Regenerative Agriculture and Organic Farming: A Dynamic Bioeconomic Model of Optimal Agricultural Soil and Pest Management

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Abstract

While the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) in agricultural production may initially improve crop yields, recent insights from soil science show that over time synthetic compounds exert an indirect negative effect on crop yields through their negative effects on soil health. We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds that accounts for these newly documented interrelationships among synthetic compound use, soil health, and crop yields. We characterize and solve for a farmer's optimal synthetic compound use strategy, and for whether and how a farmer should transition from conventional to organic farming. We find that some farmers may transition to organic management "accidentally" as their optimal trajectories eventually take them toward the certification threshold. This can happen even in the absence of an organic price premium. Others will have discrete "jump" transitions that are induced purely by the organic price premium.

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1 Introduction

Conventional agriculture has been criticized for its adverse effects on natural resources and the environment – including biodiversity loss, water pollution, and other forms of pollution posing threats to public health and climate stability - many of which are due in part to the prevalent use of synthetic compounds (e.g., synthetic fertilizers, synthetic fertilizers, pesticides, herbicides, or fungicides) that characterizes conventional management practices (Varanasi, 2019). In contrast, organic farming and regenerative agriculture practices (Chesapeake Bay Foundation, 2024; Natural Resources Defense Council, 2022) wherein farmers reduce or forego synthetic compound use are considered to be a more sustainable and more environmentally friendly alternative to conventional food production (Varanasi, 2019). This paper combines insights from economics and the natural sciences to study and inform farmer transitions from conventional to organic management and other production regimes characterized by reduced reliance on synthetic compounds.

Soil microbes benefit agricultural production and improve agricultural yields by enhancing crop nutrient use, stress tolerance, and pest resistance (Yadav et al., 2017; Yibeltie and Sahile, 2018; Righini et al., 2022). Recent insights from soil science show that the use of synthetic fertilizers and pesticides can be harmful to these beneficial soil microbes (Li et al., 2022; Blundell et al., 2020; Lori et al., 2017). Thus, while using synthetic fertilizers and pesticides may have the initial effect of increasing crop yields, over time these synthetic compounds exert an indirect negative effect on crop yields through their negative effects on soil health. This insight has implications for a farmer's optimal synthetic fertilizer and pesticide strategy, and for whether and how a farmer should transition from conventional to organic farming.

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model of crop production accounts for the newly documented interrelationships among synthetic compound use, soil health, and crop yields. By more accurately capturing the important biological processes at play, our model yields a solution that more accurately captures a farmer's optimal synthetic compound use and organic production strategy.

Notably, our model allows the producer to capture price premia associated with programs like the U.S. Department of Agriculture (USDA) National Organic Program (NOP) (USDA Agricultural Marketing Service, 2000b; Organic Produce Network, 2022). This program grants producers access to premium markets once pesticide-use rates and/or soil quality satisfy certain threshold values. Such certification programs are important to include in our production model, as they currently serve as important levers through which real-world markets reward reduced reliance on synthetic chemicals and, as such, they play a prominent role in shaping farmers' pesticide-use choices.

We use the model to explore how soil microbes' role in production should optimally affect farmers' choice in pesticide application rates, using a world in which farmers have no awareness of the role that soil microbes play in production as the relevant counterfactual. We also examine how knowledge of soil microbes may interact with other key market features, like certification programs that reward stewardship of soil health through reduced reliance on pesticides, to induce greater adoption of such certification programs than would be expected in the absence of knowledge of soil microbes.

Our objectives are the following. First, we characterize a farmer's optimal trajectory of synthetic compound use over time, given the harmful effects that these compounds have on soil bacteria, and given the beneficial effects of soil bacteria on crop yields. Second, we examine a farmer's decision of whether to adopt organic certification by lowering their use of synthetic compounds to meet certification requirements. The solution to our model describes the feasibility and optimality of organic production, given the way that soil health will respond to the transition.

Formally, the dynamic optimization problem faced by the farmer is to choose a pesticide and fertilizer input trajectory to maximize the present discounted value of their entire stream of profits from crop production. Crop yields, and therefore profits, are a function of pesticide and fertilizer input use, as well as of soil bacteria. Soil bacteria populations, in turn, are a function of per-period chemical use as well as the stock of synthetic residues that have built up in the farm's soils from past chemical use.

We characterize and solve for a farmer's optimal synthetic compound use strategy, and for whether a farmer should transition from conventional to a production regime that relies less on synthetic pesticides and more on the productivity-enhancing services generated by their soil microbiome. We find that some farmers may transition to organic management "accidentally" as their optimal trajectories eventually take them toward the certification threshold. This can happen even in the absence of an organic price premium. Others will have discrete "jump" transitions that are induced purely by the organic price premium.

Our research will help farmers improve decision-making around synthetic compound use and organic production, and has the potential to improve soil bacteria stewardship, crop yields, farmer profits, agricultural sustainability, greenhouse gas mitigation, biodiversity, resilience of the organic farming system, the protection of water and other resources, the provision of ecosystem services, and public and environmental health.

2 Literature Review

Transitioning to organic farming entails the discontinuation of pesticide use, a change that may impact farm profits. The relationship between pesticide use and farm profit has been the subject of many studies. Chambers, Karagiannis, and Tzouvelekas (2010) develop a model that measures how pesticide use affects relative returns to quasi-fixed factors of production like capital and land. Jacquet, Butault, and Guichard (2011) use a mathematical programming model to determine whether pesticide use can be reduced without affecting farmer income and find that a up to a 30 percent reduction is possible. In the long run, pesticide use may even negatively affect profits due to their effects on soil productivity through soil health. Sexton, Lei, and Zilberman (2007) acknowledge the effect that pesticide use can have on soil health through its impact on soil microbiomes. Kalia and Gosal (2011) also document the damaging effects that the application of pesticides in conventional farming has on soil microorganisms that benefit plant productivity. Jaenicke and Lengnick (1999) estimate a soil-quality index consistent with the notion of technical efficiency. van Kooten, Weisensel, and Chinthammit (1990) use a dynamic model that explicitly includes soil quality in the grower's utility function and the trade-off between soil quality (which may decline due to erosion) and net returns. Using data from rice farmers in California, Meneses et al. (2025) empirically document the insights from soil science that the use of chemical inputs may increase contemporaneous yields; and also that, over time, not using chemical inputs increases yield. Meneses et al. (forthcoming) leverage recent insights from plant and soil sciences to show how farmers' private economic incentives can be realigned with pro-carbon management practices that enhance soils' ability to store atmospheric carbon, such as those found in regenerative agriculture and organic farming.

Soil microbes benefit agricultural production and improve agricultural yields by enhancing crop nutrient use, stress tolerance, and pest resistance (Singh et al., 2016; Lori et al., 2017; Yadav et al., 2017; Yibeltie and Sahile, 2018; Blundell et al., 2020; Kalam et al., 2020; Verma et al., 2020; Righini et al., 2022; Thiebaut et al., 2022). Insights from soil science show that the use of synthetic fertilizers and pesticides can be harmful to these beneficial soil microbes (Li et al., 2022; Blundell et al., 2020; Dash et al., 2017; Lori et al., 2017; Newman et al., 2016; Kalia and Gosal, 2011; Lo, 2010; Hussain et al., 2009).

The dynamic response of soil health and productivity to the sorts of changes in pesticide use entailed by transitions to organic farming is still not well accounted for in economic assessments of the profitability of transitioning to organic farming. Stevens (2018) argues that optimal control models may be well suited for studying the economics of soil management. In this paper we argue further that dynamic optimization and dynamic programming may help shed light on the optimal rate of transition from conventional to organic farming, by allowing us to better capture the countervailing and dynamic effects that pesticide use has on profits through its effect on pest pressure and soil health.

Multiple studies have applied the dynamic optimization and programming toolkits to the study of optimal agricultural management practices. Jaenicke (2000) develops a dynamic data envelopment analysis (DEA) model of crop production to investigate the role soil capital plays in observed productivity growth and the crop rotation effect. Yeh, Gómez, and Lin Lawell (2025) develop a novel dynamic bioeconomic analysis framework that combines numerical dynamic optimization and dynamic structural econometric estimation, and apply it to analyze the optimal management strategy for Spotted Wing Drosophila, a pest affecting soft-skinned fruits. Wu (2000) develops a dynamic model and solves for the optimal time path for herbicide application. Dynamic models have also been developed to study agricultural productivity (Carroll et al., 2019), agricultural groundwater management (Sears, Lim, and Lin Lawell, 2019; Sears, Lin Lawell, and Walter, 2025), agricultural disease control (Carroll et al., 2025a), pollination input decisions by apple farmers (Wilcox et al., 2025), supply chain externalities (Carroll et al., 2025b), optimal bamboo forest management (Wu et al., 2025), fisheries management (Conrad, Lin Lawell, and Shin, 2025; Shin, Conrad, and Lin Lawell, 2025), and grapes (Sambucci, Lin Lawell, and Lybbert, 2025).

Delbridge and King (2016) use dynamic programming to address the question of why so few farmers choose to transition to organic farming, and find that the slow uptake of organic farming may be partially driven by the option value generated by the sunk costs associated with organic transition that arise from substantial reductions in gross revenue since crop yields typically fall during the organic transition and adopting an organic crop rotation often involves some diversification away from the most valuable crops. Other studies have sought to incorporate transition dynamics, such as the empirically documented initial decrease in crop yields associated with conventional to organic transitions, into profitability assessments of organic farming. Dabbert and Madden (1986) find in their multi-year simulation of a 117-hectare crop-livestock farm that the initial decrease in crop yields during an organic transition results in a 30 percentage point decrease in income in the first year of transition. The biological underpinnings of this initial decrease in productivity, and their response to farmer control variables are not made explicit.

The current study is unique in that it approaches the matter of finding an optimal transition trajectory from a bioeconomic perspective, informing its net benefit function with insights from soil sciences on how soils respond to organic management. Blundell et al. (2020) find that organic management is associated with decreased pest pressure on tomato plants. This effect is driven by an accumulation of salicylic acid in plant tissue, and is likely mediated by soil microbe communities. Lori et al. (2017) find that organic management is associated with increased microbial abundance and activity. Our net benefit function captures such soil health effects on crop productivity and farmer costs during the organic transition. We are not aware of any other studies that use a bio-economic dynamic programming approach to solving for a farm's optimal trajectory for transitioning from conventional to organic farming.

3 Dynamic Bioeconomic Model

3.1 Model

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model of crop production accounts for the newly documented interrelationships among synthetic compound use, soil health, and crop yields.

We model the farmer's field-level organic transitions decision-making problem as an infinite horizon dynamic optimization problem. We assume for the initial analysis that the farmer optimizes for each of their fields independently.

The crop production function for a field's crop output $y(t)$ at time t is given by $\tilde{f}(b(t), c(t); X)$, where crop output is a function of beneficial soil microbes (or bacteria) $b(t)$, chemical (or synthetic compound) inputs $c(t)$, and other human and natural inputs X (which may include capital, labor, soil characteristics, and land quality). The marginal product of chemical input use $c(t)$, conditional on soil microbes $b(t)$, is non-negative: $\frac{\partial \tilde{f}(b(t),c(t);X)}{\partial c} \geq 0$. Soil microbes $b(t)$ have a non-negative effect on crop production: $\frac{\partial \tilde{f}(b(t),c(t);X)}{\partial b} \geq 0$.

The biological production function for soil microbes $b(t)$ is given by $\tilde{g}(C(t), c(t); X)$, where the prevalence of beneficial soil microbes $b(t)$ decreases with the total stock of synthetic compounds in the soil $C(t)$, and with greater per-period chemical (or synthetic compound) input use $c(t)$: $\frac{\partial \tilde{g}(C(t),c(t);X)}{\partial C} \leq 0$, $\frac{\partial \tilde{g}(C(t),c(t);X)}{\partial c} \leq 0$. The total stock of synthetic compounds in the soil $C(t)$ only affects crop production through its effects on soil bacteria $b(t)$, so that $C(t)$ only appears in the crop production function $\tilde{f}(\cdot)$ through its role in the soil microbe production function $g(\cdot)$. The other inputs X to soil microbe production includes soil characteristics and other human and natural inputs that are important to how beneficial bacteria $b(t)$ respond to total synthetic compound stocks $C(t)$ and to synthetic compound use $c(t)$.

The farmer's control variable is the amount of chemical inputs $c(t)$, which may include synthetic fertilizer and pesticides (including herbicides, insecticides, fungicides, etc.), at each time t. The chemical input use $c(t)$ at any point in time does not exceed an upper bound $\bar{c}$, which may represent, for example, the maximum recommended dose for any given application, the maximum chemical input dose that is not lethal to the crop and/or to humans, the maximum chemical dose above which consumers will no longer purchase the crop, or the maximum chemical input flow at any point in time that does not destroy the farmer's land and soil. We assume the upper bound $\bar{c} > 0$.

We focus our analysis on the chemical input use $c(t)$ decision, and therefore take the other human and natural inputs X (which may include capital, labor, soil characteristics, and land quality) as given. In particular, for our stylized theory model we condition the soil microbe biological production function and output production function on the other human and natural inputs X, and allow these other human and natural inputs X to shift the marginal products of the inputs to soil microbe production and the marginal products of the inputs to crops production, but do not model changes in these other human and natural inputs X explicitly. Costs of changes in other human and natural inputs X associated with organic adoption can be captured in the organic (net) price premium.

The state variable is the stock of synthetic chemicals $C(t)$ that is present in the farm's soil at time t. The stock of synthetic chemicals increases with chemical input use $c(t)$ and decays at a constant rate $\mu(X) \geq 0$ that may depend on the soil characteristics and other human and natural inputs X:

$$\dot{C}(t) = c(t) - \mu(X)C(t). \tag{1}$$

Thus, the use of synthetic compounds $c(t)$ not only has harmful contemporaneous effects on soil microbes $b(t)$, but also has harmful effects on soil microbes $b(t)$ over time by increasing the stock of synthetic chemicals $C(t)$ that is present in the farm's soil, since both the total stock of synthetic compounds in the soil $C(t)$ and the per-period chemical input use $c(t)$ have harmful effects on soil microbes $b(t)$. As a consequence, while using synthetic fertilizers and pesticides $c(t)$ may have the initial direct effect of increasing crop yields $y(t)$, over time these synthetic compounds $c(t)$ exert an indirect negative effect on crop yields $y(t)$ through their negative effects on soil health.

Let $\overline{C}$ denote the maximum chemical stock capacity of the soil; if the stock of synthetic chemicals $C(t)$ ever exceeds this upper bound $\overline{C}$, the land and soil is destroyed forever and cannot ever be used for agricultural production again.

The national organic certification threshold for the stock of synthetic chemicals present in a farm's soil is given by C_{org}, where $0 \leq C_{org} < \overline{C}$. For the majority of our analysis, we assume (as is approximately the case in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils until they are pristine, such that $C_{org} = 0$.

The spot price of the crop in the organic market is P_{org}. The spot price of the conventionally grown crop is P_{con}. We normalize the unit price of chemical input $c(t)$ to be 1.

The optimal transition trajectory can be described by the solution to the following dynamic optimization problem:

$$\max_{\{c(t)\}} \int_{t=0}^{\infty} \big((P_{con} \cdot \mathbf{1}\{C(t) > C_{org}\} + P_{org} \cdot \mathbf{1}\{C(t) \leq C_{org}\}) \cdot \tilde{f}(b(t), c(t); X) - c(t) \big) e^{-\rho t} dt$$
$$\begin{aligned} s.t. \quad & \dot{C}(t) = c(t) - \mu(X)C(t) \\ & b(t) = \tilde{g}(C(t), c(t); X) \\ & 0 \leq c(t) \leq \bar{c} \\ & 0 \leq C(t) \leq \overline{C} \\ & C(0) = C_0\,, \end{aligned} \tag{2}$$

where $\mathbf{1}\{x\}$ is an indicator function that is equal to 1 if the condition x is true, and 0 otherwise; ρ is the interest rate; and C_0 is the initial stock of synthetic compounds in the soil.

Following Weitzman (2003), to facilitate analysis and economic interpretation, we convert our problem to prototype economic control problem form. We do this by first defining the stock of clean soil, $K(t)$, to be

$$K(t) = \overline{C} - C(t). \tag{3}$$

Net investment in clean soil stock, $I(t)$, is given by:

$$I(t) \equiv \dot{K}(t) = -\dot{C}(t). \tag{4}$$

Synthetic compound input use $c(t)$ in terms of $K(t)$ and $I(t)$ is therefore given by the following function $\tilde{c}(\cdot)$:

$$c(t) = \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t). \tag{5}$$

The constraint that $c(t) \geq 0$ can be rewritten as:

$$\mu(X)(\overline{C} - K(t)) \geq I(t) \tag{6}$$

The constraint that $c(t) \leq \bar{c}$ can be rewritten as:

$$\mu(X)(\overline{C} - K(t)) - \bar{c} \leq I(t). \tag{7}$$

We assume that $\bar{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$ and $\bar{c} > 0$ when $\mu(X) = 0$. Applying these assumptions to Equation (7), we obtain the following lower bound for net investment:

$$\begin{cases} I(t) \geq -\mu(X)K(t) & \text{if } \mu(X) > 0 \\ I(t) > \mu(X)(\overline{C} - K(t)) & \text{if } \mu(X) = 0 \end{cases} \tag{8}$$

The organic certification threshold in terms of clean soil capital is given by:

$$K_{org} = \overline{C} - C_{org}. \tag{9}$$

In terms of clean soil capital, our assumption $C_{org} = 0$ that organic certification requires that a farmer fully remediate their soils can be rewritten as $K_{org} = \overline{C}$.

We assume that the crop production function $\tilde{f}(\cdot)$ is given by:

$$\tilde{f}(b, c; X) = \alpha_b(X)b + \alpha_c(X)c + A_y(X) \tag{10}$$

where, $\forall X$, $\alpha_b \geq 0$, $\alpha_c \geq 0$, and $A_y \geq 0$.

We define a crop production function $f(\cdot)$, which is the crop production function $\tilde{f}(\cdot)$ in terms of K and I. The crop production function $f(\cdot)$ in terms of K and I is given by:

$$\begin{aligned} f(K, I; X) &= \tilde{f}(b, \tilde{c}(K, I); X) \\ &= \alpha_b(X)b + \alpha_c(X)\tilde{c}(K, I) + A_y(X) \end{aligned} \tag{11}$$

Let the soil microbe production function $\tilde{g}(\cdot)$ be given by:

$$\tilde{g}(C, c; X) = \gamma_c(X)c + \frac{1}{2}\gamma_{cc}(X)c^2 + \gamma_K(X)\left(\overline{C} - C(t)\right) + A_b(X), \tag{12}$$

where, $\forall X$, $\gamma_c \leq 0$, $\gamma_{cc} \leq 0$ (i.e., convex costs to synthetic compound use), $\gamma_K \geq 0$, and $A_b \geq 0$.

We define a soil microbe production function $g(K, I; X)$, which is the soil microbe production function $\tilde{g}(\cdot)$ in terms of K and I as follows:

$$g(K, I; X) = \gamma_c(X)\tilde{c}(K, I) + \frac{1}{2}\gamma_{cc}(X)\tilde{c}(K, I)^2 + \gamma_K(X)K + A_b(X). \tag{13}$$

Let's define the national organic certification threshold in terms of the stock of clean soil K_{org} as:

$$K_{org} = \overline{C} - C_{org}. \tag{14}$$

The initial stock of clean soil K_0 is given by:

$$K_0 = \overline{C} - C_0. \tag{15}$$

The farmer's problem can then be re-written in prototypical economic form as follows:

$$\begin{aligned}
\max_{\{I(t)\}} \int_0^\infty & \big((P_{con} \cdot \mathbf{1}\{K(t) < K_{org}\} + P_{org} \cdot \mathbf{1}\{K(t) \geq K_{org}\}) \cdot \tilde{f}(b(t), \tilde{c}(K(t), I(t)); X) \\
& \qquad - \tilde{c}(K(t), I(t))\big) \cdot e^{-\rho t} dt \\
s.t. \quad & \dot{K}(t) = I(t) \qquad : p(t) \\
& b(t) = g(K(t), I(t); X) \\
& \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t) \\
& \mu(X)(\overline{C} - K(t)) - \overline{c} \leq I(t) \leq \mu(X)(\overline{C} - K(t)) \\
& 0 \leq K(t) \leq \overline{C} \\
& K(0) = K_0 \,,
\end{aligned} \tag{16}$$

where the co-state variable $p(t)$ is the marginal value to the farmer's optimal program of an extra unit of clean soil.

3.2 What makes this optimal control problem novel and challenging to solve

The partial derivatives near the national organic certification threshold are tricky to calculate, since they involve derivatives of indicator functions. The indicator function $\mathbf{1}\{K \geq K_{org}\}$ for satisfying the national organic certification threshold is the Heaviside function $H(K - K_{org})$, where $H(x) = 1$ if $n \geq 0$ and $H(x) = 0$ if $x < 0$. The derivative of the Heaviside function $H(x)$ is the Dirac delta function $\delta(x)$:

$$\frac{dH(x)}{dx} = \delta(x), \tag{17}$$

which unfortunately is tricky to work with and interpret, as it is a function that spikes at zero.

So instead of trying to take a derivative of an indicator function, we analyze each stage of the dynamic bioeconomic model separately, and then consider possible transitions from conventional to organic management. The first stage is conventional agriculture with prices P_{con}. The second stage is organic agriculture with prices P_{org}. The second stage is reached if organic certification requirement $K(t) \geq K_{org}$ is satisfied.

4 Optimal Solution for Each Stage

We first describe behavior *within* each stage $j \in \{con, org\}$. For each stage $j \in \{con, org\}$, we solve for stationary rate of return on capital (clean soil stock) $R_j(K)$; determine whether there is a stationary solution $\hat{K}_j$; characterize direction and speed of net investment $I(t)$; and solve for the optimal trajectories $I^*(t)$ and $K^*(t)$ using the Maximum Principle.

4.1 Optimal control problem for each stage $j \in \{con, org\}$

For each stage $j \in \{con, org\}$, the farmer's dynamic optimization problem is given by:

$$\begin{aligned} V_j(K_{0j}) = \max_{\{I(t)\}} \int_0^\infty & \left(P_j \cdot \tilde{f}\left(g(K(t), I(t); X), \tilde{c}(K(t), I(t)); X\right) - \tilde{c}(K(t), I(t))\right) \cdot e^{-\rho t} dt \\ s.t. \quad & \dot{K}(t) = I(t) \qquad : p(t) \\ & \tilde{c}(K(t), I(t)) = \mu(X)(\overline{C} - K(t)) - I(t) \\ & \mu(X)(\overline{C} - K(t)) - \overline{c} \leq I(t) \leq \mu(X)(\overline{C} - K(t)) \\ & 0 \leq K(t) \leq \overline{C} \\ & K(0) = K_{0j}\,. \end{aligned} \tag{18}$$

The Hamiltonian is then:

$$H_j = G_j(K, I) + \rho I(t), \tag{19}$$

where

$$G_j(K, I) = P_j \cdot \tilde{f}\left(g(K, I), c(K, I); X\right) - \tilde{c}(K, I; X). \tag{20}$$

Given our functional form assumptions, the per-period net gain (or profits) $G_j(K, I)$ for each stage $j \in \{con, org\}$ is given by:

$$\begin{aligned} G_j(K, I) = & P_j \cdot \Bigg(\alpha_b \bigg(\gamma_c \left(\mu\left(\overline{C} - K\right) - I\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_K K + A_b\bigg) \\ & + \alpha_c \left(\mu\left(\overline{C} - K\right) - I\right) + A_y\Bigg) - \left(\mu\left(\overline{C} - K\right) - I\right) \end{aligned} \tag{21}$$

where the convex costs to synthetic compound use, as measured by the parameter $\gamma_{cc} \leq 0$, introduces non-linear investment costs. We will have a most rapid approach (MRA) policy if $\gamma_{cc} = 0$ since then $G(K, I)$ is linear in net investment I.

The solution to the farmer's dynamic optimization problem for each stage $j \in \{con, org\}$ satisfies the FOCs of the Maximum Principle:

$$[\#1]: \quad \frac{\partial H_j}{\partial I} = 0 \tag{22}$$

$$[\#2]: \quad \dot{p}(t) = -\frac{\partial \widetilde{H_j}}{\partial K}(K^*(t), p(t)) + \rho p(t) \tag{23}$$

$$[\#3]: \quad \lim_{t\to\infty} p(t)K(t)e^{-\rho t} = 0 \tag{24}$$

The FOCs of the Maximum Principle are both necessary *and sufficient* for optimality since our per-period net gain function $G_j(K, I)$ is concave and the control set is convex for each stage $j \in \{con, org\}$.

4.2 Characterizing the Optimal Solution for each stage $j \in \{con, org\}$

The present discounted value (PDV) of entire stream of marginal net benefit (MNB) of an additional unit of synthetic compound $c(t)$ today is given by:

$$P_j \cdot \alpha_c - \left(-P_j\alpha_b\left(\gamma_c + \gamma_{cc}c(t)\right) + \frac{P_j\alpha_b\gamma_K}{\mu+\rho} + 1\right), \tag{25}$$

each term of which is explained in detail in Figure 1. Owing to the convex costs of synthetic compounds on soil microbe production, as measured by the parameter $\gamma_{cc} \leq 0$, the PDV of the entire stream of marginal net benefits is decreasing in synthetic compound use $c(t)$. Since $c(t) = \mu(X)(\overline{C} - K(t)) - I(t)$ is decreasing in $K(t)$, the PDV of the entire stream of marginal net benefits is increasing in $K(t)$.

The present discounted value of the entire stream of marginal benefits from applying an additional unit of synthetic compound today, which is given by $P_j\alpha_c$, comes from the direct effect of chemical input use $c(t)$ on crop output $y(t)$ today and therefore on crop revenue today. Using an additional unit of synthetic compound today does not yield any future marginal benefits.[1]

The present discounted value of the entire stream of marginal costs of applying an additional unit of synthetic compound today consists of several components. Applying an additional unit of synthetic compound today incurs both direct and indirect marginal costs. The direct marginal cost of an additional unit of synthetic compound today is simply the unit price of chemical inputs $c(t)$, which we normalize to 1, and which is only incurred today. The indirect marginal cost of an additional unit of synthetic compound today comes from the negative effects of synthetic compounds on soil microbes $b(t)$ and their resulting negative effects on crop output $y(t)$ and therefore on crop revenue. There are two channels through which synthetic compounds have negative effects on soil microbes. First, applying an additional unit of synthetic compound today has a direct negative effect on soil microbes today through its direct negative effect on soil microbe production today. Second, applying an additional unit of synthetic compound today has an indirect negative effect on soil microbes by decreasing the stock of clean soils $K(t)$ today, which may last for multiple periods, and which in turn has a negative effect on soil microbe production over multiple periods of time; we call the PDV of the entire stream of indirect marginal costs of applying an additional unit of synthetic compound today via their indirect negative effect on soil microbes through their negative effect on stock of clean soils the "stock effect". Thus, the present discounted value of the entire

[1] Using data from rice farmers in California, Meneses et al. (2025) empirically document that the use of synthetic pesticides increases contemporaneous yields; and also that, over time, not using synthetic pesticides increases yield.

stream of marginal costs from applying an additional unit of synthetic compound today comes from the direct marginal cost of purchasing synthetic compounds today, 1; the indirect marginal cost of applying an additional unit of synthetic compound today via their direct negative effect on soil microbes today, $-P_j \alpha_b (\gamma_c + \gamma_{cc} c(t))$; and the PDV of the entire stream of indirect marginal costs of applying an additional unit of synthetic compound today via their indirect negative effect on soil microbes through their negative effect on stock of clean soils (the stock effect), $\frac{P_j \alpha_b \gamma_K}{\mu+\rho}$.

