\frac{\mu\cdot\hat{K}_{con}+\rho\cdot K_0}{\mu+\rho}-\overline{C}}_{\leq 0}\right.$$

$$\left.+\frac{\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0}+\underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\overline{C}-K_0\right)}_{\geq 0}}{\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}}\cdot\overline{C}\right)\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \tag{B.112}$$

Here we will have that $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K}\leq 0$ if K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K}\geq 0$ if both $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$) and also $\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)$ is sufficiently small.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\overline{C}}=\left(\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0}+\underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\overline{C}-K_0\right)}_{\geq 0}\right)\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}\cdot\gamma_K}_{\geq 0} \tag{B.113}$$

The farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\overline{C}}\geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\overline{C}}\leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y}=\left(\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0}+\underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\overline{C}-K_0\right)}_{\geq 0}\right)$$

$$\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}\cdot\left(\frac{1}{\alpha_b}\right)}_{\geq 0} \tag{B.114}$$

The farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} = \left(\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\overline{C}-K_0\right)}_{\geq 0}\right)\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}}_{\geq 0} \tag{B.115}$$

The farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\begin{aligned}\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} = &\left(\underbrace{\mu\left(\hat{K}_{con}-\overline{C}\right)}_{\leq 0}\cdot\left(\frac{P_{con}\alpha_c-1}{P_{con}\alpha_b^2}\right)\right.\\ &\left. + \frac{\underbrace{\frac{1}{2}\cdot(-\gamma_{cc})\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\geq 0} + \underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(K_0-\overline{C}\right)}_{\leq 0}}{\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}}\cdot\underbrace{\frac{A_y}{\alpha_b^2}}_{\geq 0}\right)\\ &\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0}\end{aligned} \tag{B.116}$$

The farmer will therefore face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} \leq 0$, for example, when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} \geq 0$ when both K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$), and the marginal revenue associated with conventional synthetic compound use is sufficiently close to the marginal cost of synthetic compound use such that $P_{con}\alpha_c \to 1$.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_{cc}} = \frac{1}{2}\cdot\left(\mu\left(\overline{C}-\hat{K}_{con}\right)\right)^2\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}\geq 0 \tag{B.117}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_{c}} = \mu\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}\geq 0 \tag{B.118}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_{c}} = \mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\frac{1}{\alpha_b}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}\geq 0 \tag{B.119}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial P_{con}} = \mu\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\frac{P_{con}^{-2}}{\alpha_b}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}\geq 0 \tag{B.120}$$

Table B.6: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}$ When Conventional Farmer Adopts OT2 (Case A2) and $K_{org}=\overline{C}$

Parameter	Full Information: A2 (OT2/3)	
ϵ	-	
ρ	$-$ $+$	For small enough K_0. For large enough K_0.
μ	$+$	
γ_K	$-$ $+$	Sufficiently large K_0. Sufficiently large $\hat{K}_{con}$ *and* small enough $\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)$.
$\overline{C}$	$-$ $+$	Sufficiently large K_0. Sufficiently large $\hat{K}_{con}$.
A_y	$-$ $+$	Sufficiently large K_0. Sufficiently large $\hat{K}_{con}$.
A_b	$-$ $+$	Sufficiently large K_0. Sufficiently large $\hat{K}_{con}$.
α_b	$-$ $+$	Sufficiently large $\hat{K}_{con}$. Sufficiently large K_0 *and* $P_{con}\alpha_c \to 1^{+}$.
γ_{cc}	$+$	
γ_c	$+$	
α_c	$+$	
P_{con}	$+$	
c_0	0	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^{*}$ when $\hat{K}_j \in [0,\overline{C}]\,\forall j \in \{con, org\}$, assuming $K_{org}=\overline{C}$ (not responsive to assumption that $\bar{c}=\mu\overline{C}$ (looks the same either way)).

B.3 Discrete Analysis for OT3' (Case F14A)

Case F14: $R(K)_{con}$ and $R(K)_{org}$ Constant with $R(K)_{con} = \rho$ and $R(K)_{org} < \rho$

Case F14A: $\gamma_{cc} = 0$, $\mu \neq 0$, $R_{con}(K) = \rho\ \forall K$ (conventional OT3'), and $R_{org}(K) < \rho$

In this case the stage 1 conventional farmer follows the following solution trajectories (OT3'):

$$K(t)_{con} = K_{org} - \epsilon\, \forall t \tag{B.121}$$

$$I(t)_{con} = 0\, \forall t \tag{B.122}$$

$$c(t)_{con} = \mu\left(\overline{C} - K(t)_{con}\right) - I(t)_{con} \tag{B.123}$$

$$= \mu\left(\overline{C} - K_{org} + \epsilon\right)\ \forall t \tag{B.124}$$

Given the assumption that $K_{org} = \overline{C}$, $K(t)_{con}$ and $c(t)_{con}$ simply to

$$K(t)_{con} = \overline{C} - \epsilon\, \forall t \tag{B.125}$$

$$c(t)_{con} = \mu \cdot \epsilon\, \forall t \tag{B.126}$$

where $\epsilon > 0$ and is determined by the equation $K(0) = K_{org} - \epsilon$.