The optimal unconstrained amount of synthetic compound $c(t)$ to apply at any time t is the synthetic compound input level c_j^{**} at which the present discounted value (PDV) of entire stream of marginal net benefit (MNB) of an additional unit of synthetic compound c today is 0. In other words, at the optimal unconstrained amount of synthetic compound c_j^{**}, the PDV of the entire stream of marginal benefits of an additional unit of synthetic compound $c(t)$ today is exactly offset by the PDV of the entire stream of marginal costs of an additional unit of synthetic compound $c(t)$ today. This optimal unconstrained amount of synthetic compound c_j^{**} is a constant that is a function of parameters but not of $K(t)$ and is given by:

$$c_j^{**} = -\frac{P_j \cdot \left(\alpha_c + \alpha_b \left(\gamma_c - \frac{\gamma_K}{\mu+\rho} \right) \right) - 1}{P_j \alpha_b \gamma_{cc}}. \tag{26}$$

As long as the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is greater than 0 (i.e., as long as $c(t) < c_j^{**}$, since the PDV of the entire stream of MNB is decreasing in $c(t)$), we would want to increase the amount of synthetic compound we use today, and will continue to do so until either (1) the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is 0 (i.e., until $c(t) = c_j^{**}$); or (2) we hit the upper bound $\bar{c}$ for synthetic compound use. If we are constrained by the upper bound $\bar{c}$ for synthetic compound use from increasing synthetic compound use $c(t)$ any further even though the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 (i.e., if $c_j^{**} > \bar{c}$, then the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is the smallest positive value that it can be.

With the farmer's problem now in prototypical economic control form, we can solve for the stationary rate of return on capital, $R_j(K)$ for each stage $j \in \{con, org\}$. The stationary rate of return on capital $R_j(K)$ is the per-period rate of return on the clean soil capital stock K from increasing net investment I a tiny bit this period from a net investment level of $I = 0$. In other words stationary rate of return on capital $R_j(\overline{K})$ is the per-period yield from increasing the clean soil capital stock K from a stationary state $\overline{K}$, to a stationary state $\overline{K} + \epsilon$, and is given by (Weitzman, 2003):

$$R_j(K) = -\frac{\frac{\partial G_j(K,0)}{\partial K}}{\frac{\partial G_j(K,0)}{\partial I}}. \tag{27}$$

Solving for the stationary rate of return $R_j(K)$ on clean soil capital for each stage $j \in \{con, org\}$, we obtain:

$$R_j(K) = -\mu + \frac{\gamma_K}{\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}. \tag{28}$$

As derived in Appendix A.1, the stationary rate of return $R_j(K)$ is weakly decreasing in clean soil capital K. We describe and discuss the stationary rate of return $R_j(K)$ in more detail in Appendix A.1.

The stationary solution $\hat{K}_j$ is the clean soil stock at which the stationary rate of return on the clean soil capital stock $R_j(\hat{K}_j)$ is equal to the rate of the return on the best alternative investment (i.e., the bank) ρ (Weitzman, 2003):

$$R_j(\hat{K}_j) = \rho \tag{29}$$

Plugging in Equation (28) for the stationary rate of return on the clean soil capital stock, $R_j(\cdot)$ into Equation (29), we obtain the following condition for the stationary solution $\hat{K}_j$:

$$P_j\alpha_c = -P_j\alpha_b\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right)\right) + \frac{P_j\alpha_b\gamma_K}{\mu + \rho} + 1 \tag{30}$$

The intuition for the condition for the stationary solution $\hat{K}_j$ is presented in Figure 2.

As seen in Equation (131) in Figure 2, at the stationary solution $\hat{K}_j$ for a given stage $j \in \{con, org\}$, the present discounted value (PDV) of the entire stream of marginal benefits from applying an additional unit of synthetic compound today equals the present discounted value of the entire stream of marginal costs of applying an additional unit of synthetic compound today.

The optimal choice of synthetic compound $c(t)$ at any time t is when the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound $c(t)$ today is 0 (or as small a non-negative number as possible).

Thus, when the stationary solution exists, the optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j \equiv \mu\left(\overline{C} - \hat{K}_j\right)$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution.

If $(\rho + \mu)\gamma_{cc}\mu \neq 0$, $\hat{K}_j$ is given by:

$$\hat{K}_j = \frac{(\rho + \mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho + \mu)\gamma_{cc}\mu} \tag{31}$$

Note that our solution for $\hat{K}_j$ may be negative or positive depending on model parameters. Also note that $K(t)$ is constrained such that $K(t) \in [0, \overline{C}]$. It is therefore possible that $\hat{K}_j$ is not feasible because $\hat{K}_j$ is not within the set of feasible K. In other words, it is possible that $\hat{K}_j$ is not feasible because either $\hat{K}_j < 0$ or $\hat{K}_j > \overline{C}$. If our solution for $\hat{K}_j$ is negative then our non-negativity constraint on $K(t)$ will bind. Importantly, we note that under this formulation of the model it is possible for (31) to be positive even in the case of special interest in which $\mu > 0$.

We conduct comparative statics for $\hat{K}_j$ in Appendix A.2. As derived in Appendix A.2, the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j: $\frac{\partial \hat{K}_j}{\partial P_j} \leq 0$. Since the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j and $P_{con} < P_{org}$, the stationary solution for the organic stage 2, if it exists, is less than the stationary solution for the conventional stage 1, if it exists.

Since our analysis using the stationary rate of return on capital $R_j(K)$ makes the assumption of the prototype economic control model that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain), we cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R(K)$ and ρ to describe the optimal solution when $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a positive effect on contemporaneous net gain starting from a net investment of $I = 0$). A farmer with $\frac{\partial G(K,0)}{\partial I} \geq 0$ would invest in the stock of clean soil, not disinvest, since there is no trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well. As shown in Appendix A.3, $\frac{\partial G(K,0)}{\partial I} \geq 0$ when $K \leq \tilde{K}_j$, where $\tilde{K}_j$ is defined as the stock of clean soils at which $\frac{\partial G(K,0)}{\partial I} = 0$. Thus, for $K \leq \tilde{K}_j$, the farmer will invest in clean soil. Also as shown in Appendix A, $\hat{K}_j \geq \tilde{K}_j$. Thus, since for $K \leq \tilde{K}_j$, the farmer will invest in clean soil, this means that for $K_{0j} \leq \tilde{K}_j$, if the stationary solution $\hat{K}_j$ exists, the farmer will continue to invest in clean soil until he reaches the stationary solution $\hat{K}_j$.

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own), then $R_j(K)$ is a constant (that does not depend on K) and $\hat{K}_j$ will not exist (nor will $\tilde{K}_j$).

We discuss how prices P_j affect the optimal solution in Appendix A.4.

4.3 Optimal Trajectories for Stage j When $\hat{K}_j$ Exists

We now solve for the farmer's optimal stage j trajectories.

In Appendix A.5, we start by solving for the unconstrained solution for each stage j by using second-order Taylor series approximations of the net gain function $G(K, I)$. Since the net gain function $G(K, I)$ is quadratic, these second-order Taylor series approximations and the solutions derived using them are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K, I)$ is an exact second-order Taylor series expansion of the net gain function $G(K, I)$.

In Appendix A.6, we then solve for the constrained optimal solution for each stage j by solving for an exact solution via direct derivation.

There are five possible types of optimal trajectories that arise when $\hat{K}_j$ exists, depending on the parameters. Taken in order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least synthetic compound use and the most investment in the stock of clean soils, these five types of optimal trajectories are as follows:

- **Optimal Trajectories 1 [OT1]**: Disinvest as fast as possible until $K = 0$ by always applying

$c(t) = \bar{c}$

- **Optimal Trajectories 2 [OT2]**: Disinvest to $\hat{K}_j < K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 3 [OT3]**: Stay at initial clean soil stock $K_0 = \hat{K}_j$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 4 [OT4]**: Invest until $\hat{K}_j > K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- **Optimal Trajectories 5 [OT5]**: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

Figure 3 presents the parameter spaces for each of the five types of optimal trajectories when $\hat{K}_j$ exists. Figure 4 plots examples of each of the five types of optimal trajectories that arise when $\hat{K}_j$ exists.

As seen in Figure 4, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the order of the optimal trajectory type by per-period yield reverses, and the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

We now derive and discuss each of the five types of optimal trajectories when $\hat{K}_j$ exists in more detail.

4.3.1 Optimal Trajectories 1 [OT1]: Disinvest as fast as possible to $K = 0$

When $\hat{K}$ exists and is negative (i.e., $\hat{K}_j < 0$), the lower-bound constraints on net investment I *always* bind (for all t), which means that the upper bound constraint on synthetic compound use always binds (i.e., $c^{**} > \bar{c}$). Thus, when $\hat{K}_j < 0$, the farmer's optimal solution is to disinvest as fast as possible until $K = 0$. The optimal synthetic compound use $c(t)$ is to apply the maximum amount possible $\bar{c}$ every period until we reach $K = 0$, at which point we stay at $K = 0$ (i.e., by applying $c(t) = \bar{c} = \mu\overline{C}$ each period).

As depicted in Figure 3, when $\hat{K}_j < 0$, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is weakly positive even when the farmer uses the maximum permissible dose $\bar{c}$ of a synthetic compound:

$$P_j\alpha_c + P_j\alpha_b\gamma_{cc} \cdot \bar{c} + P_j\alpha_b\gamma_c - P_j\alpha_b\frac{1}{(\rho+\mu)} \cdot \gamma_K - 1 \geq 0 \tag{32}$$

Since the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j, $\hat{K}_j < 0$ may occur if prices are very high.

The optimal trajectories for OT1 are as follows:

$$c_j^*(t) = \overline{c} = \mu\overline{C} \ \forall t \tag{33}$$

$$K_j^*(t) = K_{0j} \cdot e^{-\mu \cdot t} \tag{34}$$

$$I_j^*(t) = -\mu K_j^*(t) \tag{35}$$

4.3.2 Optimal Trajectories 2 [OT2]: Disinvest until $\hat{K}_j$

This case occurs when $\hat{K}_j \in [0, \overline{C}]$ *and* $K_{0j} > \hat{K}_j$. In this case the optimal solution is to disinvest until $\hat{K}_j$, and to do so at a moderate speed. The optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution:

$$c_j^*(t) = \hat{c}_j = \mu\left(\overline{C} - \hat{K}_j\right) \ \forall t \tag{36}$$

At $\hat{c}_j$, PDV of MNB $= 0$.

The optimal trajectories for OT2 are as follows:

$$c_j^*(t) = \mu\left(\overline{C} - \hat{K}_j\right) \ \forall t \tag{37}$$

$$I_j^*(t) = \mu\left(\hat{K}_j - K_j^*(t)\right) < 0 \tag{38}$$

$$K_j^*(t) = \hat{K}_j + \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{39}$$

4.3.3 Optimal Trajectories 3 [OT3]: Stay at initial clean soil stock and do not invest or disinvest

If $K_{0j} = \hat{K}_j$, we always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock. In this case it is optimal to stay at initial clean soil stock and not to invest or disinvest. In other words, in each period our chemical input use $c(t)$ should exactly offset the stock of chemicals in the soil decays on its own so that the stock of chemicals in the soil stays constant, and therefore the clean soil stock stays constant at its initial value.

Thus, for OT3, the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

$$K_j^*(t) = K_{0j} \ \forall t \tag{40}$$

$$I_j^*(t) = 0 \;\forall t \tag{41}$$

$$c_j^*(t) = \mu\left(\overline{C} - K_{0j}\right) \;\forall t \tag{42}$$

4.3.4 Optimal Trajectories 4 [OT4]: Invest until $\hat{K}_j$

This case occurs when $\hat{K}_j \in [0, \overline{C}]$ *and* $K_{0j} < \hat{K}_j$. In other words: $K_{0j} < \hat{K}_j \leq \overline{C}$. In this case the optimal solution is to invest until $\hat{K}_j$, and to do so at a moderate speed. The optimal synthetic compound use $c(t)$ is constant at the amount $\hat{c}_j$ that exactly offsets how much the stock of chemicals in the soil decays on its own at the stationary solution. At $\hat{c}_j$, PDV of MNB = 0.

The optimal trajectories for OT4 are as follows:

$$c_j^*(t) = \mu\left(\overline{C} - \hat{K}_j\right) \;\forall t \tag{43}$$

$$I_j^*(t) = \mu\left(\hat{K}_j - K_j^*(t)\right) > 0 \tag{44}$$

$$K_j^*(t) = \hat{K}_j + \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \tag{45}$$

4.3.5 Optimal Trajectories 5 [OT5]: Invest as fast as possible until $K = \overline{C}$

If $\hat{K}_j > \overline{C}$, upper-bound constraints on net investment I *always* bind (for all t). In this case the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. It is optimal not to use any synthetic compounds $c(t)$ at all.

As depicted in Figure 3, if $\hat{K}_j > \overline{C}$, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is negative even when there are no convex costs of synthetic compounds on soil microbes:

$$P_j \alpha_c < -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} - 1 \tag{46}$$

Since the stationary solution $\hat{K}_j$ is a decreasing function of prices P_j, $\hat{K}_j > \overline{C}$ may occur if prices are low.

The optimal trajectories for OT5 are as follows:

$$c_j^*(t) = 0 \;\forall t \tag{47}$$

$$K_j^*(t) = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{48}$$

$$I_j^*(t) = \mu \cdot \left(\overline{C} - K_j^*(t)\right) \tag{49}$$

4.4 Optimal Trajectories for Stage j When $R(K)$ is Constant Because $\gamma_{cc} = 0$

If $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) then $R_j(K)$ is a constant (that does not depend on K).

There are three possible types of optimal trajectories that arise when $R(K)$ is constant because $\gamma_{cc} = 0$, depending on the parameters. Taken in order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least synthetic compound use and the most investment in the stock of clean soils, these three types of optimal trajectories are as follows:

- **Optimal Trajectories 1 [OT1]**: Disinvest as fast as possible until $K = 0$ by always applying $c(t) = \bar{c}$
- **Optimal Trajectories 3' [OT3']**: Stay at initial clean soil stock K_0 by always applying the amount of synthetic compounds that exactly offsets how much the initial stock of chemicals decays on its own
- **Optimal Trajectories 5 [OT5]**: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

Figure 5 presents the parameter spaces for each of the three types of optimal trajectories when $R(K)$ is constant because $\gamma_{cc} = 0$. Figure 6 plots examples of each of the 3 types of optimal trajectories that arise when $R(K)$ is constant because $\gamma_{cc} = 0$.

As seen in Figure 6, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

4.4.1 Optimal Trajectories 1 [OT1]: Disinvest as fast as possible to $K = 0$

If $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{50}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{51}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When we have $\gamma_{cc} = 0$, our gain function is linear in the control variable I, which means that the farmer will follow a most rapid approach (MRA) policy. Thus, since $R_j(K) < \rho$, the farmer will always disinvest according to the most rapid approach (MRA) policy until he reaches $K = 0$. In other words, lower-bound constraints on net investment I *always* bind (for all t), which means the upper bound constraint on synthetic compound use always binds (i.e., $c^{**} > \bar{c}$). The optimal synthetic compound use $c(t)$ is to always apply the maximum amount possible $\bar{c}$.

As depicted in Figure 5, when $\gamma_{cc} = 0$ and $R_j(K) < \rho$, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is always positive:

$$P_j \alpha_c > -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + 1 \tag{52}$$

The optimal trajectories for OT1 are therefore:

$$c_j^*(t) = \bar{c} = \mu \overline{C} \;\forall t \tag{53}$$

$$K_j^*(t) = K_{0j} \cdot e^{-\mu \cdot t} \tag{54}$$

$$I_j^*(t) = -\mu \cdot K_{0j} \cdot e^{-\mu \cdot t} \tag{55}$$

4.4.2 Optimal Trajectories 3' [OT3']: Stay at initial clean soil stock and do not invest or disinvest

We always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock when $R_j(K)$ is constant and equal to ρ.

In this case it is optimal to stay at initial clean soil stock and not to invest or disinvest. In other words, in each period our chemical input use $c(t)$ should exactly offset the stock of chemicals in the soil decays on its own so that the stock of chemicals in the soil stays constant, and therefore the clean soil stock stays constant at its initial value.

Thus, for OT3', the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own.

$$K_j^*(t) = K_{0j} \;\forall t \tag{56}$$

$$I_j^*(t) = 0 \;\forall t \tag{57}$$

$$c_j^*(t) = \mu \left(\overline{C} - K_{0j} \right) \;\forall t \tag{58}$$

4.4.3 Optimal Trajectories 5 [OT5]: Invest as fast as possible until $K = \overline{C}$

There are two main cases in which the farmer will wish to continually invest when $R_j(K)$ is a constant (that does not depend on K).

First, if $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough to satisfy the following condition for net investment to have a non-negative effect on contemporaneous net gain (so that $R_j(K)$ is not useful for analyzing net investment):

$$P_j^{-1} \geq \alpha_b \gamma_c + \alpha_c, \tag{59}$$

then the farmer will wish to continually invest in clean soil stock.

Second, if $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough that $R_j(K) > \rho$:

$$P_j^{-1} > \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{60}$$

but also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c, \tag{61}$$

then the farmer will wish to continually invest in clean soil stock.

Moreover, when we have $\gamma_{cc} = 0$, our gain function is linear in the control variable I, which means that the farmer will follow a most rapid approach (MRA) policy. Thus, the farmer will always invest according to the most rapid approach (MRA) policy until he reaches $K = \overline{C}$. In other words upper-bound constraints on net investment I *always* bind (for all t). It is optimal not to use any synthetic compounds $c(t)$ at all.

As depicted in Figure 5, when $\gamma_{cc} = 0$ and $R_j(K) > \rho$, the PDV of the entire stream of marginal net benefits of an additional unit of synthetic compound today is negative even when there are no convex costs of synthetic compounds on soil microbes:

$$P_j \alpha_c < -P_j \alpha_b \gamma_c + \frac{P_j \alpha_b \gamma_K}{\mu + \rho} + 1 \tag{62}$$

The optimal trajectories for OT5 are therefore:

$$c_j^*(t) = 0 \ \forall t \tag{63}$$

$$K_j^*(t) = \overline{C} - \left(\overline{C} - K_{0j} \right) e^{-\mu \cdot t} \tag{64}$$

$$I_j^*(t) = \mu \cdot \left(\overline{C} - K_j(t)\right) \tag{65}$$

4.5 Optimal Trajectories for Stage j When $R(K)$ is Constant Because $\mu = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) then $R_j(K)$ is a constant (that does not depend on K).

There are three possible types of optimal trajectories that arise when $R(K)$ is constant because $\mu = 0$, depending on the parameters. Taken in order from solutions that require the most synthetic compound use and the most disinvestment in the stock of clean soil, to solutions that require the least synthetic compound use and the least disinvestment in the stock of clean soils, these three types of optimal trajectories are as follows:

- **Optimal Trajectories 1' [OT1']**: Disinvest as fast as possible by applying $c(t) = \bar{c}$ until $K = 0$ is reached
- **Optimal Trajectories 1" [OT1"]**: Disinvest by applying c_j^{**} at which PDV of MNB equals 0 until $K = 0$ is reached
- **Optimal Trajectories 3" [OT3"]**: Stay at initial clean soil stock K_0 by never applying any synthetic compounds at all

Figure 7 presents the parameter spaces for each of the three types of optimal trajectories when $R(K)$ is constant because $\mu = 0$. Figure 8 plots examples of each of the three types of optimal trajectories that arise when $R(K)$ is constant because $\mu = 0$.

As seen in Figure 8, the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the higher the initial per-period yield $y(t)$. Over time, however, the order of the optimal trajectory type by per-period yield reverses, and the more the optimal trajectory type requires synthetic compound use and disinvestment in the stock of clean soil, the lower the per-period yield $y(t)$ over the long run. Thus, while using a lot of synthetic compounds and disinvesting in the stock of clean soil may lead to higher per-period yields in the short run, doing so eventually leads to lower per-period yields in the long run.

4.5.1 Optimal Trajectories 1' [OT1']: Disinvest as fast as possible to $K = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{66}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu}\left((\rho + \mu)\gamma_c - \gamma_K\right) \tag{67}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

The lower bound to I binds when the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \bar{c}$). If the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \bar{c}$), this means that the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 at $c = \bar{c}$.

The optimal trajectories for OT1' are as follows:

$$K_j^*(t) = \begin{cases} K_{0j} - \bar{c} \cdot t & t < T_j^* \\ 0 & t \geq T_j^* \end{cases} \tag{68}$$

$$I_j^*(t) = \begin{cases} -\bar{c} & t < T_j^* \\ 0 & t \geq T_j^* \end{cases} \tag{69}$$

$$c_j^*(t) = \begin{cases} \bar{c} & t < T_j^* \\ 0 & t \geq T_j^* \end{cases} \tag{70}$$

$$T_j^* = \frac{K_{0j}}{\bar{c}} \tag{71}$$

4.5.2 Optimal Trajectories 1" [OT1"]: Disinvest to $K = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant $\tilde{R}_j$ (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b\gamma_c + \alpha_c \tag{72}$$

and the following condition for $R_j(K) < \rho$ holds:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu}\left((\rho + \mu)\gamma_c - \gamma_K\right) \tag{73}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

When $\gamma_{cc} \neq 0$ but $\mu = 0$ the gain function is non-linear in I, and therefore the optimal policy will not be MRA. If the lower corner solution for I does not bind (because $c_j^{**} \leq \bar{c}$), we will have an interior solution.

Thus, for OT1", the farmer will disinvest by applying the optimal unconstrained synthetic compound level c_j^{**} at which PDV of MNB equals 0 until $K = 0$. As derived in Appendix A.7.1, the optimal trajectories are as follows:

$$K_j^*(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - \tilde{R}_j^{-1} \right) \cdot t + K_{0j}, & \forall t \leq T_j^* \\ 0, & \forall t > T_j^* \end{cases} \tag{74}$$

$$I_j^*(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - \tilde{R}_j^{-1} \right), & \forall t \leq T_j^* \\ 0, & \forall t > T_j^* \end{cases} \tag{75}$$

$$c_j^*(t) = \begin{cases} \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - \tilde{R}_j^{-1} \right), & \forall t \leq T_j^* \\ 0, & \forall > T_j^* \end{cases} \tag{76}$$

$$T_j^* = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(\tilde{R}_j^{-1} - \rho^{-1} \right)} \geq 0 \tag{77}$$

$$\tilde{R}_j = R_j(K) = \frac{\gamma_K}{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}. \tag{78}$$

4.5.3 Optimal Trajectories 3" [OT3"]: Stay at initial clean soil stock and do not invest or disinvest

We always set $I(t) = 0$ (for all t) and stay at the initial clean soil stock when $\mu = 0$ and $R_j(K)$ is constant and greater than or equal to ρ.

If $R_j(K)$ is constant and equal to ρ, it is optimal to stay at initial clean soil stock and not to invest or disinvest.

When $\mu = 0$ the upper bound constraint on investment is equal to zero, and will always bind when $R_j(K)$ is greater than ρ. In this case, the farmer is constrained by the upper bound on I to stay at their initial capital stock K_{0j} indefinitely.

Thus, for OT3", the optimal synthetic compound use $c(t)$ is constant at the amount that exactly offsets how much the initial stock of chemicals in the soil decays on its own. Since the stock of chemicals in the soil does not decay on its own when $\mu = 0$, this means the optimal synthetic compound use $c(t)$ is constant at zero.

The optimal trajectories for OT3" are therefore the following:

$$K_j^*(t) = K_{0j} \ \forall t \tag{79}$$

$$I_j^*(t) = 0 \ \forall t \tag{80}$$

$$c_j^*(t) = 0 \ \forall t \tag{81}$$

5 Accidental Organic Transitions

The transition from conventional to organic management is "accidental" and continuous even in the absence of an organic price premium for conventional farmers whose optimal trajectories are either:

- OT5: Invest as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all

- OT4 if $\hat{K}_{con} \geq K_{org}$: Invest until $\hat{K}_{con}$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0

since in these cases the optimal solution for a conventional farmer is to invest in the stock of clean soils until $K(t)$ exceeds K_{org}.

This will happen in agricultural systems where soil microbes are sufficiently important for determining crop yields.

If $\hat{K}_{con} \in [K_{org}, \overline{C}]$, then we have Optimal Trajectories 4: Approach $\hat{K}_{con}$ at moderate speed, then the time T_{org} at which a fully informed conventional farmer makes a continuous transition to organic farming is given by:

$$T_{org} = \ln \left(\frac{K_{0,con} - \hat{K}_{con}}{K_{org} - \hat{K}_{con}} \right)^{\frac{1}{\mu}} \tag{82}$$

If $\hat{K}_{con} > \overline{C}$, or $R_{con}(K)$ is constant and greater than ρ, then we have Optimal Trajectories 5: Invest as fast as possible until $K = \overline{C}$, then the time T_{org} at which a fully informed conventional farmer makes a continuous transition to organic farming is given by:

$$T_{org} = \ln \left(\frac{\overline{C} - K_{0,con}}{\overline{C} - K_{org}} \right)^{\frac{1}{\mu}} \tag{83}$$

In the presence of an organic price premium, since $\hat{K}_j$ is a decreasing function of prices P_j and therefore $\hat{K}_{con} > \hat{K}_{org}$, all else equal a conventional farmer would invest weakly more quickly in the clean soil stock than would an organic farmer facing all the same parameters except a higher price. For example, for a farmer who is OT5 under both conventional and organic prices (e.g., if $\hat{K}_{con} > \hat{K}_{org} > \overline{C}$, or if both $R_{con}(K)$ and $R_{org}(K)$ are constant and greater than ρ), such a conventional OT5 farmer would continue to choose to invest in clean soil as fast as possible until $K = \overline{C}$ (the highest possible value of clean soil stock) by never applying any synthetic compounds at all even if they anticipated transitioning to organic. For a farmer who is OT5 under conventional prices but OT4 under organic prices (e.g., if $\hat{K}_{con} > \overline{C} > \hat{K}_{org}$), the conventional OT5 farmer would never apply any synthetic compounds while the organic OT4 farmer would always apply $\hat{c}_j$ at which PDV of MNB equals 0; thus such a conventional OT5 farmer would invest weakly more quickly in the clean soil stock if they anticipated eventually transitioning "accidentally" to organic. Similarly, for a farmer with is OT4 under both conventional and organic prices, since an OT4 farmer would always apply $\hat{c}_j$ at which PDV of MNB equals 0, and since $\hat{c}_j = \mu \left(\overline{C} - \hat{K}_j \right)$ decreases with $\hat{K}_j$

and therefore is increases with prices and is therefore higher under organic prices than conventional prices (i.e., $\hat{c}_{org} > \hat{c}_{con}$), the organic OT4 farmer all else equal would apply a higher $\hat{c}_j$; as a consequence this conventional OT4 farmer would invest weakly more quickly in the clean soil stock if they anticipated eventually transitioning "accidentally" to organic. Thus, a conventional farmer who transitions to organic "accidentally" would not make a continuous transition any faster even if they anticipated eventually transitioning "accidentally" to organic.