The stage 2 organic farmer will, conditional on having reached the organic threshold and decided to remain organic, adopt the following constrained trajectory:

$$\bar{K}(t)_{org} = \overline{C}\, \forall t \tag{B.127}$$

$$\bar{I}(t)_{org} = 0\, \forall t \tag{B.128}$$

$$\overline{C}(t)_{org} = \mu\left(\overline{C} - \bar{K}(t)_{org}\right) - \bar{I}(t)_{org} \tag{B.129}$$

$$= 0\, \forall t \tag{B.130}$$

Applying our solutions for $c(t)_{con}$ and $K(t)_{con}$ from above, as well as the assumption that $K_{org} = \overline{C}$, and $\gamma_{cc} = 0$, we can then write $V_{con}(K_{org} - \epsilon)$ as:

$$V_{con}(\overline{C}-\epsilon) = \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of "level effect"}\\ \text{of other agricultural inputs}\\ \text{at conventional prices}}} + \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\left(\overline{C}-\epsilon\right)}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} + \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{\gamma_K}{\mu+\rho}\cdot\mu\cdot\epsilon}_{\substack{\text{PDV of using synthetic compounds}\\ \text{at dynamically optimal}\\ \text{conventional rate } \mu\cdot\epsilon}} \tag{B.131}$$

When $\gamma_{cc}=0$ and $\mu\neq 0$, $R_{con}(K)=\rho\,\forall K$ implies:

$$-\mu+\frac{\gamma_K}{\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}}=\rho \tag{B.132}$$

$$\frac{\gamma_K}{\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}}=\mu+\rho \tag{B.133}$$

$$\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}=\frac{\gamma_K}{\mu+\rho} \tag{B.134}$$

On the other hand, the F14 A fully informed farmer will face the following stage 2 value function:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K\underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}P_{org}\cdot(\alpha_b A_b+A_y)}_{\substack{\text{PDV of "level effect"}\\ \text{of other agricultural inputs}\\ \text{at organic prices}}} \tag{B.135}$$

Given $K_{org}=\overline{C}$, the $\Delta^{F14\,A}(\epsilon)$ faced by the conventional F14 A farmer is given by:

$$\Delta^{F14\,A}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of value of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}}$$

$$- \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \underbrace{(\overline{C} - \epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon}_{\substack{\text{PDV of using synthetic compounts} \\ \text{at dynamically optimal} \\ \text{conventional rate } \mu \cdot \epsilon}} \tag{B.136}$$

which we can simplify as follows:

$$\Delta^{F14\,A}(\epsilon) = \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of organic price premium} \\ \text{from organic stock effect}}} + \underbrace{\frac{1}{\rho}\alpha_b \left(P_{org} - P_{con}\right)\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of organic price premium} \\ \text{from ag. base productivity}}} \tag{B.137}$$

$$+ \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \epsilon}_{\substack{\text{PDV of additional value gained from} \\ \text{microbial productivity after adopting} \\ \text{organic management practices,} \\ \text{valued at conventional prices}}} - \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \cdot \mu \cdot \epsilon}_{\substack{\text{PDV of using synthetic} \\ \text{compounds at dynamically} \\ \text{optimal conventional rate } \mu \cdot \epsilon}}$$

B.3.1 Comparative statics for $\Delta(\epsilon)$

Now we discuss the signs of $\frac{\partial \Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org} = \overline{C}$.

The results are summarized in Table B.7.