6 Premium-Induced Organic Transitions

There is no "accidental" transition from conventional to organic for conventional farmers whose optimal trajectories are:

- OT1: Disinvest as fast as possible until $K = 0$ by always applying $c(t) = \bar{c}$
- OT2: Disinvest to $\hat{K}_j < K_0$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- OT3: Stay at initial clean soil stock $K_0 = \hat{K}_j$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0
- OT3': Stay at initial clean soil stock and do not invest or disinvest
- OT3": Stay at initial clean soil stock and do not invest or disinvest
- OT4 if $\hat{K}_{con} < K_{org}$: Invest until $\hat{K}_{con}$ by always applying $\hat{c}_j$ at which PDV of MNB equals 0

For a conventional farmer to be conventional, this initial clean soil capital stock K_0 must be lower than the organic threshold K_{org}.

There is no accidental, continuous transition from conventional stage 1 to organic stage 2 when the conventional farmer stationary solution $\hat{K}_{con}$ is below K_{org}, since then a conventional farmer will tend towards the conventional farmer stationary solution $\hat{K}_{con}$, and therefore stay below K_{org} rather than become organic.

Similarly, there is no accidental, continuous transition from conventional to organic when $R_{con}(K)$ is constant and less than ρ, since then the conventional farmer will continually disinvest until $K = 0$, and therefore stay below K_{org} rather than become organic.

Nevertheless, even if there is no "accidental" transition, an organic price premium may still induce some farmers to switch to organic management. Given $P_{org} > P_{con}$, it may still be possible for fully informed conventional farmer to prefer organic farming, even when $\hat{K}_j < K_{org}$ or when $R_{con}(K)$ is constant and always less than ρ, and make a "jump" transition to the organic certification threshold. "Jump" transitions that accelerate the transition to organic are also possible in the presence of an organic price premium for conventional farmers who may have "accidental" and continuous transitions in the absence of an organic price premium.

For a "jump" transition to the organic certification threshold to occur we must have that the differential value from organic management $\Delta(\epsilon)$, defined as the difference in present discounted value of the entire stream of net benefits that a farmer will receive from organic and conventional management, is positive:

$$\Delta(\epsilon) \equiv V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon) > 0 \tag{84}$$

where $V_{org}(K_{org})$ is the present discounted value of the entire stream of net benefits that a farmer will receive from the moment they have switched to organic management, into perpetuity, assuming the organic farmer stays organic indefinitely; and $V_{con}(K_{org} - \epsilon)$ is the present discounted value of the entire stream of net benefits that a farmer will receive if they continue to produce conventionally indefinitely starting at initial clean soil stock $K_{0,con} = K_{org} - \epsilon$ for some $\epsilon > 0$.

$V_{org}(K_{org})$ is the present discounted value of the entire stream of net benefits that a farmer will receive from the moment they have switched to organic management, into perpetuity, assuming the organic farmer stays organic indefinitely. $V_{org}(K_{org})$ assumes that once in stage 2, the farmer follows the following constrained trajectories:

$$K_{org}(t) = K_{org} \,\forall t \tag{85}$$

$$I_{org}(t) = 0 \,\forall t \tag{86}$$

$$c_{org}(t) = \mu\left(\overline{C} - K_{org}\right) = 0 \,\forall t \tag{87}$$

To simplify our analysis, let's assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils, such that they will be certified organic if and only if $K = \overline{C}$.

When $K_{org} = \overline{C}$, the value $V_{org}(K_{org})$ of the farmer's optimal program for stage 2 following this constrained capital trajectory can be written as follows:

$$V_{org}(K_{org}) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} P_{org} \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of "level effect"} \\ \text{of other agricultural inputs} \\ \text{at organic prices}}} \tag{88}$$

We find the conditions on the model parameters that satisfy Equation (84).

We solve for the values of the organic price premium $\frac{P_{org}-P_{con}}{P_{con}}$ that satisfy condition (84) for a conventional farmer to want to adopt organic. Similarly, we solve for the values of ϵ, which measures how close ϵ the conventional farmer is to satisfying organic requirement K_{org} at $t = 0$, satisfy condition (84) for a conventional farmer to want to adopt organic.

Because our optimal trajectories change form depending on where we are in the parameter space,

$V_{con}(K_{org} - \epsilon)$ and, consequently, Equation (84) also change form depending on parameter space. Thus, the conditions on the organic price premium $\frac{P_{org}-P_{con}}{P_{con}}$ defining $\{\frac{P_{org}-P_{con}}{P_{con}}) : \Delta(\epsilon) > 0\}$ and also on ϵ defining $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$ do not have a general form. As a result, we must find separate conditions from Equation (84) for each part of parameter space.

6.1 Discrete Analysis for OT1

When the conventional farmer adopts OT1 solutions and $K_{org} = \overline{C}$, $V_{con}(K_{org} - \epsilon)$ is given by:

$$V_{con}(K_{org} - \epsilon) = \frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu + \rho)} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon\right) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{89}$$

With expressions for $V_{org}(K_{org})$ from Equation (88) and $V_{con}(K_{org} - \epsilon)$ from Equation (89), we can now write the differential value from organic management $\Delta(\epsilon)$, defined as the difference in present discounted value of the entire stream of net benefits that a farmer will receive from organic and conventional management, as follows when $K_{org} = \overline{C}$:

$$\Delta^{OT1}(\epsilon) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org} - P_{con}\right) \cdot \left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of organic price premium} \\ \text{on “level effect”} \\ \text{of other agricultural inputs}}} \tag{90}$$

$$- \underbrace{\frac{1}{(\mu + \rho)} \cdot P_{con}\alpha_b \cdot \gamma_K \cdot \underbrace{\left(\overline{C} - \epsilon\right)}_{=K_0}}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho} \cdot \left(P_{con} \left(\alpha_b \cdot \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c \right) + \alpha_c \right) - 1 \right) \cdot \mu\overline{C}}_{\substack{\text{PDV of using synthetic compounds} \\ \text{at dynamically optimal rate } \mu\overline{C}}} \tag{91}$$

The sign of $\frac{\partial\Delta(\epsilon)}{\partial\epsilon}$ is given by:

$$\frac{\partial\Delta(\epsilon)}{\partial\epsilon} = P_{con} \cdot \alpha_b \cdot \frac{\gamma_K}{(\mu + \rho)} \geq 0 \tag{92}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.

Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$. Note that $\Delta(\epsilon^*) = 0$. The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\epsilon^* = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K}\cdot\frac{1}{\rho}}_{\le 0}\left(\underbrace{(P_{org}-P_{con})\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\ge 0}\right.$$

$$\left. -P_{con}\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\mu-\underbrace{\left(\frac{P_{org}}{P_{con}}-\frac{\rho}{\mu+\rho}\right)\gamma_K}_{\ge 0}\right)\overline{C}\right) \tag{93}$$

This means that when $\epsilon^* \le 0$ the farmer will face $V_{org}(K_{org}) - V_{con}(K_{org}-\epsilon) > 0\,\forall\epsilon \ge 0$, and will therefore prefer to produce organically for all feasible initial capital stocks (i.e. they always prefer to produce organically).

Given $\frac{\partial\Delta(\epsilon^*)}{\partial\epsilon} \ge 0$, we will have that:

- The lower the threshold ϵ^*, the larger the set $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$
- The higher the threshold ϵ^*, the smaller the $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$

We conduct a comparative statics analysis of $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org}-\epsilon)$ to analyze how $\Delta(\epsilon)$ responds to changes in parameters ($\mu, \rho, \gamma_{cc}, \gamma_c, \gamma_K, \alpha_1, \alpha_c, P_{con},$ and P_{org}). The results are summarized in Table B.1 and the derivations are presented in Appendix B.1.1.

We similarly conduct a comparative static analysis for ϵ^*. The results are summarized in Table B.2 and the derivations are presented in Appendix B.1.2.

We also want to find how large the price premium needs to be in order to induce the fully informed farmer to prefer organic management. We derive this requirement for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ below.

The range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \ge 0$ is $\frac{P_{org}-P_{con}}{P_{con}} \ge \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} - \frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\left(\mu\overline{C}+\rho\cdot\epsilon\right)}{\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{94}$$

We also conduct a comparative statics analysis for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$. The results are summarized in Table B.3 and full derivations are presented in Appendix B.1.3.

6.2 Discrete Analysis for OT2/OT3/OT4

For the conventional farmer who adopts OT2, OT3, or OT4 solutions, assuming $K_{org} = \overline{C}$, $V_{con}(K_{org}-\epsilon)$ is given by:

$$V_{con}^{OT2,OT3,OT4}(K_{org}-\epsilon) = \underbrace{\frac{1}{\rho}\cdot P_{con}\left(\alpha_b \cdot A_b + A_y\right)}_{\substack{\text{PDV of "level effect"}\\ \text{of other agricultural inputs}\\ \text{at conventional prices}}} + \underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\left(\underbrace{(K_{org}-\epsilon)}_{=K_0} - \hat{K}_{con}\right)}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}}$$
$$+ \underbrace{\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\left(\overline{C}-\hat{K}_{con}\right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu\left(\overline{C}-\hat{K}_{con}\right)}$$

Given $K_{org} = \overline{C}$ the differential value from organic management $\Delta^{OT2,OT3,OT4}(\epsilon)$ faced by the conventional farmer with OT2/OT3/OT4 conditions is given by:

$$\Delta^{OT2,OT3,OT4}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}\left(P_{org}-P_{con}\right)\cdot\left(\alpha_b A_b + A_y\right)}_{\substack{\text{PDV of organic price premium}\\ \text{on "level effect"}\\ \text{of other agricultural inputs}}} \tag{95}$$

$$- \underbrace{\frac{1}{(\mu+\rho)}\cdot P_{con}\alpha_b\cdot\gamma_K\cdot\left(\underbrace{(\overline{C}-\epsilon)}_{=K_0} - \hat{K}_{con}\right)}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} \tag{96}$$

$$- \underbrace{\frac{1}{\rho}\cdot\left(P_{con}\left(\alpha_b\cdot\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C}-\hat{K}_{con}\right)+\gamma_c\right)+\alpha_c\right)-1\right)\cdot\mu\left(\overline{C}-\hat{K}_{con}\right)}_{\text{PDV of using synthetic compounds at dynamically optimal rate } \mu\left(\overline{C}-\hat{K}_{con}\right)} \tag{97}$$

The sign of $\frac{\partial\Delta(\epsilon)}{\partial\epsilon}$ is given by:

$$\frac{\partial\Delta(\epsilon)}{\partial\epsilon} = \frac{P_{con}\alpha_b\gamma_K}{\mu+\rho}\cdot\epsilon \geq 0 \tag{98}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.

Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$. The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where, when $K_{org} = \overline{C}$:

$$\epsilon^* = \frac{1}{\gamma_K} \cdot \frac{\mu+\rho}{\rho} \cdot \left(\underbrace{\frac{1}{2} \cdot \frac{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{\mu+\rho}\right)^2}{(-\gamma_{cc})}}_{\geq 0} - \underbrace{\left(\frac{P_{org}}{P_{con}} - 1\right) \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \right) \tag{99}$$

As shown in Appendix B.2, under OT2/OT3/OT4 conditions there is a possibility that ϵ^* exceeds $\overline{C}$. When this happens there will be no feasible ϵ for which $\Delta(\epsilon) \geq 0$, and there will therefore be no feasible capital stock for which the fully informed farmer facing OT2/OT3/OT3 conditions will prefer to produce organically. ϵ^* will be more likely to exceed $\overline{C}$ when the farmer faces small organic price premia.

We conduct a comparative statics analysis of $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$ to analyze how $\Delta(\epsilon)$ responds to changes in parameters ($\mu, \rho, \gamma_{cc}, \gamma_c, \gamma_K, \alpha_{1,}\alpha_c, P_{con},$ and P_{org}). The results are summarized in Table B.4 and the derivations are presented in Appendix B.2.1.

We similarly conduct a comparative statics analysis for ϵ^*. The results are summarized in Table B.5 and the derivations are presented in Appendix B.2.2.

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing OT2/OT3/OT4 conditions to prefer to produce organically. We derive an inequality describing the necessary conditions in Appendix B.2.3. Given the assumption that $K_{org} = \overline{C}$, and assuming conventional crop prices are not zero, the threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ is given by:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2} \cdot (-\gamma_{cc})\, \mu^2 \left(\overline{C} - \hat{K}_{con}\right)^2 - \frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \epsilon\right)}{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{100}$$

We can the determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters. The results are summarized in Table B.6 and derivations are presented in Appendix B.2.3.

6.3 Discrete Analysis for OT3'

For a conventional farmer who follows OT3' solution trajectories, under the assumption that $K_{org} = \overline{C}$, and $\gamma_{cc} = 0$, we can then write $V_{con}(K_{org} - \epsilon)$ as:

$$V_{con}(\overline{C}-\epsilon) = \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of ``level effect''}\\ \text{of other agricultural inputs}\\ \text{at conventional prices}}} + \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot(\overline{C}-\epsilon)}_{\substack{\text{PDV of value gained from}\\ \text{microbial productivity}\\ \text{under conventional management}}} + \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{\gamma_K}{\mu+\rho}\cdot\mu\cdot\epsilon}_{\substack{\text{PDV of using synthetic compounds}\\ \text{at dynamically optimal}\\ \text{conventional rate } \mu\cdot\epsilon}} \tag{101}$$

When $\gamma_{cc}=0$ and $\mu\neq 0$, $R_{con}(K)=\rho\,\forall K$ implies:

$$\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}=\frac{\gamma_K}{\mu+\rho} \tag{102}$$

Given $K_{org}=\overline{C}$, the differential value from organic management $\Delta^{OT3'}(\epsilon)$ faced by the conventional OT3' farmer is given by:

$$\Delta^{OT3'}(\epsilon) = \underbrace{\frac{1}{\rho}P_{org}\cdot\alpha_b\gamma_K\underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome}\\ \text{at organic-level capital stock and}\\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho}(P_{org}-P_{con})\cdot(\alpha_bA_b+A_y)}_{\substack{\text{PDV of organic price premium}\\ \text{on ``level effect''}\\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\underbrace{(\overline{C}-\epsilon)}_{=K_0}}_{\substack{\text{PDV of microbial productivity}\\ \text{under conventional management}}} - \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{\gamma_K}{\mu+\rho}\cdot\mu\cdot\epsilon}_{\substack{\text{PDV of using synthetic compounts}\\ \text{at dynamically optimal}\\ \text{conventional rate } \mu\cdot\epsilon}} \tag{103}$$

Assuming $P_{con}\cdot\alpha_b\cdot\gamma_K\cdot\frac{\rho}{\mu+\rho}\cdot\neq 0$, ϵ^* is given by:

$$\epsilon^*=-\frac{\mu+\rho}{\rho}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)\left(\overline{C}+\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\gamma_K^{-1}\right)\leq 0 \tag{104}$$

In Appendix B.3.1, we discuss the signs of $\frac{\partial \Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org} = \overline{C}$. The results are summarized in Table B.7.

In Appendix B.3.2, we calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$. The results are summarized in Table B.8.

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing OT3' conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below. Assuming that $P_{con} \neq 0$, and assuming that $\frac{1}{\rho}\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can write:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{105}$$

Given

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \geq 0, \tag{106}$$

$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* \leq 0$ implies that the OT3' farmer prefers organic given any non-negative price premium. Still, we may at some point be interested in how the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ responds to changes in our parameter values in this case. In Appendix B.3.3, we determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters. The results are summarized in Table B.9.

6.4 Discrete Analysis for OT3”

For a conventional farmer who follows OT3' solution trajectories, under the assumption that $K_{org} = \overline{C}$, and $\mu = 0$, we can then write $V_{con}(K_{org} - \epsilon)$ as:

$$V_{con}(K_{org} - \epsilon) = \underbrace{\frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(A_b + \frac{A_y}{\alpha_b}\right)}_{\substack{\text{PDV of “level effect”} \\ \text{of other agricultural inputs} \\ \text{at conventional prices}}} + \underbrace{\frac{1}{\rho} \cdot P_{con}\alpha_b\gamma_K\left(\overline{C} - \epsilon\right)}_{\substack{\text{PDV of value gained from} \\ \text{microbial productivity} \\ \text{under conventional management}}} \tag{107}$$

Given $K_{org} = \overline{C}$, the conventional OT3” farmer faces:

$$\Delta^{OT3''}(\epsilon) = \underbrace{\frac{1}{\rho} P_{org} \cdot \alpha_b \gamma_K \underbrace{\overline{C}}_{=K_{org}}}_{\substack{\text{PDV of stewarding soil microbiome} \\ \text{at organic-level capital stock and} \\ \text{at organic prices}}} + \underbrace{\frac{1}{\rho} (P_{org} - P_{con}) \cdot (\alpha_b A_b + A_y)}_{\substack{\text{PDV of organic price premium} \\ \text{on "level effect"} \\ \text{of other agricultural inputs}}} - \underbrace{\frac{1}{\rho} \cdot P_{con} \alpha_b \gamma_K \left(\overline{C} - \epsilon\right)}_{\substack{\text{PDV of microbial productivity} \\ \text{under conventional management}}} \tag{108}$$

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing OT3' conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below. Assuming that $P_{con} \neq 0$, and assuming that $\frac{1}{\rho}\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can write:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{109}$$

Given

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \geq 0, \tag{110}$$

$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* \leq 0$ implies that the OT3' farmer prefers organic given any non-negative price premium.

6.5 Discrete analysis for cases with accidental transition

As explained in Section 5, a conventional farmer who transitions to organic "accidentally" would not make a continuous transition any faster even if they anticipated eventually transitioning "accidentally" to organic.

Nevertheless, the accidental farmers might be induced to "jump" transition to organic by an organic price premium just as the non-accidental farmers might. Given $P_{org} > P_{con}$, it may still be possible that a fully informed conventional farmer who would otherwise transition to organic management “accidentally” might make a “jump” transition to the organic certification threshold even earlier than they would by following their optimal conventional trajectory toward leading to an “accidental” and continuous transition. Thus, “jump” transitions that accelerate the transition to organic might be possible in the presence of an organic price premium for conventional farmers who may have “accidental” and continuous transitions in the absence of an organic price premium.

6.5.1 Discrete analysis for OT5

A “jump” transition refers to conventional farmers who completely discontinue their use of synthetic compounds in order to get to the organic threshold as quickly as possible.

For an OT5 conventional farmer with $\hat{K}_{con} > \bar{C}$, the optimal conventional management plan requires applying synthetic compounds at the rate $c(t) = 0$. This is the same synthetic compound policy as in a “jump” transition. For the OT5 farmer, the optimal policy under an accidental, or continuous, transition is the same as the policy under a “jump” transition ($c(t) = 0$). Therefore, for the OT5 conventional farmer there is no threshold organic price premium that would induce the farmer to switch out of their continuous transition path (or, equivalently, out of their optimal conventional management plan), since this path is already the same as the “jump” transition path that gets the conventional farmer to organic management as quickly as possible.

7 Investment Under Organic Price Uncertainty

In this section we use dynamic programming and investment under uncertainty to derive the optimal organic switching policy (i.e., the conditions under which a conventional farmer will switch to organic).

Let the action variable a be a dummy for switching to organic.

Without loss of generality, we assume that the organic price P_{org} is uncertain and stochastic, while the the conventional price P_{con} is a fixed parameter that is not stochastic.

If the farmer stays conventional, then he gets a current period payoff (which is a function of the conventional price P_{con}, which we assume is a fixed parameter that is not stochastic) plus β times the continuation value from waiting instead of switching to organic. Let’s assume for simplicity that for all periods t for which the farmer chooses to continue producing conventionally, they will employ the pesticide policy that solves the conventional management problem conditional on conventional management being their only option.

If the farmer switches to organic, let’s assume for this investment under uncertainty analysis that the farmer can’t switch back, so the payoff to switching is a lump-sum payoff which is the value function from being organic from that time t onwards (similarly to the local discrete analysis). Thus, there is no continuation value if the farmer switches to organic since we model the farmer as having no more decisions to make after switching. We also assume that the organic price the farmer receives from that time t onwards is the organic price P_{org} at the time t of the switch to organic.

The optimal organic switching policy under uncertainty will be a threshold value of the organic price premium above which the conventional farmer will switch to organic. This threshold organic price premium will be a function of K (or ϵ) and of parameters (including parameters in the transition density for the organic price P_{org}).

Thus, for our investment under uncertainty model, we are focusing on the switch to organic, not the quantity of synthetic compound use.

The (infinite horizon) value function for a conventional farmer who has the option to switch to organic is given by:

$$v(P_{org}, K) = \max \left\{ \max_I \left(G_{con}(K, I) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K, I, a_t = 0 \right] \right), V_{org}(P_{org}) \right\}. \quad (111)$$

where P_{org} is a stochastic state variable, and where the discount factor $\beta = \frac{1}{1+\rho} \in [0, 1)$. The value function for a conventional farmer who has the option to switch to organic is the maximum of the PDV payoff from two possible options: (1) stay conventional, or (2) switch to organic. $V_{org}(K_{org})$ is the value function from organic production, and is therefore the PDV of the entire stream of net benefits from having switched to organic production (similar to the $V_{org}(K_{org})$ we use in the local discrete analysis).

We can write the following expression for $G_{con}(K, I)$:

$$G_{con}(K, I) = P_{con} \cdot f(c_t, b_t) - c_t \quad (112)$$

If the farmer remains conventional, they choose c_t according to the dynamically optimal chemical-use policy$\{c_t^*\}$ that solves their value function when managing conventionally is their only option. $\{c_t^*\}$ can be determined using optimal control theory or dynamic programming, and expressed in terms of model parameters.

We assume $K_{org} = \overline{C}$.

Right now $V_{org}(P_{org})$ is easy to work with when $K_{org} = \bar{C}$ since we can factor out P_{org} and express $V_{org}(P_{org})$ as $P_{org} \cdot f(K_{org})$ for some function $f(K_{org})$ of K_{org} (which makes it easier to solve for P_{org}^* and the threshold organic premium).

When $K_{org} = \overline{C}$, we have:

$$V_{org}(P_{org}) = \frac{1}{\rho} P_{org} \cdot \left(\alpha_b \left(\gamma_K \overline{C} + A_b \right) + A_y \right) \quad (113)$$

Next we consider

$$G_{con}(K, I) = P_{con} \cdot f(c_t, b_t) - c_t \quad (114)$$

Let's assume for simplicity that for all periods t for which the farmer chooses to continue producing conventionally, they will employ the pesticide policy that solves the conventional management problem conditional on conventional management being their only option.

When net investment $I(t)$ (and synthetic compound use $c(t)$) is chosen optimally, $G_{con}(K, I^*)$ is no longer a function of I (or c), so we can define the optimized net gain $G^*_{con}(K)$ as:

$$G^*_{con}(K) \equiv G_{con}(K, I^*) \tag{115}$$

If the farmer pursues the optimal conventional management plan as long as they remain conventional, then we can simplify our notation for $v(P_{org}, K)$ and write:

$$v(P_{org}, K) = \max\left\{G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K, a_t = 0\right], V_{org}(P_{org})\right\}, \tag{116}$$

7.0.1 Deterministic benchmark

First let's start with the no uncertainty case (i.e., the deterministic case) where P_{org} is not stochastic but instead fixed and known with certainty.

$$v_{det}(P_{org}, K) = \max\left\{V_{con}(K), V_{org}(P_{org})\right\}, \tag{117}$$

Now, the value of continuing to produce conventionally, $V_{con}(K)$, is not a function of P_{org}. The value of producing organically, $V_{org}(P_{org})$, though, is strictly increasing with respect to P_{org} (assuming $\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$.)

Therefore, assuming that $\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can find a value of P_{org} for each K at which the value of continuing to produce conventionally is equal to the value of producing organically. We will denote this value as $P^*_{org}(K)$. At $P^*_{org}(K)$ we therefore have that:

$$V_{con}(K) = V_{org}(P_{org} \mid P_{org} = P^*_{org}(K)) \tag{118}$$

$$V_{con}(K) = \frac{1}{\rho} P^*_{org}(K) \cdot \alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \tag{119}$$

where $V_{con}(K)$ comes from our solution to the conventional problem.

From Equation (119), when there is no uncertainty about P_{org} and P_{org} is not stochastic but instead known and fixed, the threshold $P^*_{org,det}(K)$ is given by:

$$P^*_{org,det}(K) = \frac{V_{con}(K)}{\frac{1}{\rho} \cdot \alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{120}$$

where $V_{con}(K)$ comes from our solutions to the conventional problem.

In Appendix C.1, we confirm, for each of OT1, OT2/OT3/OT4, and OT3', that when $K_{org} = \overline{C}$, $P^*_{org,det}(K)$ yields the same threshold organic premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ that we previously derived in our local discrete analysis when there is no uncertainty and when $K_{org} = \overline{C}$.

7.0.2 Stochastic prices

Now we will analyze the uncertainty case where P_{org} is stochastic. Let's assume that P_{org} evolves as a first-order Markov process $P'_{org} \overset{iid}{\sim} F_{P_{org}}(\cdot|P_{org})$.

For simplicity we also assume that $a_t = 0$ does not give the farmer information about the distribution of P_{org} because, for example, the distribution of P_{org} is known to the farmer ahead of the starting period and is not affected by their adoption decision in the current period. In this case we can write that

$$v(P_{org}, K) = \max\left\{G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right], V_{org}(P_{org})\right\}, \tag{121}$$

Now, the value of continuing to produce conventionally, $G^*_{con}(K)+\beta\cdot\mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]$, is not a function of P_{org}. The value of producing organically, $V_{org}(P_{org})$, though, is strictly increasing with respect to P_{org} (assuming $\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$.)

Therefore, assuming that $\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can find a value of P_{org} for each K at which the value of continuing to produce conventionally is equal to the value of producing organically. We will denote this value as $P^*_{org}(K)$. At $P^*_{org}(K)$ we therefore have that:

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] = V_{org}(P_{org} \mid P_{org} = P^*_{org}(K)) \tag{122}$$

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] = \frac{1}{\rho}P^*_{org}(K) \cdot \alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \tag{123}$$

From condition (123) we have that at $P^*_{org}(K)$ the following condition holds when $K_{org} = \overline{C}$:

$$G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] = \frac{1}{\rho}P^*_{org}(K) \cdot \alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \tag{124}$$

where the RHS comes from the continuous time value of $V_{org}(P_{org})$, conditional on remaining organic and conditional on $K_{org} = \overline{C}$.

From this equation we can write:

$$\begin{aligned} P^*_{org}(K) = &\frac{V_{con}(K)}{\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \\ &+ \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \end{aligned} \tag{125}$$

where $K^*(t)$ is the solution for a conventional farmer (who stays conventional and has no option of becoming organic); where $T_{org}(P_{org}, K)$ is the time at which the farmer adopts organic (i.e., the first time t when $a(P_{org}, K) = 1$, and therefore the first time t when $V_{org}(P_{org}) > G^*_{con}(K) + \beta \cdot$

$\mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]$); and where a conventional farmer will adopt organic ($a = 1$) the first time when:

$$a(P_{org}, K) = \mathbf{1}\{V_{org}(P_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]\} \tag{126}$$

As derived in Appendix C.2 for each of Cases OT1, OT2/OT3/OT4, and OT3', we can write an expression for the organic price premium required to induce adoption of organic management when the farmer faces uncertainty in the value of P_{org}:

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} - \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \tag{127}$$
$$\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \geq 0$$

We therefore have that the threshold organic price premium higher when there is uncertainty:

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} \geq \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} \tag{128}$$

The argument for why

$$\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right] \geq 0 \tag{129}$$

is satisfied is as follows. A conventional farmer that has the option to switch permanently to organic does weakly better than a conventional farmer who must stay conventional. In other words $G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] \geq G^*_{con}(K) + \beta \cdot \mathbb{E}\left[V_{con}(K') \mid P_{org}, K\right]$ for all K, which means $G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] \geq V_{con}(K)$ for all K.

$T_{org}(P_{org}, K)$ is the first time t when $a(P_{org}, K) = 1$, and therefore the first time t when $V_{org}(P_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]$. Since $G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right] \geq V_{con}(K)$ for all K, this means that at time $t = T_{org}(P_{org}, K)$, $V_{org}(P_{org}) \geq V_{con}(K)$.

Thus, consistent with the real options theory for investment under uncertainty (Dixit and Pindyck, 1994), when there is uncertainty over the organic price premium, there is an option value to waiting before adopting organic management, and the farmer therefore requires a higher threshold organic price premium before adopting organic management.

8 Application to Organic Standards in US and elsewhere

8.1 USDA Organic Standards

In the United States, the National Organic Program (NOP), which is directed by the U.S. Department of Agriculture (USDA) Agricultural Marketing Service (AMS) and became effective on February

20, 2001, oversees and enforces the integrity of the rigorous USDA organic standards and the accreditation of organic certifiers (USDA Agricultural Marketing Service, 2000b; Organic Produce Network, 2022). Organic is one of the most heavily regulated and closely monitored food systems in the U.S. Any product labeled as organic must be USDA certified (Organic Produce Network, 2022). The National Organic Program (NOP) establishes national standards for the production and handling of organically produced products, including a National List of substances approved for and prohibited from use in organic production and handling; as well as requirements for labeling products as organic and containing organic ingredients. Under the National Organic Program (NOP), certifying agents certify production and handling operations in compliance with the requirements of this regulation and initiate compliance actions to enforce program requirements (USDA Agricultural Marketing Service, 2000b).

The organic production and handling requirements of the National Organic Program (NOP) include the requirement that production practices implemented must maintain or improve the natural resources of the operation, including soil and water quality, as well as the requirement that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop. The on-side inspection must verify that prohibited substances have not been and are not being applied to the operation through means which, at the discretion of the certifying agent, may include the collection and testing of soil; water; waste; seeds; plant tissue; and plant, animal, and processed products samples (USDA Agricultural Marketing Service, 2000a). Thus, becoming certified organic under the USDA National Organic Program may entail being subject to soil testing (USDA Agricultural Marketing Service, 2000a; Baier and Ahramjian, 2012; USDA Agricultural Marketing Service, 2018).

In our model, we model organic certification requirements as a clean soil stock threshold K_{org} (or, equivalently, a threshold for the stock of synthetic compounds in the soil C_{org}). Our choice to make certification contingent on a stock threshold at least partially captures the main features of the US National Organic Program, including the requirements that practices implemented must maintain or improve the natural resources of the operation, including soil and water quality, and that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop. In order to verify that practices implemented must maintain or improve the natural resources of the operation, including soil and water quality and that prohibited substances have not been applied by a period of 3 years, the certifying agent may collect and test the soil; as a consequence, the certification requirement essentially amounts to the requirement that the stock of synthetic compounds in the soil must not exceed a threshold C_{org} (or, equivalently, clean soil stock must meet a clean soil stock threshold K_{org}).

To see this, consider that a farmer's organic certification agent ultimately reserves the right to reject a farmer's application if they find evidence of prohibited substances in the farmer's soils (as per the USDA's certification requirements) that exceed a threshold C_{org} (or, equivalently, a clean soil stock that does not meet a clean soil stock threshold K_{org}). Therefore, the farmer needs to ensure that the stock of synthetic residues remaining in their soils meets this threshold C_{org}. If the stock of synthetic residues remaining in a farmer's soils after the required 3-year period of not using synthetic compounds has not fallen below the threshold C_{org}, the farmer's application will be rejected, even if the farmer claims to have not used prohibited substances for the last three years. Therefore, our use of a capital stock threshold is justified, because satisfying this synthetic residue stock requirement is a meaningful/necessary part of becoming eligible for certification.

We can additionally impose the requirement that the field or farm parcel must have had no prohibited substances applied to it for a period of 3 years immediately preceding harvest of the crop as a requirement that the optimal trajectory for synthetic compound input use $c(t) = 0$ for all t. Continuous transitions to organic management that can involve $c(t) = 0$ for at least 3 years include:

1. Optimal Trajectories 5 (OT5): Invest as fast as possible until $K = \overline{C}$
2. Optimal Trajectories 4 (OT4) if $\hat{K}_j = \overline{C}$: Approach $\hat{K}_j = \overline{C}$ at moderate speed; and
3. Optimal Trajectories 4 (OT4, Approach $\hat{K}_j$ at moderate speed) if $\mu = 0$ (i.e., the stock of synthetic compounds in the soil does not decay on its own) and $\hat{K}_j \geq K_{org}$.

For discrete "jump" transitions, we can additionally impose the requirement that any discrete "jump" transition must have $c(t) = 0$ for at least 3 years as part of the discrete "jump" .

For the majority of our analysis, we also assume (as we approximately have in all real-world organic certification programs known to the authors) that organic certification requires that a farmer fully remediate their soils, such that they will be certified organic if and only if $K = \overline{C}$. In this case we have that $K_{org} = \overline{C}$.

When $K_{org} = \overline{C}$, this means that, for cases in which there is no "accidental" , continuous transition to organic, a farmer that becomes organic and stays organic will choose not to apply any synthetic compounds, since:

$$c_{org}(t) = \mu \left(\overline{C} - K_{org} \right) = 0 \, \forall t \tag{130}$$

Thus, under the assumption that $K_{org} = \overline{C}$, $c(t) = 0$ for organic farmers, which corresponds to the organic production requirements of the National Organic Program (NOP).

8.2 Organic Standards Elsewhere

Great Britain's organic standards are currently the same as EU standards. Basically, no synthetic fertilizers or pesticides may be used. Approved fertilizers and pesticides can only be used if other management methods are not working by themselves. Even then their use has to be justified. The certification program establishes nutrient caps (such that farmers cannot apply more than X amount of nitrogen equivalents per Y area of land). Farmers must also demonstrate that they are making efforts to increase the ecological/environmental soundness of their operation (e.g. by minimizing the destruction of important natural habitats, etc) (Soil Association, 2023a). There is a 2- to 3-year transition period before organic premium can be claimed (Soil Association, 2023a). Testing will be performed at the end of the transition period to determine whether farmers need to pursue a longer transition period, and soil samples may be taken to determine if the conversion period need to be extended (Soil Association, 2023b).

In Australia, ACO Certification LTD, Australia's largest organic certifier, tests farmers' soils for pesticide residues (ACO Certification Ltd, 2023a,b)[2].

9 Examples of crop-synthetic compound-soil microbe systems

One type of beneficial soil microbe is Acidobacteria, which have been shown to affect soils in a number of ways that are important for crop production for a variety of crops (Kalam et al., 2020), including corn (Li et al., 2019), rice (Liu et al., 2021; Huang et al., 2020), and tomatoes. Evidence suggests that Acidobacteria promote plant growth through production of certain beneficial compounds. Some Actinobacteria, like Micrococcales, have been shown to increase the abundance of salicylic acid, which improves crop pest resistance (Blundell et al., 2020). It is hypothesized that Acidobacteria facilitate water and nutrient uptake by plants; that they are very important in nutrient cycling (including carbon, nitrogen and sulfur cycling); and that they function as a "keystone taxa", vital for shaping how a soil ecosystem works (Kalam et al., 2020). Newman et al. (2016) finds that Acidobacteria are negatively affected by the use of Glyphosate (Round Up), a pesticide used for corn and tomatoes.

Dash et al. (2017) shows that the use of the synthetic pesticide benthiocarb harms cyanobacteria in agricultural soils used in the cultivation of rice, as indicated by falling measures of the biomass, acetylene reduction activity (ARA), and N-yield of cyanobacteria present in soils. Findings suggest that benthiocarb harms the ability of the cyanobacteria present in agricultural soils to fix nitrogen and to make that nitrogen available to a farmer's rice crops.

Li et al. (2022) show that 50% substitution of synthetic fertilizer with organic fertilizer outperforms synthetic fertilizers alone, and organic fertilizers alone, over 7 years. Several studies have shown that long-run rice yields have been improved by using less synthetic fertilizer and in some instances substituting with organic fertilizers (Md Mozammel Haque and Kabir, 2019; Meng et al., 2009; Moe et al., 2019; Singh et al., 2019).

Ji et al. (2018) show that tea plantations can become more productive by substituting away some fraction of the synthetic the synthetic nitrogen fertilizer typically used in production. This shift is associated with a shift in the microbial communities found in these plantations. Arafat et al. (2020) find that decreased productivity in tea plantations is associated with a shift in microbial communities. Tao et al. (2015) found that bacteria of the genus Bacillus (which seem to do better in environments with less synthetic nitrogen, phosphorous, and potassium fertilizers) may suppress soil-born pathogens, which may benefit a plantation's productivity.

[2] This was also confirmed via personal communication with ACO's Technical Officer, Ruwi Jayasuriya, January 2024

Ding et al. (2020) show that sweet-potatoes in certain low-nitrogen environments actually perform better (i.e. have higher yields) when less than standard doses of synthetic nitrogen fertilizer are used. This is a finding that replicates in at least one other study (Satpathy and Singh, 2021). Ding et al. (2020) also show that the decrease in nitrogen use is associated with a shift in their soil's microbial communities in favor of (several) plant-growth promoting bacteria. Several studies in related literatures corroborate the idea that the finding of Ding et al. (2020) is mediated by synthetic fertilizers' effects on soil bacteria (Bill et al., 2021; Wang et al., 2022; Wei et al., 2017; Yoneyama, Terakado-Tonooka, and Minamisawa, 2017). Tangapo, Astuti, and Aditiawati (2018) shed light on which particular soil bacteria may be driving these effects.

Liu et al. (2020) show that long-term organic management of kiwifruit orchards results in a yield increase vis-à-vis comparable orchards managed conventionally. Improved fruit yields were associated with changes in the organic orchard's microbial community, and an increase in the abundance of potentially beneficial microbes.

Vico et al. (2020) find that some organic management regimes result in slightly higher spinach yields than does their conventional control. This is the case even after just one growing system. Other studies have found that organically managed spinach can perform just as well as conventionally managed spinach after just one season as well (Bharad et al., 2013). If we assume that a farmer's soil ecology continues to improve season over season, we might expect that the margin between organic and conventional management systems will eventually widen. Note however, that other short-term studies of organically managed spinach have found that conventionally managed spinach significantly outperforms organically managed spinach, at least in the short-run (Zhuang et al., 2019).

Lead arsenate was commonly used to control pests in US apple orchards in the early to mid-1900s (Gamble, Givens, and Sparks, 2018). Arsenic has been shown to reduce bacterial activity of nitrogen fixers present in agricultural soils (Arindam Chakraborty and Islam, 2017). Nitrogen fixing bacteria are commonly understood to be critical for the proper cycling of nitrogen (a vital resource for agricultural crops) in soils. When lead arsenate was in use as an insecticide in US agriculture, the USDA still did not have an organic certification program (this was established after the passing of the Organic Foods Production Act of 1990). So we should think of this as being an organic system in which a farmer, sometime between 1890 and 1988 (when the EPA finally banned the use of lead arsenate for insecticidal purposes) was considering whether or not to apply for an international ecological certification, such as from Demeter International, which started its certification program in 1928 (Demeter, 1999), or some other ecological certification program.

Arsenic has been shown to reduce bacterial activity of nitrogen fixers present in agricultural soils (Arindam Chakraborty and Islam, 2017). Nitrogen-fixing bacteria (diazotrophs) are important to wine production because the production of high-quality wine depends on grapes having a high enough nitrogen content (Bell and Henschke, 2005; Verdenal et al., 2021). Diazotrophs can also help crops through the regulation of phytohormones that can coordinate plant responses to environmental factors (Thiebaut et al., 2022). Common diazotrophs in grape orchards include: Rhizobium, which function as nitrogen fixers (Wright et al., 2022; Verma et al., 2020); Pseudomonas, which also fix nitrogen, and protect against disease (Wright et al., 2022; Preston, 2022), Alpha-Proteobacteria - Bradyrhizobium (Gamalero et al., 2020), and Beta-Proteobacteria - Burkholderia (Gamalero et al., 2020).

Organochlorides were another class of historical pesticides that were highly persistent. One such compound, DDT, which was used as a pesticide in US agriculture between 1945 and 1972, has a half life in soil anywhere from 2 to 15 years, depending on the soil type (Agency for Toxic Substances and Disease Registry [ATSDR], 2005).

10 Synthetic Compound Decay Rates

The decay rate $\mu(X)$ of a synthetic compound vary depending on the synthetic compound. Thiobencarb, a synthetic pesticide which is widely used on rice, for example, has a half-life in soils of about 21 days (U.S. Department of Agriculture [USDA], 1995). So 21 days after applying 4 lb of active ingredient over an acre (4 lb per acre being the EPA's maximum recommended dose in a one-year period (U.S. Environmental Protection Agency [EPA], 2022)), just 2 lb will remain. After 42 days only 1 lb will remain. After 63 days 0.5 lb remain, and so on and so forth. By the end of the year the farmer has about 0.00002344 lb, or 0.01063221 grams of thiobencarb left on their acre of land.

While many modern day pesticides degrade relatively quickly, with half-lives on the order of a couple of days to a couple of weeks, some pesticides still in use continue to be longer-lived. For instance, depending on environmental conditions the insecticide Clothianidin, which is applied to crops like corn and canola, may have a soil half-life of up to 1,155, or even 6,931 days (U.S. Environmental Protection Agency [EPA], 2003; DeCant and Barrett, 2010). The more conservative upper bound implies that up to half of the clothianidin applied to a farmer's field today would still be present in their soils three years from now.

At the far extreme we have a number of discontinued pesticides which were used as part of historical farming practices, and which contained heavy metals that were very persistent in soils. Lead arsenate, for instance, was widely used to combat pests in US apple orchard through the 1940's (Hood, 2006). The half-life of lead arsenate in soil is estimated to be approximately 16 years (Eisler, 1988), such that roughly 3% of the lead arsenate applied in orchard in the year 1949 was still present in the orchard's soils in the year 2024.

Depending on the pesticides being considered, therefore, there is certainly scope for the time t in our model to be measured on the scale of years.

11 Discussion and Conclusion

We develop a dynamic bioeconomic model of a farmer's decisions regarding the use of synthetic compounds (e.g., synthetic fertilizers and pesticides) and the transition from conventional to organic management. Our model accounts for newly documented interrelationships among synthetic compound use, soil health, and crop yields. In particular, new insights from soil science show that the use of synthetic compounds can be harmful to beneficial soil microbes that improve agricultural yields by enhancing crop nutrient use, stress tolerance, and pest resistance.

We characterize and solve for a farmer's optimal synthetic compound use strategy, and for whether and how a farmer should transition from conventional to organic farming. Results show that some farmers may transition to organic management "accidentally" as their optimal trajectories gradually take them toward the certification threshold (this can happen even in the absence of an organic price premium). Other transitions may be induced by the organic price premium.

In particular, under full information, some conventional farmers (OT5) prefer organic management, even if there is no organic price premium. Some conventional farmers (OT4 and OT5) will pursue active stewardship and invest in clean soil stock, even there is no organic price premium. Among those still preferring conventional farming, some conventional farmers (OT2, OT3, and OT4) will use less than the maximum level of synthetic compounds, so as to cultivate, and benefit from, soil microbes. Other farmers (OT1) will prefer conventional and will choose the maximum level of synthetic compound use no matter how large the organic price premium. Farmers who prefer to produce conventionally in absence of an organic price premium (OT1, OT2, OT3, OT4) can be induced to prefer organic (via a "jump" transition if the organic price premium is high enough.

Our study sheds light on the importance to farmers of optimally acting upon an accurate understanding of the role that soil bacteria play in crop production and the sensitivity of that bacteria to the application of synthetic compounds. This will help open the way to educational extension programs that could result in improved yields for conventional farmers, organic farmers, and farmers transitioning to organic farming.

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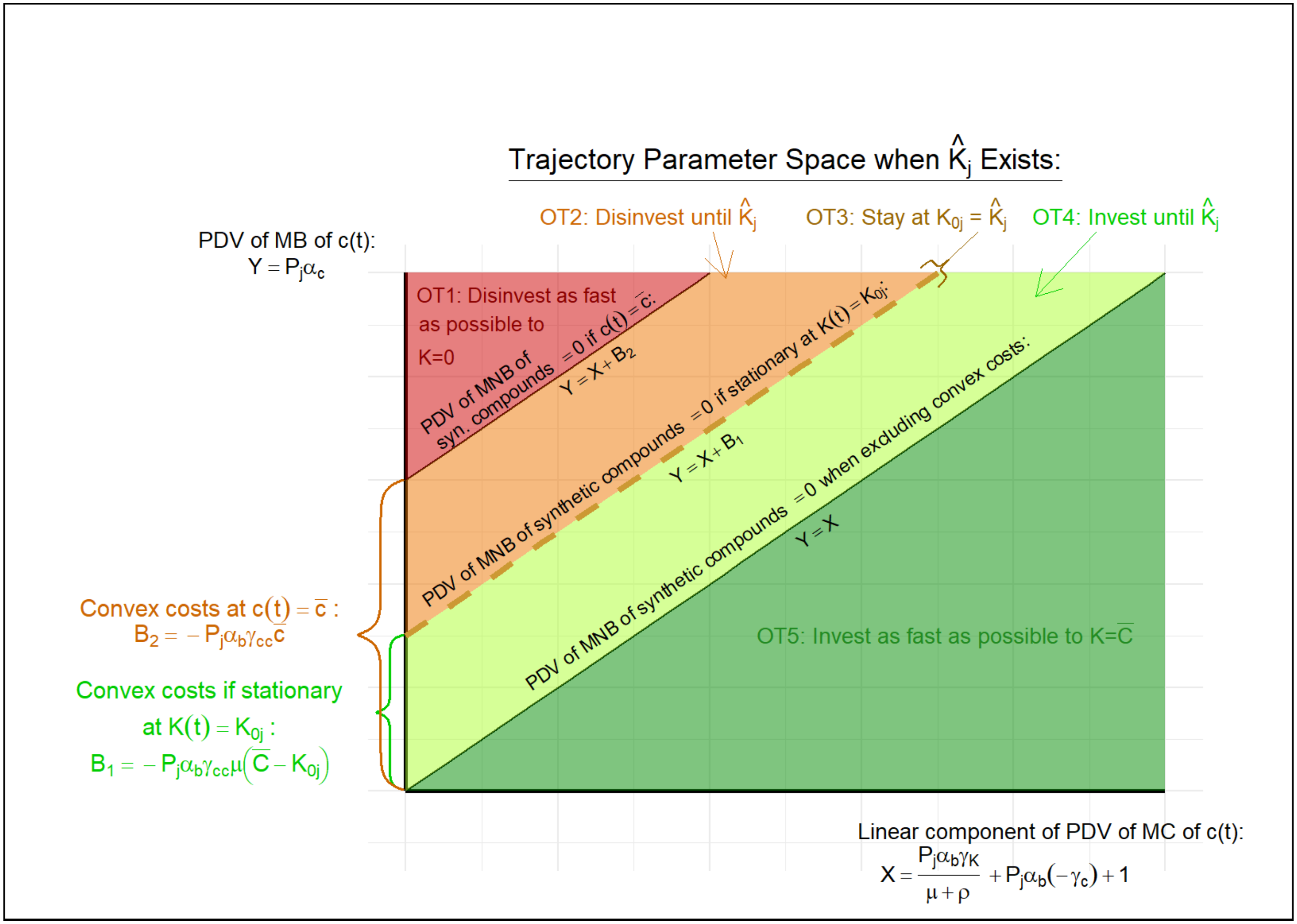
Figure 1: PDV of entire stream of marginal net benefit of additional unit of synthetic compound $c(t)$ today

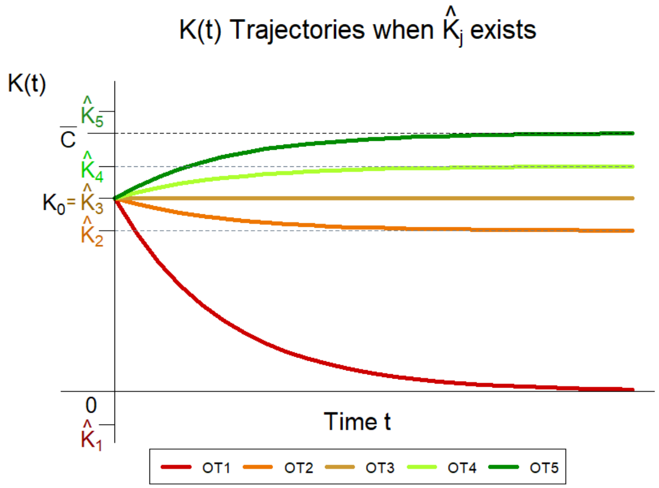
$$P_j \cdot \underbrace{\underbrace{\alpha_c}_{\substack{\geq 0\\ \text{direct effect}\\ \text{of } c(t)\\ \text{on crop output}}}}_{\substack{\text{PDV of entire stream}\\ \text{of marginal benefit}\\ \text{of additional unit}\\ \text{of synthetic compound today}}} - \underbrace{\left(\underbrace{\underbrace{-P_j \underbrace{\alpha_b\left(\underbrace{\gamma_c}_{\text{linear}} + \underbrace{\gamma_{cc}c(t)}_{\text{nonlinear}}\right)}_{\substack{\leq 0\\ \text{indirect effect of } c(t)\\ \text{on crop output}\\ \text{via their direct negative effect}\\ \text{on soil microbes}}}}_{\substack{\geq 0\\ \text{indirect marginal cost of } c(t)\\ \text{via its direct negative effect}\\ \text{on soil microbes}}} + \underbrace{\frac{P_j \overbrace{\alpha_b\gamma_K}^{\substack{\text{indirect effect of}\\ \text{clean soil stock}\\ \text{on crop output}\\ \text{via its direct positive effect}\\ \text{on soil microbes}}}}{\mu+\rho}}_{\substack{\geq 0\\ \text{PDV of entire stream of indirect marginal cost}\\ \text{of unit of synthetic compound today}\\ \text{via its indirect negative effect on soil microbes}\\ \text{thru its negative effect on stock of clean soils}\\ \text{(stock effect)}}}}_{\substack{\text{PDV of entire stream of indirect marginal costs}\\ \text{of additional unit of synthetic compound today}\\ \text{via its negative effects on soil microbes}}} + \underbrace{\underbrace{1}_{\substack{\geq 0\\ \text{unit price}\\ \text{of } c(t)}}}_{\substack{\text{direct marginal cost}\\ \text{of unit of}\\ \text{synthetic compound today}}} \right)}_{\text{PDV of entire stream of marginal costs of additional unit of synthetic compound today}}$$

Figure 2: Condition for stationary solution $\hat{K}_j$ for each stage $j \in \{con, org\}$