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \rho} = -\frac{1}{\rho^2} \cdot P_{con} \cdot \alpha_b \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) + \left((\rho - 1) \cdot \frac{\left(\frac{\mu+\rho-1}{\rho-1} \right) \cdot \mu + \rho}{(\mu + \rho)^2} \right) \cdot \gamma_K \cdot \epsilon \right) \tag{B.138}$$

So we have the following *sufficient* conditions

- $\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \rho} \leq 0$ for
  - Sufficiently large $\frac{P_{org}-P_{con}}{P_{con}}$
- $\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \rho} \geq 0$ for
  - Sufficiently small $\frac{P_{org}-P_{con}}{P_{con}}$ and μ, and $\rho < 1$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \mu} = -P_{con} \cdot \alpha_b \cdot \frac{1}{(\mu + \rho)^2} \cdot \gamma_K \cdot \epsilon \leq 0 \tag{B.139}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial P_{org}} = \frac{1}{\rho} \cdot \alpha_b \gamma_K \overline{C} + \frac{1}{\rho} \alpha_b \left(A_b + \frac{A_y}{\alpha_b} \right) \geq 0 \tag{B.140}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial P_{con}} = -\frac{1}{\rho} \cdot \alpha_b \cdot \left(A_b + \frac{A_y}{\alpha_b} + \gamma_K \cdot \left(\frac{\mu}{\mu + \rho} + \underbrace{\frac{\overline{C}}{\epsilon}}_{\geq 1} -1 \right) \cdot \epsilon \right) \leq 0 \tag{B.141}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \alpha_b} = \frac{1}{\rho} \cdot P_{con} \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \left(\gamma_K \overline{C} + A_b \right) + \frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon \right) \geq 0 \tag{B.142}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \gamma_K} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \overline{C} + \frac{\rho}{\mu + \rho} \cdot \epsilon \right) \geq 0 \tag{B.143}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \overline{C}} = \frac{1}{\rho} \left(P_{org} - P_{con} \right) \cdot \alpha_b \gamma_K \geq 0 \tag{B.144}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial A_b} = \frac{1}{\rho} \alpha_b \left(P_{org} - P_{con} \right) \geq 0 \tag{B.145}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial A_y} = \frac{1}{\rho} \left(P_{org} - P_{con} \right) \geq 0 \tag{B.146}$$

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \epsilon} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(1 - \underbrace{\frac{\mu}{\mu + \rho}}_{\leq 1} \right) \geq 0 \tag{B.147}$$

B.3.2 Comparative statics for ϵ^*

We now calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$.

The results are summarized in Table B.8.

Next we consider the partials of ϵ^* w.r.t. our model parameters.

Assuming $P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \frac{\rho}{\mu+\rho} \cdot \neq 0$, we can then write:

$$\Rightarrow \epsilon^* = -\frac{\mu+\rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}}\right)\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}\right) \leq 0 \tag{B.148}$$

Given

$$\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial \epsilon} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \left(1 - \frac{\mu}{\mu+\rho}\right) \geq 0, \tag{B.149}$$

$\epsilon^* \leq 0$ implies that the F14A farmer prefers organic given any initial capital stock. Still, we may at some point be interested in how the value of ϵ^* responds to changes in our parameter values in this case, so we will calculate the partials of ϵ^* wrt to these model parameters.

$$\frac{\partial \epsilon^*}{\partial \rho} = \frac{1}{\rho} \cdot \underbrace{\left(\frac{\mu+\rho}{\rho} - 1\right)}_{\geq 0} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}}\right)\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}\right) \geq 0 \tag{B.150}$$

$$\frac{\partial \epsilon^*}{\partial \mu} = -\left(\frac{P_{org} - P_{con}}{P_{con}}\right)\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}\right) \leq 0 \tag{B.151}$$

$$\frac{\partial \epsilon^*}{\partial P_{org}} = -\frac{\mu+\rho}{\rho} \cdot \left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}\right) \leq 0 \tag{B.152}$$

$$\frac{\partial \epsilon^*}{\partial P_{con}} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{1}{P_{con}} + \frac{P_{org} - P_{con}}{P_{con}^2} \right) \underbrace{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)}_{\geq 0} \geq 0 \tag{B.153}$$

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\frac{A_y}{\alpha_b^2} \cdot \gamma_K^{-1} \right) \geq 0 \tag{B.154}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-2} \geq 0 \tag{B.155}$$

$$\frac{\partial \epsilon^*}{\partial \overline{C}} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \leq 0 \tag{B.156}$$

$$\frac{\partial \epsilon^*}{\partial A_b} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\gamma_K^{-1} \right) \leq 0 \tag{B.157}$$

$$\frac{\partial \epsilon^*}{\partial A_y} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\frac{1}{\alpha_b} \cdot \gamma_K^{-1} \right) \leq 0 \tag{B.158}$$

B.3.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*$

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing F14A conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below.