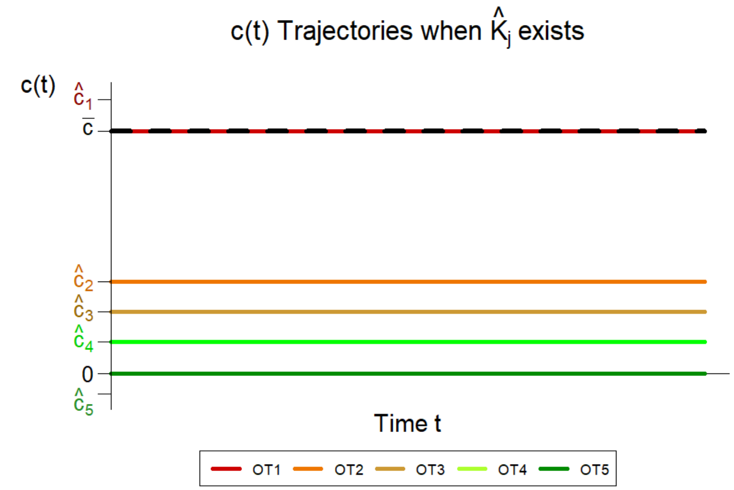
$$P_j \cdot \underbrace{\underbrace{\alpha_c}_{\substack{\geq 0\\ \text{direct effect}\\ \text{of synthetic compounds}\\ \text{on crop output}}}}_{\substack{\text{PDV of entire stream of marginal benefit}\\ \text{of additional unit of synthetic compound today}}} = \underbrace{\underbrace{\underbrace{\underbrace{-P_j\,\alpha_b\left(\underbrace{\gamma_c}_{\text{linear component}} + \underbrace{\gamma_{cc}\,\mu\underbrace{\left(\overline{C}-\hat{K}_j\right)}_{\equiv\hat{c}_j}}_{\text{nonlinear component}}\right)}_{\substack{\leq 0\\ \text{indirect effect of synthetic compounds}\\ \text{on crop output}\\ \text{via their direct negative effect on soil microbes}}}}_{\substack{\geq 0\\ \text{indirect marginal cost}\\ \text{of additional unit of synthetic compound today}\\ \text{via its direct negative effect on soil microbes}}} + \underbrace{P_j\frac{\overbrace{\alpha_b\gamma_K}^{\substack{\geq 0\\ \text{indirect effect of clean soil stock}\\ \text{on crop output}\\ \text{via its direct positive effect on soil microbes}}}}{\mu+\rho}}_{\substack{\geq 0\\ \text{PDV of entire stream of indirect marginal cost}\\ \text{of unit of synthetic compound today}\\ \text{via its indirect negative effect on soil microbes}\\ \text{thru its negative effect on stock of clean soils}\\ \text{(stock effect)}}}}_{\substack{\text{PDV of entire stream of indirect marginal costs}\\ \text{of additional unit of synthetic compound today}\\ \text{via its negative effects on soil microbes}}} + \underbrace{\underbrace{1}_{\substack{\geq 0\\ \text{unit price}\\ \text{of synthetic compound}}}}_{\substack{\text{direct marginal cost}\\ \text{of unit of synthetic compound today}}}}_{\substack{\text{PDV of entire stream of marginal costs}\\ \text{of additional unit of synthetic compound today}}} \tag{131}$$

Figure 3: Parameter Space for Optimal Trajectories When $\hat{K}_j$ Exists

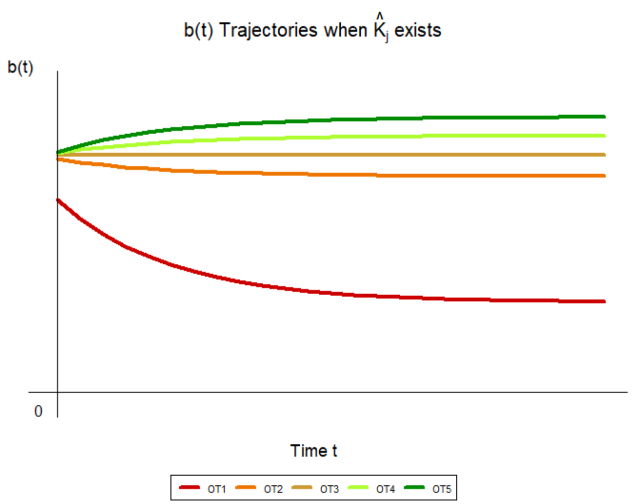




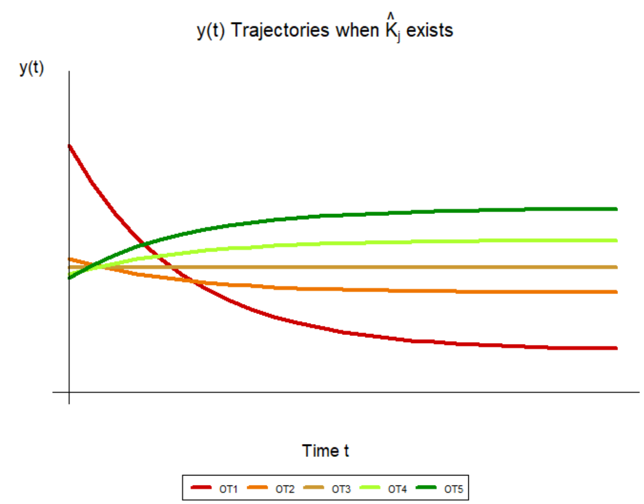
(a) Clean soil stock $K(t)$



(b) synthetic compound use $c(t)$



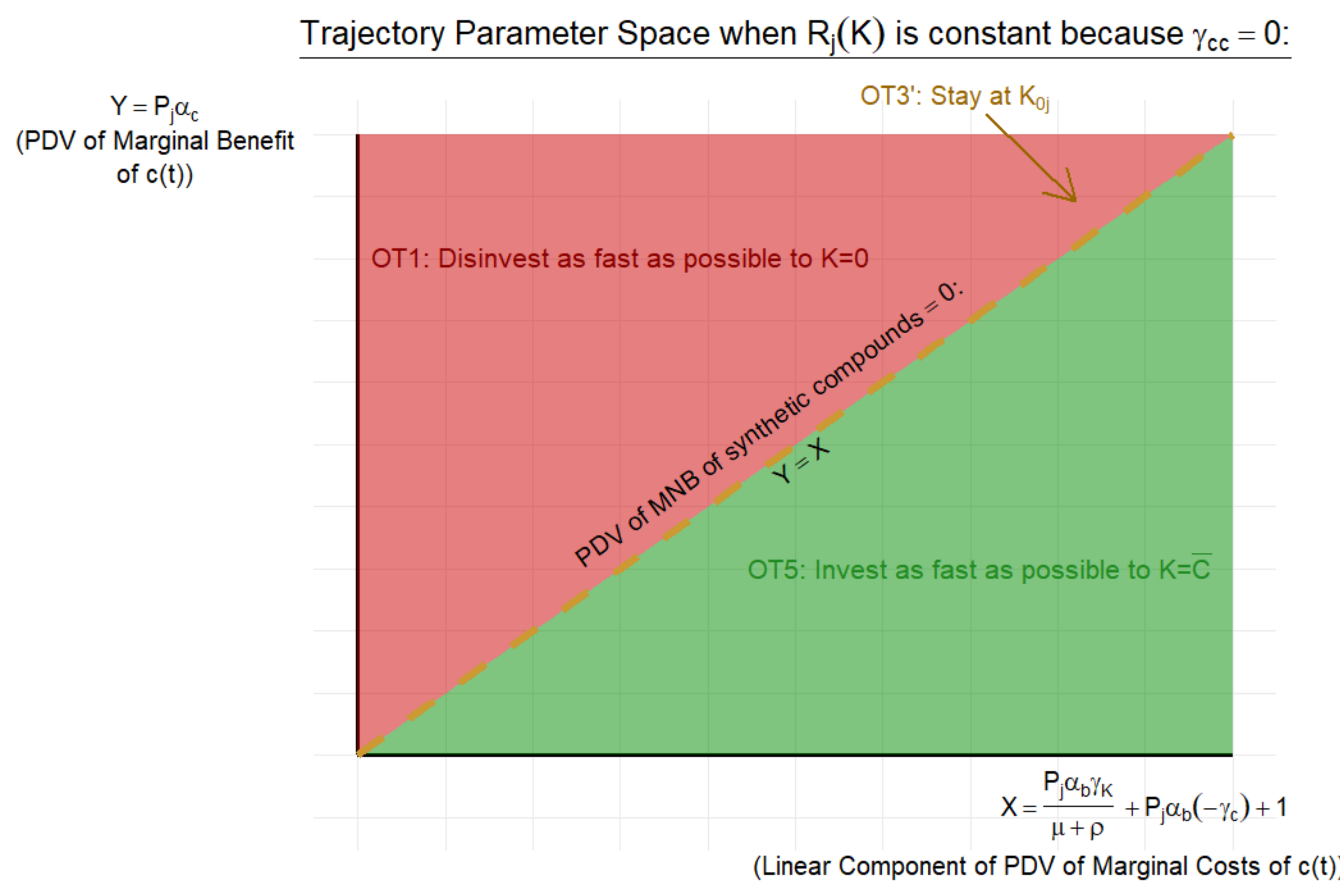
(c) Microbes $b(t)$

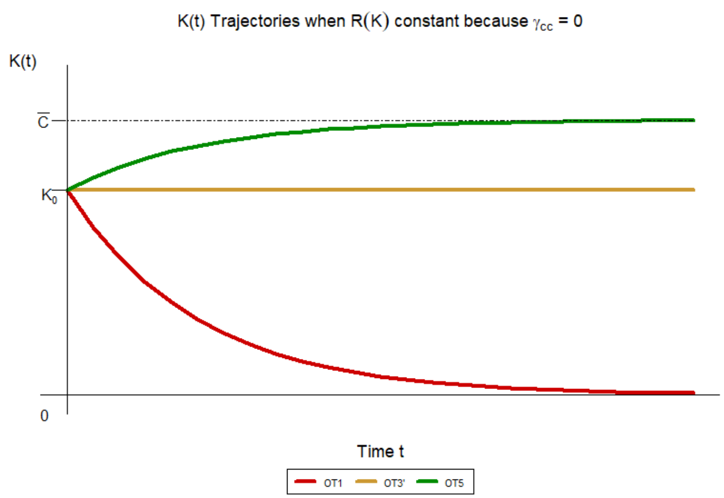


(d) Yield $y(t)$

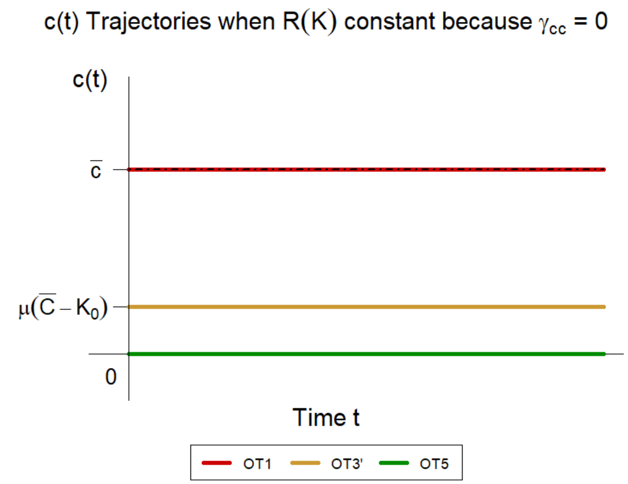
Figure 4: Optimal Trajectories When $\hat{K}_j$ Exists
Note: We assume $\overline{c} = \mu\overline{C}$.

Figure 5: Parameter Space for Optimal Trajectories When $R(K)$ is Constant Because $\gamma_{cc} = 0$

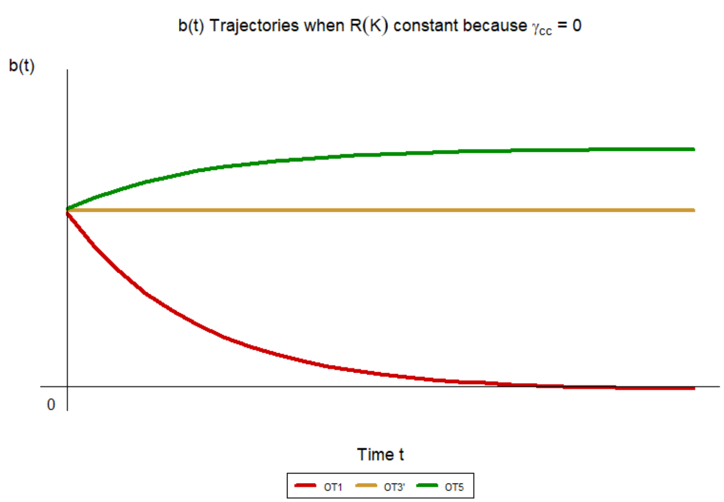




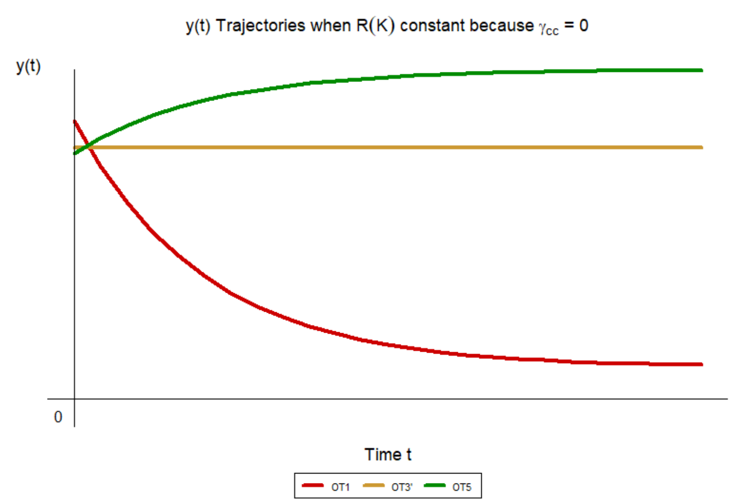
(a) Clean soil stock $K(t)$



(b) synthetic compound use $c(t)$



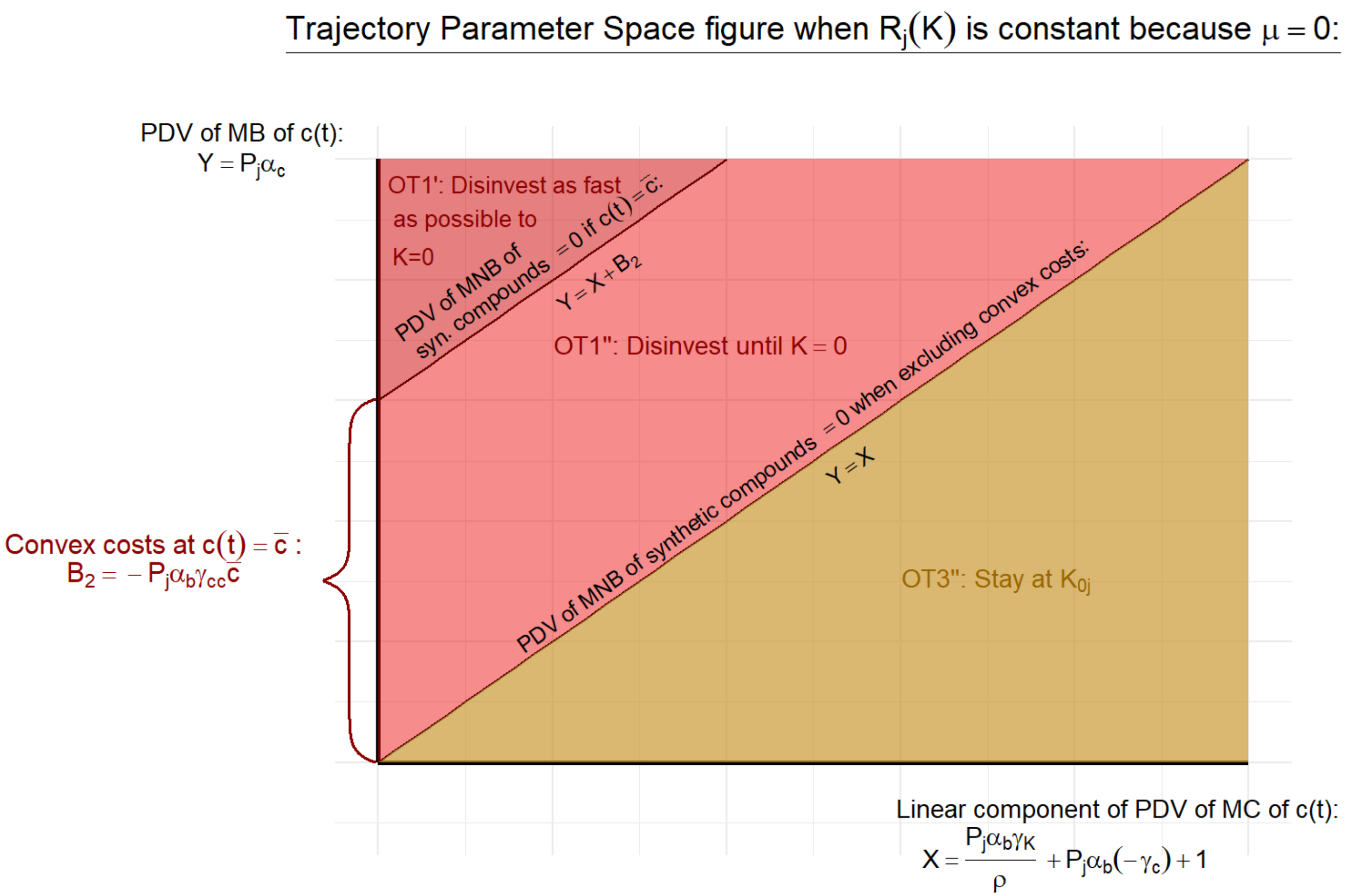
(c) Microbes $b(t)$

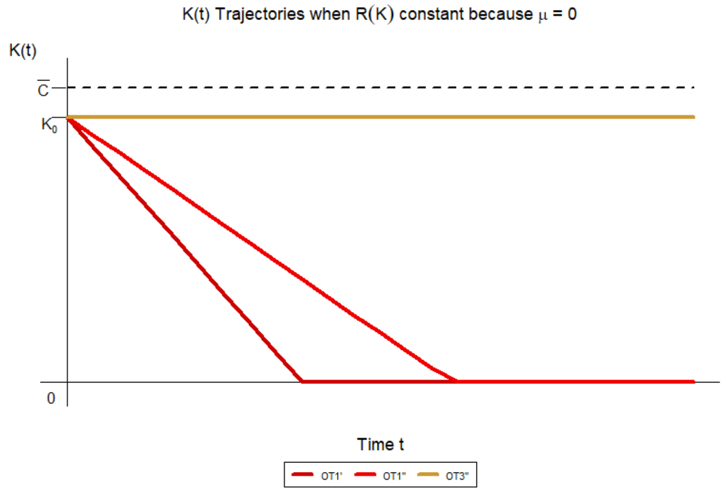


(d) Yield $y(t)$

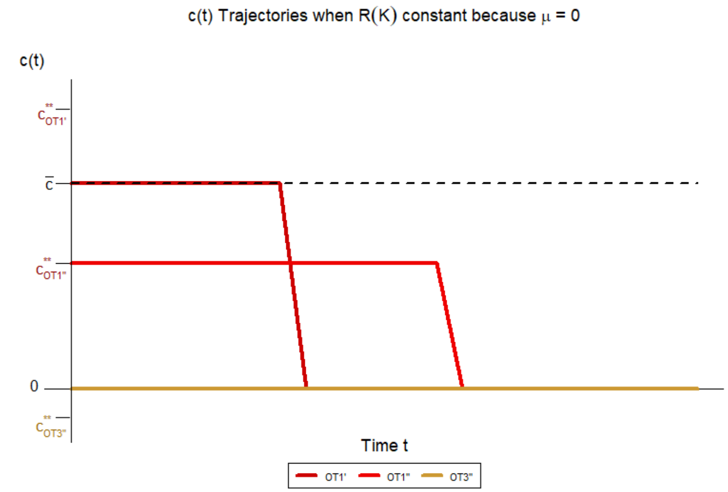
Figure 6: Optimal Trajectories When $R(K)$ is constant because $\gamma_{cc} = 0$
Note: We assume $\mu \neq 0$ and $\bar{c} = \mu\overline{C}$.

Figure 7: Parameter Space for Optimal Trajectories When $R(K)$ is Constant Because $\mu = 0$

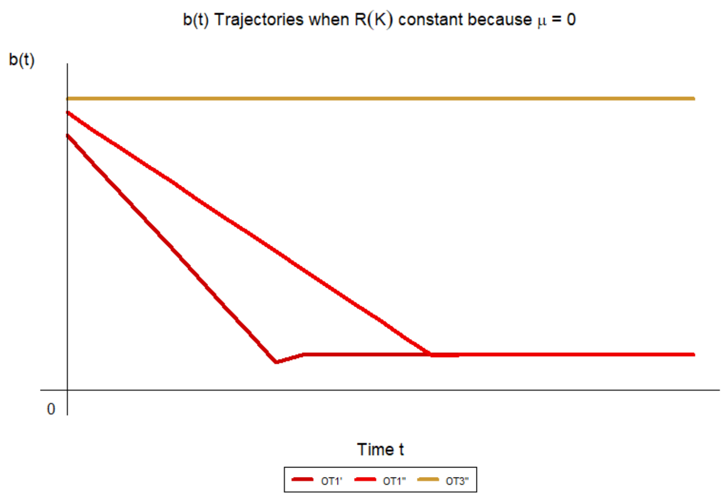




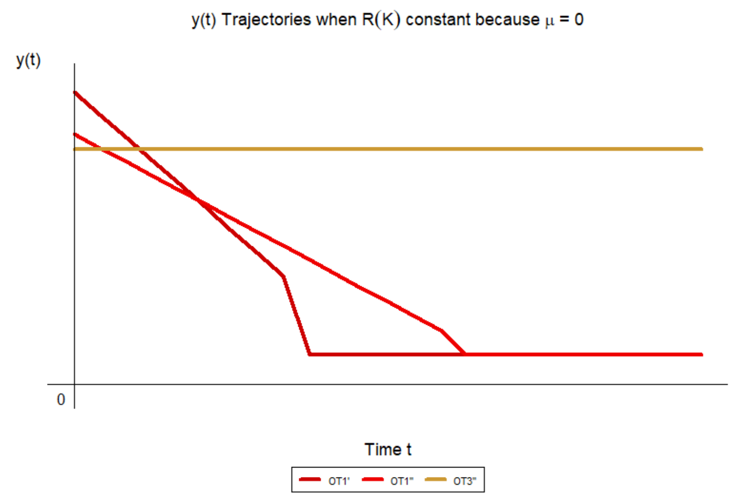
(a) Clean soil stock $K(t)$



(b) synthetic compound use $c(t)$



(c) Microbes $b(t)$



(d) Yield $y(t)$

Figure 8: Optimal Trajectories When $R(K)$ is constant because $\mu = 0$
Note: We assume $\gamma_{cc} \neq 0$.

Appendix

A Optimal Solution for Each Stage

A.1 Stationary rate of return $R_j(K)$

The stationary rate of return $R_j(K)$ on clean soil capital for each stage $j \in \{con, org\}$is given by:

$$R_j(K) = -\mu + \frac{\gamma_K}{\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}. \tag{A.1}$$

The slope of $R_j(K)$ is:

$$R_j'(K) = \frac{\gamma_K}{\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}\gamma_{cc}\mu \tag{A.2}$$

which we can sign as follows:

$$R_j'(K) = \frac{\underbrace{\gamma_K}_{\geq 0}}{\underbrace{\left(\gamma_c + \gamma_{cc}\mu\left(\overline{C} - K\right) + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}_{\geq 0}}\underbrace{\gamma_{cc}}_{\leq 0}\underbrace{\mu}_{\geq 0} \leq 0 \tag{A.3}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then the constant $R_j(K)$ is lower when prices are higher:

$$\frac{\partial R_j(K)}{\partial P_j} = -\frac{\gamma_K}{\left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right)^2}\frac{P_j^{-2}}{\alpha_b} \leq 0 \tag{A.4}$$

A.2 Comparative Statics for Stationary Solution $\hat{K}_j$

We evaluate the effect of each parameter on the stationary solution $\hat{K}_j$ by calculating the partials of $\hat{K}_j$, where:

$$\hat{K}_j = \frac{(\rho + \mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho + \mu)\,\gamma_{cc}\mu} \tag{A.5}$$

A.2.1 Crop price P_j

$$\frac{\partial \hat{K}_j}{\partial P_j} = \frac{1}{\underbrace{P_j^2\alpha_b}_{\geq 0}\underbrace{\mu}_{\geq 0}\underbrace{\gamma_{cc}}_{\leq 0}} \leq 0 \tag{A.6}$$

The effect of crop price on the stationary solution is weakly negative: $\frac{\partial \hat{K}_j}{\partial P_j} \leq 0$. The stock of clean soil at the stationary solution is smaller at higher crop prices. Thus, when the farmer faces greater incentives to produce, the stationary solution $\hat{K}_j$ at which the stationary rate of return of clean stock capital equals the interest rate ρ is lower:

Note that $\frac{\partial \hat{K}_j}{\partial P_j}$ becomes more negative, and therefore we have a greater decrease in the stock of clean soil at the stationary solution for a unit increase in crop prices, when: the direct effect that soil bacteria have on crop production (α_b) is smaller, the rate at which synthetic compounds decompose (μ) is smaller, synthetic compounds are less detrimental to soil bacteria (γ_{cc} is less negative), and when crop prices P_j are lower.

A.2.2 Interest rate ρ

$$\frac{\partial \hat{K}_j}{\partial \rho} = \frac{\overbrace{\gamma_K}^{\geq 0}}{\underbrace{\mu}_{\geq 0} \underbrace{\gamma_{cc}(\mu+\rho)^2}_{\leq 0}} \leq 0 \tag{A.7}$$

The stock of clean soil at the stationary solution decreases as the interest rate increases: $\frac{\partial \hat{K}_j}{\partial \rho} \leq 0$. This is as expected, since as the payoff from the best alternative investment increases, we would expect the farmer to invest a greater amount in the best alternative investment, and therefore a lesser amount in the stock of clean soil.

Note that $\frac{\partial \hat{K}_j}{\partial \rho}$ becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when the benefit of the clean soil stock to the soil microbiome (γ_K) is greater. This is likely because when γ_K is greater the farmer requires less clean soil in order to have a desired positive effect of any given size on the soil bacteria. $\frac{\partial \hat{K}_j}{\partial \rho}$ also becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when the quadratic part of synthetic compound use's effect on soil bacteria (γ_{cc}) is less detrimental (such that γ_{cc} is smaller in magnitude). This is because as γ_{cc} grows smaller in magnitude, not investing in the stock of clean soils (by increasing per-period synthetic compound use) becomes less costly, allowing the farmer to forgo greater amounts of K as the outside option becomes more attractive. $\frac{\partial \hat{K}_j}{\partial \rho}$ becomes more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, as $\mu \geq 0$ becomes smaller. This is as expected, since as soils become less able to clean themselves up on their own it becomes more costly for the farmer to achieve a stock of clean soils of any given size, since the farmer no longer benefits from as much "free capital" as the capital stock's ability to grow on its own shrinks. Finally $\frac{\partial \hat{K}_j}{\partial \rho}$ is more negative, and therefore increasing ρ reduces $\hat{K}_j$ more, when ρ is smaller in magnitude. Thus the effect of increasing ρ is higher when ρ is smaller.

A.2.3 Maximum chemical stock capacity of soil $\overline{C}$

$$\frac{\partial \hat{K}_j}{\partial \overline{C}} = 1 \tag{A.8}$$

A one unit increase in the soil's ability to tolerate synthetic compounds results in a one unit increase in the stock of clean soil, or a one unit decrease in the stock of synthetic compounds, at the stationary solution. This result stems from the fact that the amount of clean soil is defined as the difference between the soil's ability to tolerate synthetic compounds and the actual stock of dirty soil. Therefore increasing the soils ability to tolerate synthetics, even while keeping actual stocks of synthetic compounds fixed, will by definition result in an 1 to 1 increase in the stock of clean soil.

A.2.4 Effect of per-period synthetic compound use on crop production α_c

$$\frac{\partial \hat{K}_j}{\partial \alpha_c} = \frac{1}{\underbrace{\mu}_{\geq 0}\underbrace{\alpha_b \gamma_{cc}}_{\leq 0}} \leq 0 \tag{A.9}$$

The stock of clean soil at the stationary solution decreases as the effect $\alpha_c$that per-period synthetic compound use has on crop production increases $\frac{\partial \hat{K}_j}{\partial \alpha_c} \leq 0$. This makes intuitive sense, since if, all else equal, fertilizers and pesticides yield greater productivity boosts the farmer will choose to use more of these compounds. This results in a smaller stock of clean soils. Note that $\frac{\partial \hat{K}_j}{\partial \alpha_c}$ becomes less negative as α_b increases in value. This makes sense, since if soil bacteria, which depend on the stock of clean soils, are more important to crop production, then we should be less willing to erode our stock of clean soils as synthetic compounds become more productive. Note also that $\frac{\partial \hat{K}_j}{\partial \alpha_c}$ becomes less negative as γ_{cc} becomes more negative. Again, this makes sense, since if per-period application of synthetic compounds becomes more detrimental to soil bacteria, which help crop production, then we should be less willing to apply synthetic compounds, and thus erode our stock of clean soils, as synthetic compounds become more productive.

A.2.5 Linear part γ_c of the effect that per-period synthetic compound use has on soil bacteria

$$\frac{\partial \hat{K}_j}{\partial \gamma_c} = \frac{1}{\underbrace{\mu}_{\geq 0}\underbrace{\gamma_{cc}}_{\leq 0}} \leq 0 \tag{A.10}$$

The stock of clean soil at the stationary solution decreases as per-period synthetic compound use becomes less detrimental to soil bacteria health: $\frac{\partial \hat{K}_j}{\partial \gamma_c} \leq 0$.

$\frac{\partial \hat{K}_j}{\partial \gamma_c} \leq 0$ makes sense intuitively, because if per-period application of synthetic compounds becomes less harmful to soil bacteria, which are themselves beneficial to crop production, then in any given period farmers will face less of an incentive not to use synthetic compounds, and will therefore choose to apply these compounds at greater rates. All else equal this should, in turn, result in greater stocks of synthetic compound at any given period, including the in the period at which the farmer reaches the stationary solution. If the farmer accumulates a greater stock of synthetic compounds at the stationary solution, by definition they accumulate smaller stocks of clean soils.

A.2.6 Soil bacteria's effect α_b on crop production

$$\frac{\partial \hat{K}_j}{\partial \alpha_b} = \frac{\frac{1}{P_{\mathrm{j}}} - \alpha_c}{\underbrace{\mu}_{\geq 0} \underbrace{\alpha_b^2 \gamma_{cc}}_{\leq 0}} \tag{A.11}$$

We see that soil bacteria's effect α_b on crop production has an ambiguous effect on the stock $\hat{K}_j$ of clean soil at the stationary solution. Its effect is mediated by crop prices P_{j} and the effect α_c of per-period synthetic compound application has on crop production.

We are more likely to have $\frac{\partial \hat{K}_j}{\partial \alpha_b} \geq 0$ such that the stock of clean soils at the stationary solution increases as soil bacteria become more important to production, when crop prices P_{j} are higher, since a greater crop price incentivizes the farmer to produce more, and since as α_b increases they are better able to produce more by increasing their stock of clean soils.

The stock of clean soils at the stationary solution also increases as soil bacteria become more important to production when the effect α_c of per-period synthetic compound use on production is higher, since then a farmer does not have to apply as much fertilizer or pesticide in order to produce at any given level of production.

Given a large enough α_c so that $\frac{1}{P_{\mathrm{j}}} - \alpha_c \leq 0$, then $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ is also more positive when the effect α_b of soil bacteria on crop production is smaller in magnitude. Intuitively, this means that when synthetic compounds are sufficiently important to production, then the stock of clean soil at the stationary solution increases, but at a diminishing rate, as soil bacteria's effect on production increases. This diminishing nature of $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ arises because as α_b increases the farmer does not have to increase the stock of clean soil by as much in order to achieve a given level of productivity gain from their soil bacteria. The farmer has an incentive not to increase $\hat{K}_j$ by too much, because this would mean losing out on the productive effects of their synthetic compounds, which are significant, since we have assumed α_c to be large.

Given a large enough α_c so that $\frac{1}{P_{\mathrm{j}}} - \alpha_c \leq 0$, $\frac{\partial \hat{K}_j}{\partial \alpha_b}$ is also more positive when the effect γ_{cc} of chemical inputs on soil microbe production is smaller in magnitude. Thus, when chemical inputs have less of a detrimental effect γ_{cc} on soil microbe production, then the larger the effect α_b that soil microbes have on crop production, the higher the stock of clean soils at the stationary solution. This is because the grower can still benefit from the productive effects of chemical inputs without harming soil microbe production as much, and also benefit from the productive effects of soil microbes.

A.2.7 Other comparative statics with ambiguous sign

$$\frac{\partial \hat{K}_j}{\partial \mu} = \left(\underbrace{\frac{1}{\rho+\mu} + \frac{1}{\mu}}_{\geq 0} \right) \left(\underbrace{\overline{C} - \hat{K}_j}_{\geq 0} \right) + \underbrace{\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{(\rho+\mu)\gamma_{cc}\mu}}_{\leq 0} \tag{A.12}$$

$$\frac{\partial \hat{K}_j}{\partial \gamma_{cc}} = \frac{1}{\mu \gamma_{cc}^2} \left(\frac{1}{\underbrace{\alpha_b}_{\geq 0}} \left(\frac{1}{P_{\mathrm{j}}} - \alpha_c \right) + \frac{\overbrace{\gamma_K}^{\geq 0}}{(\mu+\rho)} - \underbrace{\gamma_c}_{\leq 0} \right) \tag{A.13}$$

A.3 Stock of clean soils $\tilde{K}_j$ below which there is no trade-off involved with net investment

The stationary rate of return on capital $R_j(K)$ is undefined when $\frac{\partial G(K,0)}{\partial I} = 0$. The condition that $\frac{\partial G(K,0)}{\partial I} = 0$ simplifies to:

$$\left(P_j \cdot \left(\alpha_b \left(\gamma_c + \gamma_{cc}\mu \left(\overline{C} - K \right) \right) + \alpha_c \right) - 1 \right) = 0 \tag{A.14}$$

Let $\tilde{K}_j$ be defined as the stock of clean soils at which $\frac{\partial G(K,0)}{\partial I} = 0$. In other words, at the stock of clean soils $\tilde{K}_j$, the marginal effect of net investment I on the net gain function $G(K, I)$ when net investment is 0 is 0. $\tilde{K}_j$ is given by:

$$\tilde{K}_j = \frac{\frac{\alpha_c - P_j^{-1}}{\alpha_b} + \gamma_c}{\gamma_{cc}\mu} + \overline{C} \tag{A.15}$$

For $K < \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} > 0$ (i.e., net investment has a positive effect on contemporaneous net gain starting from a net investment of $I = 0$), and for $K \leq \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$). Since our analysis using the stationary rate of return on capital $R(K)$ assumes that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain), we cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R_j(K)$ and ρ to describe the optimal solution when $K \leq \tilde{K}_j$.

To see this, we find that for $K < \tilde{K}_j$, $R_j(K) \leq -\mu \leq 0$:

$$R_j(K) = \underbrace{-\mu}_{\leq 0} + \frac{\overbrace{\gamma_K}^{\geq 0}}{\underbrace{\gamma_{cc}\mu \left(\overline{C} - K \right) + \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}_{<0}} \leq -\mu \leq 0, \tag{A.16}$$

which would suggest that for $K < \tilde{K}_j$, since $K < \tilde{K}_j$, $R_j(K) \leq -\mu < \rho$, the farmer will always disinvest in clean soil until K is driven down to $K = 0$. But disinvesting would not make sense when $\frac{\partial G(K,I)}{\partial I} > 0$ (i.e., net investment has a positive effect on contemporaneous net gain), which is the case when $K < \tilde{K}_j$, since net investment increases the future stock K of clean soil and the stock K of clean soil has a positive effect on net gain (i.e., $\frac{\partial G(K,0)}{\partial K} \geq 0$):

$$\frac{\partial G(K,0)}{\partial K} = \underbrace{\mu \frac{\partial G(K,0)}{\partial I}}_{\geq 0} + \underbrace{P_j \cdot \alpha_b \gamma_K}_{\geq 0} \geq 0. \tag{A.17}$$

Thus, since our analysis using the stationary rate of return on capital $R_j(K)$ makes the assumption of the prototype economic control model that $\frac{\partial G(K,I)}{\partial I} < 0$ (i.e., net investment has a strictly negative effect on contemporaneous net gain), we cannot use the stationary rate of return on capital $R_j(K)$ and the comparison between the stationary rate of return on capital $R(K)$ and ρ to describe the optimal solution when $K \leq \tilde{K}_j$.

Instead, based on the feature that for $K \leq \tilde{K}_j$, net investment I has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$, $\frac{\partial G(K,0)}{\partial I} \geq 0$, and moreover that net investment increases the future stock K of clean soil and the stock K of clean soil has a positive effect on net gain (i.e., $\frac{\partial G(K,0)}{\partial K} \geq 0$), then we would expect that a farmer with $K \leq \tilde{K}_j$ would invest in the stock of clean soil, not disinvest, since there is no trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well.

Thus, for $K \leq \tilde{K}_j$, the farmer will invest in clean soil.

Although $\tilde{K}_j$ does not actually matter that much for describing investment behavior, it has important economic content and intuition, since $\tilde{K}_j$ (if it exists) is the threshold below which net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$. A farmer with $K \leq \tilde{K}_j$ does not face any trade-off involved with net investment: net investment not only increases future net gain, but also current net gain as well. In addition, another important thing about $\tilde{K}_j$ was to note that our standard interpretation of how the relative value of $R(K)$ determines investment behavior becomes invalid for $K < \tilde{K}_j$. Another important aspect of $\tilde{K}_j$ is it determines which parameter space we are in.

We confirm that $\hat{K}_j \geq \tilde{K}_j$:

$$\hat{K}_j = \tilde{K}_j - \frac{\gamma_K}{(\rho + \mu)\,\gamma_{cc}\mu} \tag{A.18}$$

$$\hat{K}_j = \tilde{K}_j - \frac{\overbrace{\gamma_K}^{\geq 0}}{\underbrace{\underbrace{(\rho + \mu)}_{\geq 0}\,\underbrace{\gamma_{cc}}_{\leq 0}\,\underbrace{\mu}_{\geq 0}}_{\leq 0}} \geq \tilde{K}_j \tag{A.19}$$

Thus, since for $K \leq \tilde{K}_j$, the farmer will invest in clean soil, this means that for $K_{0j} \leq \tilde{K}_j$, if the stationary solution $\hat{K}_j$ exists, the farmer will continue to invest in clean soil until he reaches the stationary solution $\hat{K}_j$.

$\tilde{K}_j$ is a decreasing function of prices P_j:

$$\frac{\partial \tilde{K}_j}{\partial P_j} = \frac{1}{\underbrace{\alpha_b}_{\geq 0} \underbrace{\gamma_{cc}}_{\leq 0} \underbrace{\mu}_{\geq 0} \underbrace{P_j^2}_{>0}} \leq 0 \tag{A.20}$$

The intuition is as follows. The threshold $\tilde{K}_j$ is such that for all $K \leq \tilde{K}_j$, $\frac{\partial G(K,0)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$). A higher level of net investment $I(t)$ in the stock of clean soil affects the farmer in the following ways. In the current period t, given the clean soil stock $K(t)$ for that period t, a higher level of net investment $I(t)$ means a lower level of chemical input use $c(t)$. The time-t benefits of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would forego) come through the beneficial effects of chemical input use $c(t)$ on crop output $f(\cdot)$. There are two time-t costs of chemical input use $c(t)$ (which a farmer who wishes to increase net investment $I(t)$ in the stock of clean soil would no longer incur). First, there is a unit price to chemical input use, which we normalize to 1. Second, chemical input use decreases beneficial soil microbes $b(t)$, and this decrease in beneficial soil microbes $b(t)$ may then have an adverse impact to crop output $f(\cdot)$. In addition to the time-t costs and benefits of a lower level of chemical input use $c(t)$, a higher level of net investment $I(t)$ also means a higher level of future clean soil stock K.

When prices P_j are lower, the threshold $\tilde{K}_j$ below which net investment has a non-negative effect on contemporaneous net gain starting from a net investment of $I = 0$ is higher because with lower prices P_j, a farmer who net invests would have less revenue to forego from the foregone beneficial effects of chemical input use $c(t)$ on crop revenue $f(\cdot)$, and thus the costs of net investing are lower and more likely to be outweighed by its benefits, which include foregoing the price of chemical input use as well as the adverse effect of chemical input use on soil microbes.

Neither $\tilde{K}_j$ nor $\hat{K}_j$ will exist if either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own).

If either $\gamma_{cc} = 0$ or $\mu = 0$, then the condition that $\frac{\partial G(K,I)}{\partial I} \geq 0$ (i.e., net investment has a non-negative effect on contemporaneous net gain) simplifies to:

$$P_j^{-1} \geq \alpha_b \gamma_c + \alpha_c \tag{A.21}$$

A.4 Effects of Price P_j on Optimal Solution for each stage $j \in \{con, org\}$

As shown above, $\hat{K}_j$ and $\tilde{K}_j$ are both decreasing functions of price P_j.

Also as shown above, if either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then the constant $R_j(K)$ is lower when prices are higher.

A.4.1 Very low P_j

If $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then since $\tilde{K}_j$ is a decreasing function of P_j , for very low P_j we will have $\tilde{K}_j > \overline{C}$, and therefore $K \leq \tilde{K}_j \leq \hat{K}_j$ for all feasible K, and the farmer will continue to invest such that K approaches $\hat{K}_j$ until K reaches its upper bound $\overline{C}$.

The condition for $\tilde{K}_j > \overline{C}$ simplifies to:

$$P_j^{-1} > -\alpha_b \left(\gamma_{cc}\mu - \gamma_c\right) + \alpha_c \tag{A.22}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough to satisfy the following condition for net investment to have a non-negative effect on contemporaneous net gain (so that $R_j(K)$ is not useful for analyzing net investment):

$$P_j^{-1} \geq \alpha_b\gamma_c + \alpha_c, \tag{A.23}$$

then the farmer will again invest in clean soil until $K = \overline{C}$.

A.4.2 Very high P_j

Alternatively, if $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then for very high P_j , we will have $\tilde{K}_j < 0$, and therefore $K \geq \tilde{K}_j$ for all feasible K.

In this case, the farmer's capital stock will approach the stationary solution and reaches it if $\hat{K}_j \in [0, \overline{C}]$. If $\hat{K}_j > \overline{C}$, then the farmer will approach $\hat{K}_j$ from below until they reach the blocked state $K = \overline{C}$, where they will stay indefinitely. If $\hat{K}_j < 0$ (and therefore does not exist since it is less than 0), the farmer will approach $\hat{K}_j$ from above until they reach the blocked state $K = 0$, where they will stay indefinitely.

The condition for $\tilde{K}_j < 0$ simplifies to:

$$P_j^{-1} < -\alpha_b \left(-\gamma_{cc}\mu\overline{C} - \gamma_c\right) + \alpha_c \tag{A.24}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are high enough to satisfy the condition that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c \tag{A.25}$$

as well as the following condition for $R_j(K) < \rho$:

$$P_j^{-1} < \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{A.26}$$

then the farmer will always disinvest until he reaches $K = 0$ since $R_j(K) < \rho$.

A.4.3 Intermediate P_j

Finally, if $\gamma_{cc} \neq 0$ and $\mu \neq 0$ so that both $\tilde{K}_j$ and $\hat{K}_j$ exist, then for certain intermediate values of P_j, it will be possible to have $\tilde{K}_j \in [0, \overline{C}]$. For $K_{0j} \leq \tilde{K}_j$, the farmer will continue to invest in clean soil and approach the stationary solution $\hat{K}_j$ until he reaches the stationary solution $\hat{K}_j \leq \overline{C}$. For $K_{0j} > \tilde{K}_j$, the farmer will approach and eventually reach the stationary solution $\hat{K}_j \leq \overline{C}$ by either investing, as in the case in which $K_{0j} < \hat{K}_j$; or by disinvesting, as in the case in which $K_{0j} > \hat{K}_j$.

The condition for $\tilde{K}_j \in [0, \overline{C}]$ is given by:

$$-\alpha_b \left(\gamma_{cc}\mu - \gamma_c \right) + \alpha_c \leq P_j^{-1} \leq -\alpha_b \left(-\gamma_{cc}\mu\overline{C} - \gamma_c \right) + \alpha_c \tag{A.27}$$

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices are low enough that $R_j(K) > \rho$:

$$P_j^{-1} > \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{A.28}$$

but also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c, \tag{A.29}$$

then the farmer will always invest until he reaches $K = \overline{C}$ since $R_j(K) > \rho$.

If either $\gamma_{cc} = 0$ (i.e., the negative effects of chemical input use $c(t)$ on beneficial soil microbes $b(t)$ are linear rather than convex) or $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) so that $R_j(K)$ is a constant (that does not depend on K), then if prices satisfy the condition that $R_j(K) = \rho$:

$$P_j^{-1} = \frac{\alpha_b}{\rho + \mu} \left((\rho + \mu)\gamma_c - \gamma_K \right) \tag{A.30}$$

but are also high enough that net investment has a negative effect on contemporaneous net gain (so that $R_j(K)$ is useful for analyzing net investment):

$$P_j^{-1} < \alpha_b \gamma_c + \alpha_c, \tag{A.31}$$

then the farmer will will stay at $K = K_{0j}$ since $R_j(K) = \rho$.

A.5 Unconstrained Solution for Stage j when $\hat{K}_j$ Exists: Deriving using Taylor series expansion

To solve for the farmer's optimal stage j trajectories, we start by solving for the unconstrained solution for each stage j by using second-order Taylor series approximations of the net gain function $G(K, I)$. Since the net gain function $G(K, I)$ is quadratic, these second-order Taylor series approximations and the solutions derived using them are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K, I)$ is an exact second-order Taylor series expansion of the net gain function $G(K, I)$.

When $\hat{K}_j$ exists, we can write a Taylor Series approximation of $G(K, I)$ around the stationary solution $\left(\hat{K}_j, 0\right)$, and use this approximation to solve for K^* and I^*.

The net gain function $G(K, I)$ is quadratic, so the second-order Taylor series approximation and the solutions derived using it are exact. In other words, the second-order Taylor series approximations of the net gain function $G(K, I)$ is an exact second-order Taylor series expansion of the net gain function $G(K, I)$.

We define the following values:

$$G_1 = \frac{\partial G(\hat{K}_j, 0)}{\partial K} = -\mu \left(P_j \cdot \left(\alpha_b \left(\gamma_{cc} \mu \left(\overline{C} - \hat{K}_j \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) + P_j \cdot \alpha_b \left(\gamma_K \right) \tag{A.32}$$

$$G_2 = \frac{\partial G(\hat{K}_j, 0)}{\partial I} = -\left(P_j \cdot \left(\alpha_b \left(\gamma_{cc} \mu \left(\overline{C} - \hat{K}_j \right) + \gamma_c \right) + \alpha_c \right) - 1 \right) \tag{A.33}$$

$$G_{11} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial K^2} = P_j \cdot \alpha_b \left(\gamma_{cc} \mu^2 \right) \tag{A.34}$$

$$G_{22} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial I^2} = P_j \alpha_b \gamma_{cc} \tag{A.35}$$

$$G_{12} = G_{21} = \frac{\partial^2 G(\hat{K}_j, 0)}{\partial K \partial I} = P_j \cdot \alpha_b \mu \gamma_{cc} \tag{A.36}$$

With the first and second order partials of G in now hand, the second order Taylor series approximation of the gain function $G(K, I)$ around a stationary solution $(\hat{K}_j, 0)$ can be written as:

$$G(K,I) \approx G(\hat{K}_j,0)+G_1\cdot(K-\hat{K}_j)+G_2\cdot(I-0)+G_{11}\cdot\frac{(K-\hat{K}_j)^2}{2}+G_{22}\cdot\frac{(I-0)^2}{2}+G_{12}\cdot(K-\hat{K}_j)\cdot(I-0) \tag{A.37}$$

With this Taylor series approximation of the gain function we can, in turn, derive an explicit closed-form solution to the second-order approximation of the optimal control problem for $K(t)$ and $I(t)$. To this end, we begin by noting that the Hamiltonian can now be approximated as:

$$H = G(\hat{K}_j,0)+G_1\cdot(K-\hat{K}_j)+G_2\cdot(I)+G_{11}\cdot\frac{(K-\hat{K}_j)^2}{2}+G_{22}\cdot\frac{(I)^2}{2}+G_{12}\cdot(K-\hat{K}_j)\cdot(I)+pI \tag{A.38}$$

Where p is the shadow price of capital. Letting

$$X = K - \hat{K}_j, \tag{A.39}$$

we can re-write the Hamiltonian as

$$H = G(\hat{K}_j,0) + G_1\cdot X + G_2\cdot I + G_{11}\cdot\frac{X^2}{2} + G_{22}\cdot\frac{I^2}{2} + G_{12}\cdot X\cdot I + pI. \tag{A.40}$$

The first criteria of the maximum principle,

$$\frac{\partial H}{\partial I} = 0 \tag{A.41}$$

implies

$$G_2 + G_{22}\cdot I + G_{12}\cdot X + p = 0, \tag{A.42}$$

so that

$$\widetilde{I}(X,p) = -\frac{(G_2 + G_{12}\cdot X + p)}{G_{22}}. \tag{A.43}$$

From this expression for the maximized investment value we get the maximized Hamiltonian:

$$\widetilde{H} = G(\hat{K}_j,0)+G_1\cdot X+G_2\cdot\widetilde{I}(X,p)+G_{11}\cdot\frac{X^2}{2}+G_{22}\cdot\frac{\widetilde{I}(X,p)^2}{2}+G_{12}\cdot X\cdot\widetilde{I}(X,p)+p\widetilde{I}(X,p) \tag{A.44}$$

In preparation for examining the second criteria of the maximum principle, we'll first derive expressions for $\frac{\partial\widetilde{H}}{\partial K}$, $p(t)$, $\dot{p}(t)$, and ρ. Beginning with $\frac{\partial\widetilde{H}}{\partial K}$, note that $\frac{\partial\widetilde{H}}{\partial K} = \frac{\partial\widetilde{H}}{\partial X}\cdot\frac{\partial X}{\partial K} = \frac{\partial\widetilde{H}}{\partial X}$, since $\frac{\partial X}{\partial K} = \frac{\partial(K-\hat{K}_j)}{\partial K} = 1$. Then we have

$$\frac{\partial\widetilde{H}}{\partial K} = G_1+G_2\cdot\frac{\partial\widetilde{I}(X,p)}{\partial X}+G_{11}\cdot X+G_{22}\cdot\widetilde{I}(X,p)\frac{\partial\widetilde{I}(X,p)}{\partial X}+G_{12}\cdot\widetilde{I}(X,p)+G_{12}\cdot X\cdot\frac{\partial\widetilde{I}(X,p)}{\partial X}+p\frac{\partial\widetilde{I}(X,p)}{\partial X} \tag{A.45}$$

$$\Rightarrow \frac{\partial \widetilde{H}}{\partial K} = G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p) + [\frac{G_2 + G_{12} \cdot X + p}{G_{22}} + \widetilde{I}(X,p)]\frac{\partial \widetilde{I}(X,p)}{\partial X} \cdot G_{22} \tag{A.46}$$

But from (A.43) we know that $\frac{G_2+G_{12} \cdot X+p}{G_{22}} + \widetilde{I}(X,p) = 0$. So we have

$$\frac{\partial \widetilde{H}}{\partial K} = G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p) \tag{A.47}$$

To get our expressions for $p(t)$ and $\dot{p}(t)$, we solve (A.43) for $p(t)$:

$$p(t) = -(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \tag{A.48}$$

and taking the time derivative of (A.48) we get

$$\dot{p}(t) = -G_{22} \cdot \dot{I}(t) - G_{12} \cdot \dot{X}(t) \tag{A.49}$$

And to get an expression for ρ remember that a stationary solution must satisfy

$$R(\hat{K}_j) = \rho \tag{A.50}$$

$$\Rightarrow \frac{-\frac{\partial G(\hat{K}_j,0)}{\partial K}}{\frac{\partial G(\hat{K}_j,0)}{\partial I}} = \rho \tag{A.51}$$

$$\Rightarrow \rho = -\frac{G_1}{G_2} \tag{A.52}$$

Now, we continue by examining the second criteria of the maximum principle. Remember this criteria requires that:

$$\dot{p}(t) = -\frac{\partial \widetilde{H}}{\partial K} + \rho p(t) \tag{A.53}$$

Plugging in (A.47), (A.48), and (A.49) gives:

$$\begin{aligned} -G_{22} \cdot \dot{I}(t) - G_{12} \cdot \dot{X}(t) = & \\ & -(G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) - \rho(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \end{aligned} \tag{A.54}$$

$$\begin{aligned} \Rightarrow G_{22} \cdot \dot{I}(t) + G_{12} \cdot \dot{X}(t) = & \\ & (G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) + \rho(G_2 + G_{22} \cdot I(t) + G_{12} \cdot X(t)) \end{aligned} \tag{A.55}$$

Note that $\dot{X} = \frac{d(K-\widehat{K})}{dt} = \frac{d(K)}{dt} = \dot{K} = I$. Evaluating at $I = \widetilde{I}$ gives

$$G_{22} \cdot \dot{\tilde{I}}(t) + G_{12} \cdot \tilde{I}(t) = \\ (G_1 + G_{11} \cdot X + G_{12} \cdot \widetilde{I}(X,p)) + \rho(G_2 + G_{22} \cdot \widetilde{I}(t) + G_{12} \cdot X(t)) \tag{A.56}$$

Plugging (A.52) into the RHS and simplifying then gives

$$G_{22} \cdot \dot{\tilde{I}}(t) = G_{11} \cdot X + \rho G_{22} \cdot \tilde{I}(t) + \rho G_{12} \cdot X(t) \tag{A.57}$$

$$\Rightarrow \dot{\tilde{I}}(t) - \rho \tilde{I}(t) - \frac{G_{11} \cdot X + \rho G_{12} \cdot X(t)}{G_{22}} = 0 \tag{A.58}$$

But remembering

$$\tilde{I}(t) = \frac{\partial K}{\partial t} = \frac{\partial (K - \widehat{K})}{\partial t} = \frac{\partial X}{\partial t} \tag{A.59}$$

and that therefore also

$$\dot{\tilde{I}}(t) = \frac{\partial \tilde{I}(t)}{\partial t} = \frac{\partial^2 X}{\partial t^2} \tag{A.60}$$

we can rewrite (A.58) as the following second-order linear ODE

$$\frac{\partial^2 X(t)}{\partial t^2} - \rho \frac{\partial X(t)}{\partial t} - \frac{G_{11} + \rho G_{12}}{G_{22}} X(t) = 0 \tag{A.61}$$

We guess a solution to (A.61) of the form:

$$X(t) = X(0)e^{-at} \tag{A.62}$$

then (A.61) becomes

$$X(0)e^{-at}a^2 + \rho X(0)e^{-at}a - \frac{G_{11} + \rho G_{12}}{G_{22}} X(0)e^{-at} = 0 \tag{A.63}$$

Note that if we assume a finite t and a, and that $K_{0j} \neq \widehat{K}$ so that $X(0) \neq 0$, then we can divide both sides of the above equation by $X(0)e^{-at}$ to get

$$a^2 + \rho a - \frac{G_{11} + \rho G_{12}}{G_{22}} = 0 \tag{A.64}$$

Applying the quadratic formula, and taking the positive root since we want $X(t) = K(t) - \hat{K}_j$ to converge to 0 as $K \to \hat{K}_j$. we get the following solution for a:

$$a = \sqrt{\left(\frac{\rho}{2}\right)^2 + \frac{G_{11} + \rho G_{12}}{G_{22}}} - \frac{\rho}{2} \tag{A.65}$$

$$\Rightarrow a = \sqrt{\left(\frac{\rho}{2}\right)^2 + \mu^2 + \rho\mu} - \frac{\rho}{2} \tag{A.66}$$

$$\Rightarrow a = \mu \tag{A.67}$$

The rate of approach to the stationary solution equals the rate at which synthetic compounds decompose from the farmer's soils. Since the speed of approach $a \geq 0$ in order for $X(t) = K(t) - \hat{K}_j$ to converge to 0 as $K \to \hat{K}_j$, this means that, in order for $K \to \hat{K}_j$, we must have $\mu \geq 0$. Moreover, $\frac{\partial a}{\partial \mu} = 1 > 0$, which means that the farmer will reach the stationary solution faster when the rate of decay is greater.