Assuming that $P_{con} \neq 0$, and assuming that $\frac{1}{\rho}\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \neq 0$, we can write:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = -\frac{1}{\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}}\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\leq 0 \tag{B.159}$$

Given

$$\frac{\partial\Delta^{F14\,A}(\epsilon)}{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)} = \frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)\geq 0, \tag{B.160}$$

$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* \leq 0$ implies that the F14A farmer prefers organic given any non-negative price premium. Still, we may at some point be interested in how the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ responds to changes in our parameter values in this case, so we will calculate the partials of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ wrt to these model parameters.

Then we can determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters by examining the signs of the partials below. The results are summarized in Table B.9.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\epsilon} = -\frac{1}{\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}}\cdot\frac{\rho}{\mu+\rho}\leq 0 \tag{B.161}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\rho} = -\frac{1}{\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}}\cdot\frac{\mu}{(\mu+\rho)^2}\cdot\epsilon\leq 0 \tag{B.162}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\mu} = \frac{1}{\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}}\cdot\frac{\rho}{(\mu+\rho)^2}\cdot\epsilon\geq 0 \tag{B.163}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial P_{con}} = 0 \tag{B.164}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_b}=\frac{1}{\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)^2}\cdot\left(-\frac{A_y}{\alpha_b^2}\cdot\gamma_K^{-1}\right)\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\leq 0 \tag{B.165}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K}=\frac{1}{\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)^2}\cdot\left(-\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-2}\right)\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\leq 0 \tag{B.166}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y}=\frac{1}{\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)^2}\cdot\left(\frac{1}{\alpha_b}\cdot\gamma_K^{-1}\right)\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\geq 0 \tag{B.167}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b}=\frac{1}{\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)^2}\cdot\gamma_K^{-1}\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\geq 0 \tag{B.168}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\overline{C}}=\frac{1}{\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)^2}\cdot\frac{\rho}{\mu+\rho}\cdot\epsilon\geq 0 \tag{B.169}$$

Table B.7: Comparative Statics for $\Delta^{F14\,A}(\epsilon)$ For Case F14 A When $K_{org} = \overline{C}$

Summary of $\frac{\partial \Delta^{F14\,A}(\epsilon)}{\partial i}$		
i	Sign	Condition
ρ	$-$	Large $\frac{P_{org}-P_{con}}{P_{con}}$.
	$+$	Small $\frac{P_{org}-P_{con}}{P_{con}}$ and μ, and ρ¡1.
μ	-	
P_{org}	$+$	
2 P_{con}	-	
α_b	$+$	
α_c	N/A	N/A
γ_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	$+$	
A_y	$+$	
A_b	$+$	
$\overline{C}$	$+$	
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon)$, assuming that $K_{org} = \overline{C}$ and $R_{con}(K) = \rho \forall K$, and assuming $K_{org} = \overline{C}$ (Does not depend on whether or not $\overline{c} = \mu\overline{C}$). A conventional farmer is said to prefer producing organically when $\Delta(\epsilon) > 0$.

* Does not depend on whether or not $\overline{c} = \mu\overline{C}$

Table B.8: Comparative Statics for ϵ^* For Case F14A When Conventional $K_{org} = \overline{C}$

Summary of $\frac{\partial \epsilon^*}{\partial i}$		
i	Sign	Condition
ρ	+	
μ	-	
P_{org}	-	
P_{con}	+	
α_b	+	
α_c	N/A	N/A
α_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	+	
A_y	-	
A_b	-	
c_0	N/A	N/A
C_0	N/A	N/A
$\overline{C}$	-	

Notes: Table reports comparative statics for ϵ^* assuming that $K_{org} = \overline{C}$ and $R_{con}(K) = \rho \forall K$, and assuming $K_{org} = \overline{C}$ (Does not depend on whether or not $\overline{c} = \mu\overline{C}$). A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ^* lower than K_{org}.

Table B.9: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ For Case F14 A When $K_{org}=\overline{C}$

Summary of $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial i}$		
i	Sign	Condition
ϵ	-	
ρ	-	
μ	+	
P_{con}	0	
α_b	-	
α_c	N/A	N/A
γ_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	-	
A_y	+	
A_b	+	
c_0	N/A	N/A
C_0	N/A	N/A
$\overline{C}$	+	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ assuming that $K_{org}=\overline{C}$ and $R_{con}(K)=\rho\forall K$, and assuming $K_{org}=\overline{C}$ (Does not depend on whether or not $\overline{c}=\mu\overline{C}$).

* Does not depend on whether or not $\overline{c}=\mu\overline{C}$