Given our solution for a we can now solve for the optimal policy for capital by rearranging equation (A.62) as follows:

$$X(t) = X(0)e^{-at} \tag{A.68}$$

$$\Rightarrow K_j(t) = \hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-\mu t} \tag{A.69}$$

We get our optimal policy for investment by taking the time derivative of the equation for $K_j(t)$ above and simplifying as follows:

$$I_j(t) = \frac{dK_j(t)}{dt} = -a(K_{0_{S_j}} - \hat{K}_j)e^{-at} \tag{A.70}$$

$$\Rightarrow I_j(t) = \mu(\hat{K}_j - K_j(t)) \tag{A.71}$$

Since our gain function is quadratic, our Taylor series approximation of the farmer's interior solution is the same as the exact interior solution. Therefore our unconstrained Taylor series approximation will be the same as our unconstrained exact solution.

In the next section we derive the exact solution and also apply the upper and lower bound constraints that the farmer faces on $I(t)$. These are the same constrained trajectories that we would get from applying the upper and lower bound constraints on investment to our Taylor series approximation of the farmer's interior solution, since, again, we assume our gain function to be quadratic, and therefore our Taylor series approximation is exact.

A.6 Solution for Stage j when $\hat{K}_j$ Exists: Directly Deriving Exact Solution and Imposing Lower and Upper Bound Constraints

We now derive the exact solution directly when $\hat{K}_j$ exists, show that the exact unconstrained solution is the same as what we derived using a second-order Taylor series approximation, and then impose the lower and upper bound constraints on $I(t)$ to obtain the constrained optimal solution for each stage j.

The upper-bound constraint $M(K)$ on net investment I comes from constraint that chemical input use $c(t)$ is non-negative.

The lower-bound constraint $m(K)$ on net investment I comes from upper bound $\bar{c}$ to chemical input use $c(t)$. The upper bound $\bar{c}$ to chemical input use $c(t)$ may depend on the total stock of synthetic compounds present in the soil $C(t)$, and which may represent, for example, the maximum recommended dose for any given application; the maximum chemical input dose that is not lethal to crops and/or humans; the maximum chemical dose above which consumers will no longer purchase the crop; and/or the maximum chemical input flow at any point in time that does not destroy the farmer's land and soil.

We assume that $\bar{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$ and $\bar{c} > 0$ when $\mu(X) = 0$. Since investment is bounded from below by $\mu(X)(\overline{C} - K(t)) - \bar{c}$, the trajectory for net investment must satisfy:

$$\begin{cases} I(t) \geq -\mu(X)K(t) & \text{if } \mu(X) > 0 \\ I(t) > \mu(X)(\overline{C} - K(t)) & \text{if } \mu(X) = 0 \end{cases} \tag{A.72}$$

Our derivation of the exact solution is as follows. The Hamiltonian is given by:

$$\begin{aligned} H =& P_j \cdot f\left((g(K, I)), \left(\mu(\overline{C} - K) - I\right)\right) - \left(\mu(X)(\overline{C} - K) - I\right) + pI & \text{(A.73)} \\ &+ \lambda_1 \left(I - \left(\mu(\overline{C} - K) - \bar{c}\right)\right) + \lambda_b \left(\mu(\overline{C} - K) - I\right) & \text{(A.74)} \\ & & \text{(A.75)} \end{aligned}$$

where

$$\begin{aligned} \lambda_1 \left(I - \left(\mu(\overline{C} - K) - \bar{c}\right)\right) = 0 \\ \lambda_b \left(\mu(\overline{C} - K) - I\right) = 0 \end{aligned}$$

A.6.1 Interior solution for $I(t)$ (OT2/OT3/OT4)

When $I(t)$ is interior we will have $\lambda_1 = 0$ and $\lambda_b = 0$.

With

$$f(b, c) = \alpha_b b + \alpha_c c + A_y \tag{A.76}$$

so that $\frac{\partial f}{\partial b} = \alpha_b$ and $\frac{\partial f}{\partial c} = \alpha_c$, and

$$g(K, I) = \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right)^2 + \gamma_c\left(\mu\left(\overline{C} - K\right) - I\right) + \gamma_K K + A_b \tag{A.77}$$

so that

$$\frac{\partial g(K,I)}{\partial K} = \left(-\mu\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\mu+\gamma_K\right) \tag{A.78}$$

and

$$\frac{\partial g(K,I)}{\partial I} = \left(-\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\right) \tag{A.79}$$

the first two conditions of the maximum principle can be re-expressed as follows:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}(-1)\right) + 1 + p = 0 \tag{A.80}$$

$$\Rightarrow \left(-\left(P_j \cdot \left(\alpha_b\left(-\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\right)-\alpha_c\right)+1\right)\right) = p \tag{A.81}$$

and

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu\right) + \rho p(t) \tag{A.82}$$

$$\Rightarrow \dot{p}(t) = -\left(P_j \cdot \left(\alpha_b\left(-\mu\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\mu+\gamma_K\right)-\mu\cdot\alpha_c\right)+\mu\right)+\rho p(t) \tag{A.83}$$

and the transversality condition requires that

$$\lim_{t\to\infty} p(t)K^*(t)e^{-\rho t} = 0. \tag{A.84}$$

We solve this system of equations as follows. Note that taking the time derivative of the first condition of the maximum principle we get

$$\dot{p} = -P_j \cdot \alpha_b\gamma_{cc}\left(\mu\dot{K} + \dot{I}\right) \tag{A.85}$$

We can substitute this new identity for $\dot{p}$, as well as our identity for p from the first condition of the maximum principle, into our expression for $\dot{p}$ from the second condition of the maximum principle, we get the following second order differential equation for $K(t)$:

$$(\mu+\rho)\,\mu\left(K(t)-\hat{K}_j\right) + \rho\cdot\dot{K}(t) - \ddot{K}(t) = 0 \tag{A.86}$$

This is a second order differential equation with solution:

$$K(t) = c_1e^{-\mu\cdot t} + c_2e^{(\mu+\rho)\cdot t} + \hat{K}_j \tag{A.87}$$

Our initial condition requires that:

$$K_{0j} = c_1e^{-\mu\cdot 0} + c_2e^{(\mu+\rho)\cdot 0} + \hat{K}_j \tag{A.88}$$

$$\Rightarrow c_1 + c_2 + \hat{K}_j = K_{0j} \tag{A.89}$$

$$\Rightarrow c_2 = K_{0j} - \hat{K}_j - c_1 \tag{A.90}$$

Note that we can take the time derivative of our preliminary solution for $K(t)$ to derive the following preliminary equation for $\dot{K}(t)$:

$$\dot{K}(t) = -\mu \cdot c_1 e^{-\mu \cdot t} + (\mu + \rho) \cdot c_2 e^{(\mu+\rho)\cdot t} \tag{A.91}$$

Plugging in Equation (A.90) into the above, we obtain:

$$K(t) = \left(c_1 e^{-\mu \cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t} + \hat{K}_j\right) \tag{A.92}$$

$$I(t) = \dot{K}(t) = \left(-\mu \cdot c_1 e^{-\mu \cdot t} + (\mu + \rho) \cdot \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t}\right) \tag{A.93}$$

Therefore

$$p(t) = \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right) - \alpha_c\right) + 1\right)\right) \tag{A.94}$$

can now be written as:

$$p(t) = \left(-P_j \alpha_b \gamma_{cc} (2\mu + \rho) \cdot \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t} - P_j \cdot \left(\alpha_b \left(-\gamma_{cc}\left(\mu\overline{C} - \mu\hat{K}_j\right) - \gamma_c\right) - \alpha_c\right) - 1\right) \tag{A.95}$$

We can use this expression, together with

$$K(t) = \left(c_1 e^{-\mu \cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{(\mu+\rho)\cdot t} + \hat{K}_j\right) \tag{A.96}$$

to determine the range of values for c_1 that satisfy the transversality condition:

$$\lim_{t\to\infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.97}$$

$$\lim_{t\to\infty} \left(c_1 a \left(K_{0j} - \hat{K}_j - c_1\right) + a\left(K_{0j} - \hat{K}_j - c_1\right)^2 e^{(2\mu+\rho)\cdot t} + a\hat{K}_j \left(K_{0j} - \hat{K}_j - c_1\right) e^{\mu\cdot t} + \left(K_{0j} - \hat{K}_j - c_1\right) e^{\mu \cdot t} \cdot b\right) \tag{A.98}$$

with $a = -P_j \alpha_b \gamma_{cc} (2\mu + \rho)$. Assuming $a \neq 0$, then the transversality condition is satisfied if and only if

$$c_1 = \left(K_{0j} - \hat{K}_j\right) \tag{A.99}$$

So the farmer's optimal trajectory is:

$$K_j(t) = \hat{K}_j + (K_{0_{S_j}} - \hat{K}_j)e^{-\mu t} \tag{A.100}$$

$$I_j(t) = \mu(\hat{K}_j - K_j(t)) \tag{A.101}$$

To derive trajectories for $c(t)$ and $C(t)$ under an interior solution for $I(t)$, we can write:

$$c(K, I) = \mu\left(\overline{C} - K(t)\right) - I(t) \tag{A.102}$$

$$\Rightarrow c(K, I) = \mu\left(\overline{C} - \hat{K}_j\right) \forall t \geq 0, \tag{A.103}$$

and

$$C(K, I) = \overline{C} - K(t) \tag{A.104}$$

$$\Rightarrow C(K, I) = \overline{C} - \hat{K}_j - \left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}. \tag{A.105}$$

We also have the following expressions for soil microbes $b(t)$ and crop output $y(t)$ under an interior solution for $I(t)$:

$$g(K, I; X) = \left(\gamma_c \mu\left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b + \gamma_K\left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t}\right), \tag{A.106}$$

$$\begin{aligned} \tilde{f}(b, c; X) = \alpha_b &\left(\gamma_c \mu\left(\overline{C} - \hat{K}_j\right) + \frac{1}{2}\gamma_{cc}\left(\mu\left(\overline{C} - \hat{K}_j\right)\right)^2 + \gamma_K \hat{K}_j + A_b\right) \\ &+ \alpha_c \mu\left(\overline{C} - \hat{K}_j\right) + A_y + \alpha_b \gamma_K\left(K_{0j} - \hat{K}_j\right) \cdot e^{-\mu \cdot t} \end{aligned} \tag{A.107}$$

A.6.2 Lower corner solution for $I(t)$ (OT1)

On the other hand if $I(t)$ has a lower corner solution (such that the lower bound constraint on $I(t)$ binds, but the upper bound constraint does not) we will have $\lambda_1 \geq 0$ and $\lambda_b = 0$.

In this case, the Hamiltonian is given by:

$$H = P_j \cdot f\left(\left(g(K,I)\right), \left(\mu(\overline{C}-K)-I\right)\right) - \left(\mu(X)(\overline{C}-K)-I\right) + pI \tag{A.108}$$

$$+ \lambda_1 \left(I - \left(\mu(\overline{C}-K)-\overline{c}\right)\right) \tag{A.109}$$

$$\tag{A.110}$$

Condition [#1] of the Maximum Principle will then yield:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}(-1)\right) + 1 + p + \lambda_1 = 0 \tag{A.111}$$

$$\Rightarrow -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} - \frac{\partial f}{\partial c}\right) + 1\right) - \lambda_1 = p \tag{A.112}$$

$$\Rightarrow \left(-\left(P_j \cdot \left(\alpha_b\left(-\gamma_{cc}\left(\mu\left(\overline{C}-K\right)-I\right)-\gamma_c\right)-\alpha_c\right)+1\right)-\lambda_1\right) = p \tag{A.113}$$

and

$$\frac{\partial H}{\partial \lambda_1} = \left(I + (\mu - \kappa) K\right) = 0 \tag{A.114}$$

$$\frac{\partial H}{\partial \lambda_1} = 0 \Rightarrow -\left(\mu(\overline{C}-K)-\overline{c}\right) = 0. \tag{A.115}$$

Given:

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}(-\mu)\right) + \mu + \lambda_1 \mu\right) \tag{A.116}$$

[#2] yields:

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu + \lambda_1 \mu\right) + \rho p(t) \tag{A.117}$$

The transversality condition [#3] requires that

$$\lim_{t\to\infty} p(t)K^*(t)e^{-\rho t} = 0. \tag{A.118}$$

We can use (A.115) to find the farmer's optimal trajectories when the lower bound constraint on $I(t)$ binds as follows:

$$I(t) = \left(\mu(\overline{C}-K)-\overline{c}\right) \tag{A.119}$$

Remember that the upper bound constraint on per-period synthetic compound use, $\overline{c}$, is assumed to satisfy the following conditions:

- $\overline{c} = \mu(X)\overline{C}$ when $\mu(X) > 0$
- and $\overline{c} > 0$ when $\mu(X) = 0$

When $\mu(X) = 0$, then $\hat{K}_j$ does not exist, so the case when $\mu(X) = 0$ does not apply when $\hat{K}_j$ exists.

When $\mu \neq 0$ the farmer's optimal constrained trajectory is:

$$K(t) = K(0) \cdot e^{-\mu \cdot t} \, \forall \, t \tag{A.120}$$

$$I(t) = -\mu K(t) \, \forall \, t \tag{A.121}$$

$$c(t) = \overline{c} = \mu \overline{C} \, \forall \, t \tag{A.122}$$

$$C(t) = \overline{C} - K(0) \cdot e^{-\mu \cdot t} \, \forall \, t \tag{A.123}$$

Given $\tilde{g}(C(t), c(t)) = \gamma_c c + \frac{1}{2}\gamma_{cc} c^2 + \gamma_K \left(\overline{C} - C(t)\right) + A_b$:

$$b(t) = \max\{\gamma_c \overline{c} + \frac{1}{2}\gamma_{cc}\overline{c}^2 + \gamma_K \cdot \left(K(0) \cdot e^{-\mu \cdot t}\right) + A_b, \, 0\} \, \forall \, t \tag{A.124}$$

Given $f(c(t), b(t)) = \alpha_c c(t) + \alpha_b b(t) + A_y$:

$$y(t) = \alpha_c \cdot \overline{c} + \alpha_b \cdot b(t)_{LC} + A_y \, \forall \, t \tag{A.125}$$

The lower bound to I binds when the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$). If the optimal unconstrained synthetic compound level c_j^{**} exceeds the upper bound for synthetic compound use (i.e., if $c_j^{**} > \overline{c}$), this means that the PDV of the entire stream of MNB of an additional unit of synthetic compound $c(t)$ today is still greater than 0 at $c = \overline{c}$.

When $\hat{K}_j$ exists, the optimal unconstrained synthetic compound level c_j^{**} is given by $c^{**} = \hat{c}_j$. If $c^{**} = \hat{c}_j > \overline{c}$, this means $\hat{K}_j < 0$. A negative $\hat{K}_j$ means that even at $K = 0$, the marginal net benefit of synthetic compound use at $c = \overline{c}$ is positive.

So a sufficient condition for the farmer to adopt a lower corner solution when $\mu \neq 0$ and $\hat{K}_j$ exists, assuming that $K_{0_j} \neq \hat{K}_j$, is for the marginal net benefit of synthetic compound use to be positive, even when we are accounting for convex costs, and even when those convex costs are evaluated at $c(t) = \overline{c}$.

When the lower bound constraint on investment binds, the multiplier λ_1 on the lower bound constraint is given by:

$$\lambda_1(t) = \underbrace{P_j \cdot \left(\alpha_c + \alpha_b \left(\gamma_{cc} \cdot c(t) + \gamma_c - \frac{1}{(\mu+\rho)}\gamma_K\right)\right) - 1}_{\text{MNB of c(t)}} \tag{A.126}$$

If $\hat{K}_j$ is positive and greater than $K(t)$, then the farmer is *never at the lower corner solution for* $I(t)$. This makes sense since if $\hat{K}_j > K(t)$, then means we will be investing in the stock of clean soil to increase $K(t)$ and approach $\hat{K}_j$ from below.

The lower bound binds when $\hat{K}_j < 0$. A negative $\hat{K}_j$ means that even at $K = 0$, the marginal net benefit of synthetic compound use at $c = \bar{c}$ is positive. In this case the farmer is *always at the lower corner solution for* $I(t)$, and the farmer's capital trajectory converges to $K(t) = 0$ from above.

So a sufficient condition for the farmer to adopt a lower corner solution when $\mu \neq 0$, assuming that $K_{0_j} \neq \hat{K}_j$, is for the marginal net benefit of synthetic compound use to be positive, even when we are accounting for convex costs, and even when those convex costs are evaluated at $c(t) = \bar{c}$.

A.6.3 Upper corner solution for $I(t)$ (OT5)

On the other hand if $I(t)$ has an upper corner solution (such that the upper bound constraint on $I(t)$ binds, but the lower bound constraint does not) we will have $\lambda_1 = 0$ and $\lambda_b \geq 0$. Then the Maximum Principle will yield:

[#1]:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} + \frac{\partial f}{\partial c}(-1)\right) + 1 + p - \lambda_b = 0 \tag{A.127}$$

$$\Rightarrow -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial I} - \frac{\partial f}{\partial c}\right) + 1\right) + \lambda_b = p \tag{A.128}$$

$$\Rightarrow \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc}\left(\mu\left(\overline{C} - K\right) - I\right) - \gamma_c\right) - \alpha_c\right) + 1\right) + \lambda_b\right) = p \tag{A.129}$$

and

$$\frac{\partial H}{\partial \lambda_b} = \left(\mu(\overline{C} - K) - I\right) = 0 \tag{A.130}$$

$$\mu\left(\overline{C} - K(t)\right) - I(t) = 0 \tag{A.131}$$

[#2]:

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}(-\mu)\right) + \mu - \lambda_b \mu\right) \tag{A.132}$$

so that

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu - \lambda_b \mu\right) + \rho p(t) \tag{A.133}$$

and [#3]: the transversality condition requires that

$$\lim_{t\to\infty} p(t)K^*(t)e^{-\rho t} = 0 \tag{A.134}$$

We can use (A.131) to find the farmer's optimal trajectories when the upper bound constraint on $I(t)$ binds as follows:

$$\mu\overline{C} - \mu K(t) - I(t) = 0 \tag{A.135}$$

$$\Rightarrow \mu K(t) + \dot{K}(t) = \mu\overline{C} \tag{A.136}$$

$$\Rightarrow \left(\mu K(t) + \dot{K}(t)\right) \cdot e^{\mu\cdot t} = \mu\overline{C} \cdot e^{\mu\cdot t} \tag{A.137}$$

$$\Rightarrow \int_{s=0}^{t} \left(\mu K(s) + \dot{K}(s)\right) \cdot e^{\mu\cdot s} ds = \int_{s=0}^{t} \mu\overline{C} \cdot e^{\mu\cdot s} ds \tag{A.138}$$

$$\Rightarrow K(s) \cdot e^{\mu\cdot s}]_0^t = \overline{C} \cdot e^{\mu\cdot s}\Big]_0^s \tag{A.139}$$

$$\Rightarrow K(t) \cdot e^{\mu\cdot t} - K_{0j} = \overline{C} \cdot e^{\mu\cdot t} - \overline{C} \tag{A.140}$$

$$\Rightarrow K(t) = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu\cdot t} \tag{A.141}$$

$$I(t) = \mu \cdot \left(\overline{C} - K_{0j}\right) e^{-\mu\cdot t} \tag{A.142}$$

$$\Rightarrow I(t) = \mu \cdot \left(\overline{C} - K(t)\right) \tag{A.143}$$

To determine when upper bound for $I(t)$ binds or not, ewe compare the interior solution for $I(t)$:

$$I(K(t))_{Int} = \mu(\hat{K}_j - K(t)_{Int}) \tag{A.144}$$

to the upper corner solution for $I(t)$:

$$I(K(t))_{UC} = \mu \cdot \left(\overline{C} - K(t)_{UC}\right) \tag{A.145}$$

in order to determine the conditions under which the interior solution falls above the upper corner solution, at which point the upper bound constraint on $I(t)$ will bind: we examine the following inequality

$$I(K(t))_{UC} < I(K(t))_{Int} \tag{A.146}$$

$$\mu \cdot \left(\overline{C} - K(t)_{UC}\right) < \mu(\hat{K}_j - K(t)_{Int}) \tag{A.147}$$

$$\Rightarrow \overline{C} < \hat{K}_j \tag{A.148}$$

Note that if $\overline{C} < \hat{K}_j$, then we have $I(t)_{UC} < I(t)_{Int}$ for all t . In this case the farmer is stuck at the upper corner solution indefinitely. On the other hand if $\overline{C} \geq \hat{K}_j$, then we have $I(t)_{UC} \geq I(t)_{Int}$ for all t . In this case the farmer's investment trajectory will never be constrained by its upper bound, and we will have $\lambda_b = 0 \, \forall t$.

If $\overline{C} < \hat{K}_j$, then we are stuck at the upper corner solution indefinitely such that we will have $\lambda_b(t) \geq 0$ indefinitely, so we are able to use the transversality condition to get more information about $\lambda_b(t)$ and derive the closed form solution above.

When $\hat{K}_j > \overline{C}$, the optimal solution is to continue to invest as fast as possible until $K = \overline{C}$. In this case, the upper bound on $I(t)$ always binds and the farmer's optimal solutions take the form:

$$K_j^*(t) = K(t)_{UC,S_j} = \overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.149}$$

$$I_j^*(t) = I(t)_{UC,S_j} = \mu \cdot \left(\overline{C} - K(t)_{UC,S_j}\right) \tag{A.150}$$

$$c_j^*(t) = 0 \tag{A.151}$$

$$C_j^*(t) = \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t} \tag{A.152}$$

$$b_j^*(t) = \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) \tag{A.153}$$

$$y_j^*(t) = \alpha_b \left(\gamma_K \left(\overline{C} - \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) + A_b\right) + A_y \tag{A.154}$$

To determine λ_b:

For the intervals of time over which we have an upper corner solution for $I(t)$, we have a continuous $\lambda_b(t)$. In that we can solve for $\lambda_b(t)$ as follows. From (A.129) we have:

$$p(t) = \left(-\left(P_j \cdot \left(\alpha_b \left(-\gamma_{cc} \left(\mu \left(\overline{C} - K(t)\right) - I(t)\right) - \gamma_c\right) - \alpha_c\right) + 1\right) + \lambda_b(t)\right) \tag{A.155}$$

or, given (A.141) and (A.143):

$$p(t) = \left(P_j \cdot (\alpha_b \gamma_c + \alpha_c) - 1 + \lambda_b(t)\right) \tag{A.156}$$

Taking the time derivative of both sides of the equation above we get:

$$\dot{p}(t) = \dot{\lambda}_2(t) \tag{A.157}$$

But from the second condition of the maximum principle, [#2], we have:

$$\dot{p}(t) = -\frac{\partial H}{\partial K} + \rho p(t) \tag{A.158}$$

where

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} + \frac{\partial f}{\partial c}(-\mu)\right) + \mu - \lambda_b \mu\right), \tag{A.159}$$

so that

$$\dot{p}(t) = -\left(P_j \cdot \left(\frac{\partial f}{\partial b}\frac{\partial g}{\partial K} - \mu \cdot \frac{\partial f}{\partial c}\right) + \mu - \lambda_b \mu\right) + \rho p(t). \tag{A.160}$$

Given our assumed functional forms for f and g, the equation above can be written as:

$$\dot{p}(t) = -\left(P_j \cdot \left(\alpha_b \left(-\mu \gamma_{cc} \left(\mu \left(\overline{C} - K\right) - I\right) - \gamma_c \mu + \gamma_K\right) - \mu \cdot \alpha_c\right) + \mu - \lambda_b \mu\right) + \rho p(t) \tag{A.161}$$

We can substitute (A.141), (A.143), (A.156) and (A.157) into the above to get:

$$\begin{aligned} \dot{\lambda}_2(t) = & -\left(P_j \cdot \left(\alpha_b \left(-\mu \gamma_{cc} \left(\mu \left(\overline{C} - \left(\overline{C} + \left(K_{0j} - \overline{C}\right) e^{-\mu \cdot t}\right)\right)\right.\right.\right.\right. \\ & \left.\left.\left.\left. -\mu \cdot \left(\overline{C} - K_{0j}\right) e^{-\mu \cdot t}\right) - \gamma_c \mu + \gamma_K\right) - \mu \cdot \alpha_c\right) + \mu - \lambda_b \mu\right) \\ & + \rho \left(P_j \cdot (\alpha_b \gamma_c + \alpha_c) - 1 + \lambda_b(t)\right) \end{aligned} \tag{A.162}$$

We simplify the equation above and solve the resulting second order ODE for $\lambda_b(t)$:

$$\lambda_b(t) = \left(P_j \alpha_b \gamma_{cc} \mu \left(\hat{K}_j - \overline{C}\right)\left(\cdot e^{(\mu+\rho)\cdot t} - 1\right) + \lambda_b(0) \cdot e^{(\mu+\rho)\cdot t}\right) \tag{A.163}$$

Note that we can make use of the transversality condition to find $\lambda_b(0)$ because as we previously showed, whenever the upper bound on investment binds, it will bind for all $t \geq 0$.

So, substituting $p(t) = (P_j \cdot (\alpha_b\gamma_c + \alpha_c) - 1 + \lambda_b(t))$, (A.163), and $K(t)_{UC} = \left(\overline{C} + \left(K_{0j} - \overline{C}\right) \cdot e^{-\mu \cdot t}\right)$ into our transversality condition, we get

$$\lim_{t\to\infty} \left(\overline{C} P_j \alpha_b \left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \right) e^{-\rho t} + \overline{C} \left(P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) + \lambda_b(0) \right) \cdot e^{\mu \cdot t} \right.$$
$$+ P_j \alpha_b \left(K_{0j} - \overline{C} \right) \left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \right) e^{-(\rho+\mu)t}$$
$$\left. + \left(K_{0j} - \overline{C} \right) \left(P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) + \lambda_b(0) \right) \right) = 0$$

which is equivalent to:

$$\lim_{t\to\infty} \left(\overline{C} \left(P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) + \lambda_b(0) \right) \cdot e^{\mu \cdot t} \right.$$
$$\left. + \left(K_{0j} - \overline{C} \right) \left(P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) + \lambda_b(0) \right) \right) = 0$$

which is satisfied if

$$\lambda_b(0) = -P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \tag{A.164}$$

So we have

$$\lambda_b(t) = \left(P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \left(\cdot e^{(\mu+\rho)\cdot t} - 1 \right) - P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \cdot e^{(\mu+\rho)\cdot t} \right) \tag{A.165}$$

$$\Rightarrow \lambda_b(t) = -P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \ \forall t \geq 0 \tag{A.166}$$

$$p(t) = \left(P_j \cdot (\alpha_b\gamma_c + \alpha_c) - 1 - P_j \alpha_b \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \right) \tag{A.167}$$

$$\Rightarrow p(t) = \alpha_b \left(\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \gamma_{cc}\mu \left(\hat{K}_j - \overline{C} \right) \right) \tag{A.168}$$

A.7 Optimal Solution for Stage j When $R(K)$ is Constant Because $\mu = 0$

If $\mu = 0$ (i.e., synthetic compounds in the soil do not decay on their own) then $R_j(K)$ is a constant (that does not depend on K).

A.7.1 Optimal Trajectories 1" [OT1"]: Disinvest to $K = 0$

When $\gamma_{cc} \neq 0$ but $\mu = 0$ the gain function is non-linear in I, and therefore the optimal policy will not be MRA. If the lower corner solution for I does not bind (because $c_j^{**} \leq \bar{c}$), we will have an interior soluion.

If neither constraint on $I(t)$ binds and $I(t)^*$ is interior, the conditions of the Maximum Principle yield:

[#1]:

$$\frac{\partial H}{\partial I} = P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial I} + \frac{\partial f}{\partial c} (-1) \right) + 1 + p = 0 \tag{A.169}$$

$$\Rightarrow p(t) = -P_j \cdot (\alpha_b (-\gamma_c) - \alpha_c) - P_j \alpha_b \gamma_{cc} \cdot I(t) - 1 \tag{A.170}$$

$$\Rightarrow I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_{cc}} - \frac{1}{P_j \alpha_b \gamma_{cc}} p(t) \tag{A.171}$$

[#2]:

$$-\frac{\partial H}{\partial K} = -\left(P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} + \frac{\partial f}{\partial c} (-\mu) \right) + \mu \right) \tag{A.172}$$

or, given $\mu = 0$,

$$-\frac{\partial H}{\partial K} = -P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} \right) \tag{A.173}$$

so that

$$\dot{p}(t) = -P_j \cdot \left(\frac{\partial f}{\partial b} \frac{\partial g}{\partial K} \right) + \rho p(t) \tag{A.174}$$

$$\Rightarrow \dot{p}(t) = -\left(P_j \cdot (\alpha_b \gamma_K) \right) + \rho p(t) \tag{A.175}$$

and [#3]: the transversality condition requires that

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.176}$$

Using (A.175), we can solve for $p(t)$ as follows:

$$\dot{p}(t) - \rho p(t) = -P_j \alpha_b \gamma_K \tag{A.177}$$

$$\Rightarrow p(t) = \frac{P_j \alpha_b \gamma_K}{\rho} + \left(p(0) - \frac{P_j \alpha_b \gamma_K}{\rho} \right) \cdot e^{\rho \cdot t} \tag{A.178}$$

Substituting (A.178) into (A.171) yields:

$$I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} - \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \cdot e^{\rho \cdot t} \tag{A.179}$$

And integrating the above yields:

$$\begin{aligned} K(t) = \Bigg(& \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} t - \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \cdot e^{\rho \cdot t} \\ & + \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) + K_{0j} \Bigg) \end{aligned} \tag{A.180}$$

where K_{0j} is given.

We then substitute (A.171) and (A.180) into our transversality condition to see if we can learn more about $p(0)$.

$$\lim_{t \to \infty} p(t) K^*(t) e^{-\rho t} = 0 \tag{A.181}$$

$$\begin{aligned} \Rightarrow \lim_{t \to \infty} \Bigg(& \frac{P_j \alpha_b \gamma_K}{\rho} \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} t + \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) + K_{0j} \right) e^{-\rho t} \\ & - \frac{P_j \alpha_b \gamma_K}{\rho} \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \\ & + \left(p(0) - \frac{P_j \alpha_b \gamma_K}{\rho} \right) \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} t + \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) + K_{0j} \right) e^{-2\rho t} \\ & - \frac{1}{\rho P_j \alpha_b \gamma_{cc}} \left(p(0) - \frac{P_j \alpha_b \gamma_K}{\rho} \right)^2 \cdot e^{-\rho \cdot t} \Bigg) = 0 \end{aligned} \tag{A.182}$$

Applying l'Hospital's rule we see that this limit is equivalent to:

$$\lim_{t \to \infty} \left(-\frac{P_j \alpha_b \gamma_K}{\rho} \frac{1}{\rho} \left(\frac{p(0)}{P_j \alpha_b \gamma_{cc}} - \frac{\gamma_K}{\rho \gamma_{cc}} \right) \right) = 0 \tag{A.183}$$

So we see that the transversality condition will be satisfied if:

$$p(0) = \frac{P_j \alpha_b \gamma_K}{\rho} \tag{A.184}$$

Let's make this assumption. Then we have that

$$p(t) = \frac{P_j \alpha_b \gamma_K}{\rho} \tag{A.185}$$

$$I(t) = \frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{1}{\rho}\gamma_K}{\gamma_{cc}} \tag{A.186}$$

$$\Rightarrow I(t) = \frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} \tag{A.187}$$

$$\Rightarrow I(t) = \frac{\gamma_K}{\gamma_{cc}} \left(R(K)^{-1} - \rho^{-1} \right) \tag{A.188}$$

$$\Rightarrow I(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \tag{A.189}$$

and

$$K(t) = \left(\frac{\gamma_K \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho} \right)}{\gamma_{cc}} t + K_{0j} \right) \tag{A.190}$$

$$\Rightarrow K(t) = \frac{\gamma_K}{\gamma_{cc}} \left(R(K)^{-1} - \rho^{-1} \right) \cdot t + K_{0j} \tag{A.191}$$

$$\Rightarrow K(t) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \cdot t + K_{0j} \tag{A.192}$$

However, note that if $R(K)$ is constant (as when $\mu = 0$), and $R(K) \leq \rho$, as we've assumed, then the equation for $K(t)$ that we have derived above implies K is weakly decreasing in t, and *strictly* decreasing when $R(K) < \rho$ and $\gamma_K \neq 0$. When this is the case, $K(t)^*$ will eventually fall below zero, and the farmer will have to switch to a constrained optimal solution so as to prevent K from violating our non-negativity condition. We solve for the first moment at which $K(t)^* = 0$, which we will denote $T(\mu)_{K=0}$, below:

$$K(T(\mu)_{K=0}) = \frac{\gamma_K}{(-\gamma_{cc})} \left(\rho^{-1} - R(K)^{-1} \right) \cdot T(\mu)_{K=0} + K_{0j} = 0 \tag{A.193}$$

$$T(\mu)_{K=0} = \frac{-K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right)} \tag{A.194}$$

$$\Rightarrow T(\mu)_{K=0} = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(R(K)^{-1} - \rho^{-1}\right)} \geq 0 \tag{A.195}$$

$$\Rightarrow T(\mu)_{K=0} = \frac{K_{0j}}{\frac{\gamma_K}{(-\gamma_{cc})} \cdot \left(\frac{\gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b}}{\gamma_K} - \frac{1}{\rho}\right)} \geq 0 \tag{A.196}$$

So $\forall t \leq T(\mu)_{K=0}$ the farmer adopts the unconstrained optimal solution:

$$K(t)^* = \left(\frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t + K_{0j}\right) \forall t \leq T(\mu)_{K=0} \tag{A.197}$$

and

$$I(t)^* = \frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right) \forall t \leq T(\mu)_{K=0}. \tag{A.198}$$

Otherwise, when $t > T(\mu)_{K=0}$ the farmer adopts the constrained optimal solution:

$$K(t) = 0, \forall t > T(\mu)_{K=0} \tag{A.199}$$

and

$$I(t) = 0, \forall t > T(\mu)_{K=0}. \tag{A.200}$$

We therefore have the overall solution:

$$K(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right) \cdot t + K_{0j}, & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.201}$$

and

$$I(t) = \begin{cases} \frac{\gamma_K}{(-\gamma_{cc})}\left(\rho^{-1} - R(K)^{-1}\right), & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.202}$$

Given this solution, we can solve for $c(t)$ as follows:

$$c(t) = \underbrace{\mu}_{=0}\left(\overline{C} - K(t)\right) - I(t) \tag{A.203}$$

$$\Rightarrow c(t) = -I(t) \tag{A.204}$$

$$\Rightarrow c(t) = \begin{cases} \frac{\gamma_K}{\gamma_{cc}} \cdot \left(\rho^{-1} - R(K)^{-1}\right), & \forall t \leq T(\mu)_{K=0} \\ 0, & \forall > T(\mu)_{K=0} \end{cases} \tag{A.205}$$

We solve for $C(t)$ as follows:

$$C(t) = \overline{C} - K(t) \tag{A.206}$$

$$\Rightarrow C(t) = \begin{cases} C_{0j} + \frac{\gamma_K}{\gamma_{cc}} \left(\rho^{-1} - R(K)^{-1}\right) \cdot t & \forall t \leq T(\mu)_{K=0} \\ \overline{C} & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.207}$$

We solve for $b(t)$ as follows:

$$b(t) = \gamma_c c(t) + \frac{1}{2}\gamma_{cc} c(t)^2 + \gamma_K K(t) + A_b \tag{A.208}$$

$$b(t) = \begin{cases} \gamma_K \cdot \left(\left(\left(\frac{\gamma_c - \gamma_K \cdot t}{\gamma_{cc}} \right) - \frac{1}{2} \cdot (\gamma_{cc})^{-1} \cdot \gamma_K \cdot \left(\rho^{-1} - R(K)^{-1}\right) \right) \cdot \left(\rho^{-1} - R(K)^{-1}\right) \right. \\ \left. + K_{0j} \right) + A_b, & \forall t \leq T(\mu)_{K=0} \\ \\ A_b, & \forall t > T(\mu)_{K=0} \\ \end{cases} \tag{A.209}$$

We solve for $y(t)$ as follows:

$$y(t) = \alpha_c c(t) + \alpha_b b(t) + A_y \tag{A.210}$$

$$y(t) = \begin{cases} \alpha_b \gamma_K \left(\frac{1}{\gamma_{cc}} \cdot \left(\frac{\alpha_c}{\alpha_b} + \gamma_c - \gamma_K \cdot \left(t + \frac{1}{2} \cdot \left(\rho^{-1} - R(K)^{-1}\right)\right)\right) \right. \\ \left. \cdot \left(\rho^{-1} - R(K)^{-1}\right) + K_{0j} + \frac{A_b + \frac{A_y}{\alpha_b}}{\gamma_K} \right), & \forall t \leq T(\mu)_{K=0} \\ \\ \alpha_b A_b + A_y, & \forall t > T(\mu)_{K=0} \end{cases} \tag{A.211}$$

B Discrete Transitions

B.1 Discrete Analysis for OT1

The sign of $\frac{\partial \Delta(\epsilon)}{\partial \epsilon}$ is given by:

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} \geq \underbrace{P_{con} \cdot \alpha_b \cdot \frac{1}{(\mu+\rho)} \cdot \gamma_K}_{\geq 0} \tag{B.1}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.

Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$. Note that $\Delta(\epsilon^*) = 0$. The range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\begin{aligned}\epsilon^* = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho}}_{\leq 0} \Bigg(& \underbrace{(P_{org} - P_{con})\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \\ & - P_{con}\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\mu - \underbrace{\left(\frac{P_{org}}{P_{con}} - \frac{\rho}{\mu+\rho}\right)\gamma_K}_{\geq 0}\right)\overline{C}\Bigg)\end{aligned} \tag{B.2}$$

This means that when $\epsilon^* \leq 0$ the farmer will face $V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon) > 0 \, \forall \epsilon \geq 0$, and will therefore prefer to produce organically for all feasible initial capital stocks (i.e. they always prefer to produce organically).

Given $\frac{\partial \Delta(\epsilon^*)}{\partial \epsilon} \geq 0$, we will have that:

- The lower the threshold ϵ^*, the larger the set $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$
- The higher the threshold ϵ^*, the smaller the $\{K_{0,con} = K_{org} - \epsilon : \Delta(\epsilon) > 0\}$

B.1.1 Comparative statics for $\Delta(\epsilon)$

First we do a comparative static exercise for $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$. The results are summarized in Table B.1 and derived below.

$$\frac{\partial\Delta(\epsilon)}{\partial P_{con}} = \frac{1}{\rho}\cdot\alpha_b\cdot\left(\underbrace{-\left(A_b+\frac{A_y}{\alpha_b}\right)}_{\leq 0}+\underbrace{\gamma_K\cdot\left(\frac{\rho}{(\mu+\rho)}\cdot\epsilon-\overline{C}\right)}_{\leq 0}\right. \tag{B.3}$$

$$\left. + \underbrace{\left(-\left(\underbrace{\mu\gamma_{cc}\cdot\underbrace{\hat{K}_{con}}_{\leq 0}}_{\geq 0}+\underbrace{\frac{1}{2}\left(-\gamma_{cc}\right)\mu\overline{C}}_{\geq 0}+\underbrace{\frac{P_{con}^{-1}}{\alpha_b}}_{\geq 0}\right)\right)\cdot\mu\overline{C}}_{\leq 0}\right)\leq 0$$

$$\frac{\partial\Delta(\epsilon)}{\partial P_{org}} = \underbrace{\frac{1}{\rho}\cdot\alpha_b\cdot\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\geq 0 \tag{B.4}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\rho} = -P_{con}\cdot\alpha_b\cdot\frac{1}{\rho^2}\cdot\left(\underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq}\right.$$

$$+\underbrace{\frac{\rho}{(\mu+\rho)}\cdot\gamma_K\cdot\left(\frac{\rho}{\mu+\rho}\cdot\epsilon+\mu\overline{C}\cdot\left(\frac{1}{\rho}+\frac{1}{(\mu+\rho)}\right)\right)}_{\geq 0}$$

$$\left.-\underbrace{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C}}_{\geq 0}\right)$$

So that for large enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ we will have $\frac{\partial\Delta(\epsilon)}{\partial\rho}\leq 0$, and for small enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ and large enough α_c we will have $\frac{\partial\Delta(\epsilon)}{\partial\rho}\geq 0$.

$$\frac{\partial\Delta(\epsilon)}{\partial\mu} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\underbrace{\left(\underbrace{\frac{1}{(\mu+\rho)^2}}_{\geq 0}\cdot\gamma_K\cdot\left(\rho\cdot\epsilon+\mu\cdot\overline{C}\right)+\mu\underbrace{\gamma_{cc}\cdot\hat{K}_{con}}_{\geq 0}\cdot\overline{C}\right)}_{\geq 0}\leq 0$$

$$\frac{\partial\Delta(\epsilon)}{\partial\overline{C}} = \underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b}_{\geq 0}\cdot\left(\underbrace{\underbrace{(\gamma_K)}_{\geq 0}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}+\underbrace{(-\gamma_{cc})\,\mu^2\hat{K}_{con}}_{\leq 0}\right) \tag{B.5}$$

So we will have $\frac{\partial\Delta(\epsilon)}{\partial\overline{C}}\geq 0$ for large enough $\frac{P_{org}-P_{con}}{P_{con}}$, and $\frac{\partial\Delta(\epsilon)}{\partial\overline{C}}\leq 0$ for negative enough $\hat{K}_{con}$ (as we might have when α_c is sufficiently large)

$$\begin{aligned}\frac{\partial\Delta(\epsilon)}{\partial\alpha_b} = &\underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\left(\gamma_K\cdot\overline{C}+A_b\right)}_{\geq 0}\cdot\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}+\underbrace{P_{con}\cdot\frac{1}{(\mu+\rho)}\cdot\gamma_K}_{\geq 0}\cdot\epsilon\\ &+\underbrace{\frac{1}{\rho}\cdot P_{con}\cdot\left(\frac{1}{2}(-\gamma_{cc})\,\mu\overline{C}+(-\gamma_c)+\frac{1}{(\mu+\rho)}\cdot\gamma_K\right)\cdot\mu\overline{C}}_{\geq 0}\geq 0\end{aligned}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_{cc}} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\frac{1}{2}\cdot\left(\mu\overline{C}\right)^2\leq 0 \tag{B.6}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_c} = -\frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\mu\overline{C}\leq 0 \tag{B.7}$$

$$\frac{\partial\Delta(\epsilon)}{\partial\gamma_K} = \frac{1}{\rho}\cdot P_{con}\cdot\alpha_b\cdot\left(\overline{C}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)+\underbrace{\frac{1}{(\mu+\rho)}}_{\geq 0}\cdot\left(\rho\epsilon+\mu\overline{C}\right)\right)\geq 0 \tag{B.8}$$

which means that as the marginal product γ_K of an additional unit of clean soil increases, the farmer becomes more likely to prefer organic production.

$$\frac{\partial \Delta(\epsilon)}{\partial A_y} = (-P_{con} \cdot \alpha_b) \cdot \frac{P_{\text{con}} - P_{\text{org}}}{\rho P_{\text{con}} \alpha_b} \geq 0 \tag{B.9}$$

$$\frac{\partial \Delta(\epsilon)}{\partial A_b} = (-P_{con} \cdot \alpha_b) \cdot \frac{P_{\text{con}} - P_{\text{org}}}{\rho P_{\text{con}}} \geq 0 \tag{B.10}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} = (-P_{con} \cdot \alpha_b) \cdot \left(-\frac{\gamma_K}{\mu + \rho}\right) \geq 0 \tag{B.11}$$

Table B.1: Comparative Statics for $\Delta(\epsilon)$ When Conventional Farmer Adopts OT1

Parameter	Full Information: OT1	
ρ	$+$	Small enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ and large enough α_c.
	$-$	Large enough $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$
μ	$-$	
P_{org}	$+$	
P_{con}	$-$	
α_b	$+$	
α_c	$-$	
γ_c	$-$	
γ_{cc}	$-$	
γ_K	$+$	
A_y	$+$	
A_b	$+$	
$\overline{C}$	$+$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$.
	$-$	Large enough α_c
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon) = V_{org}(K_{org}) - V_{con}(K_{org} - \epsilon)$ when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically when $\Delta(\epsilon) > 0$.

B.1.2 Comparative statics for ϵ^*

Now we do comparative statics of ϵ^* for the parameters: $\mu, \rho, \gamma_{cc}, \gamma_c, \gamma_K, \alpha_1, \alpha_c, P_{con},$ and P_{org}.

The results are summarized in Table B.2.

$$\frac{\partial \epsilon^*}{\partial P_{org}} = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho}}_{\leq 0} \left(\underbrace{\underbrace{\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} + \underbrace{\gamma_K \overline{C}}_{\geq 0}}_{\geq 0} \right) \leq 0 \tag{B.12}$$

$$\frac{\partial \epsilon^*}{\partial \overline{C}} = \underbrace{-\frac{\mu+\rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho}}_{\leq 0} \left(\underbrace{\left(P_{org} - P_{con}\right)\gamma_K}_{\geq 0} - \mu^2 P_{con}\gamma_{cc}\hat{K}_{con} \right) \tag{B.13}$$

So for sufficiently large organic price premiums ($(P_{org} - P_{con})$ large) we'll have $\frac{\partial \epsilon^*}{\partial \overline{C}} \leq 0$. On the other hand when synthetic compounds are very effective at increasing yields (such that we have very large $\mu^2 P_{con}\gamma_{cc}\hat{K}_{con}$), we'll have $\frac{\partial \epsilon^*}{\partial \overline{C}} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial \mu} = \left(\frac{1}{\mu+\rho}\right)\epsilon^* + \left(\frac{\mu+\rho}{P_{con}\gamma_K}\right) \cdot \frac{1}{\rho}\left(P_{con}\left(\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \left(\frac{\rho}{(\mu+\rho)^2}\right)\gamma_K\right)\overline{C}\right) \tag{B.14}$$

Since for a conventional OT1 farmer we have that $\hat{K}_{con} < 0$ (and $\hat{K}_{org} < 0$), we have that:

$$\hat{K}_{con} = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\gamma_{cc}\mu} \leq 0 \tag{B.15}$$

Since $\rho > 0$, $\mu \geq 0$, and $\gamma_{cc} \leq 0$, we know that $(\rho+\mu)\gamma_{cc}\mu \leq 0$ (and in particular if $\hat{K}_{con}$ is well defined and satisfies $\hat{K}_{con} \leq 0$, it must be the case that $(\rho+\mu)\gamma_{cc}\mu < 0$). Then if $\hat{K}_{con} \leq 0$ and $(\rho+\mu)\gamma_{cc}\mu \leq 0$ it must be the case that:

$$(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K \geq 0 \tag{B.16}$$

Given $\gamma_K \geq 0$, we know that $\gamma_K \geq \frac{\rho}{\rho+\mu} \cdot \gamma_K \geq 0$ (since $0 \leq \frac{\rho}{\rho+\mu} \leq 1$). Thus

$$(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \frac{\rho}{(\rho+\mu)}\gamma_K \geq 0 \tag{B.17}$$

Thus:

$$\frac{\partial \epsilon^*}{\partial \mu} = \left(\frac{1}{\mu+\rho}\right)\epsilon^* + \underbrace{\left(\frac{\overline{C}}{\gamma_K}\right)\cdot\frac{1}{\rho}}_{\geq 0}\cdot\left(\underbrace{(\mu+\rho)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)-\frac{\rho}{(\mu+\rho)}\gamma_K}_{\geq 0}\right) \geq 0 \quad \text{(B.18)}$$

where:

$$\begin{aligned}\epsilon^* = & \left(\frac{\mu+\rho}{\gamma_K}\cdot\left(\frac{1}{2}\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-1\right)\cdot\frac{\mu}{\rho}\cdot\overline{C} \\ & -\frac{1}{\gamma_K}\cdot\underbrace{\frac{\mu+\rho}{\rho}\cdot\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\left(\frac{P_{org}-P_{con}}{P_{con}}\right)\end{aligned}$$

$$\begin{aligned}\frac{\partial \epsilon^*}{\partial \mu} = & \frac{1}{\rho}\cdot\left(\frac{1}{\gamma_K}\cdot\left(\frac{1}{2}\cdot(-\gamma_{cc})\cdot\overline{C}-\frac{1}{(\mu+\rho)^2}\cdot\gamma_K\right)\cdot\mu^2\cdot\overline{C}\right. \\ & + \underbrace{\left(-\frac{1}{\gamma_K}\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}\cdot\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)}_{\geq 0}\right)}_{\leq 0} \\ & \left. + \underbrace{\frac{2\mu+\rho}{\mu+\rho}\cdot\left(\frac{\overline{C}}{\gamma_K}\right)\cdot\left((\mu+\rho)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\frac{\rho}{(\mu+\rho)}\gamma_K\right)}_{\geq 0}\right)\end{aligned} \quad \text{(B.19)}$$

We have then that $\frac{\partial \epsilon^*}{\partial \mu} \leq 0$ for large enough organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$, or large enough $A_b + \frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields. We will have $\frac{\partial \epsilon^*}{\partial \mu} \geq 0$, on the other hand, if both the organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$ are sufficiently small and also $(\mu+\rho)$ is sufficiently large.

$$\frac{\partial \epsilon^*}{\partial \rho} = \underbrace{\left(\frac{1}{\rho}-\frac{1}{\mu+\rho}\right)}_{\geq 0}\left(\overline{C}-\epsilon^*\right) \quad \text{(B.20)}$$

The sign of $\left(\overline{C}-\epsilon^*\right)$ is ambiguous, and will depend on other parameter values. What we can say is that $\left(\overline{C}-\epsilon^*\right) \geq 0$ is a necessary condition for the feasible set of initial capital stock for which the farmer prefers to produce organically to be non-empty. So if the feasible set of initial capital stocks for which the farmer prefers to produce organically to be non-empty then we will have $\frac{\partial \epsilon^*}{\partial \rho} \geq 0$, such that increasing the interest rate (so that the farmer cares less about the future) increases ϵ^*, and contracts the set of initial capital stock at which the farmer prefers to produce organically.

$$\frac{\partial \epsilon^*}{\partial \gamma_{cc}} = \frac{\mu+\rho}{\gamma_K} \cdot \frac{1}{\rho}\left(\frac{1}{2}\left(\mu\overline{C}\right)^2\right) \geq 0 \tag{B.21}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_c} = \frac{\mu+\rho}{\gamma_K} \cdot \frac{\mu}{\rho}\overline{C} \geq 0 \tag{B.22}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \underbrace{\frac{1}{\gamma_K^2} \cdot \frac{\mu+\rho}{\rho}}_{\geq 0} \cdot \left(\underbrace{\left(\frac{P_{org}-P_{con}}{P_{con}}\right)\left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} + \underbrace{\underbrace{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)}_{\geq 0} \cdot \underbrace{\left(-\mu\overline{C}\right)}_{\leq 0}}_{\leq 0} \right), \tag{B.23}$$

where $\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \geq 0$ comes from the fact that for a conventional OT1 farmer we have $\hat{K}_{con}$. So we see that we will have:

- $\frac{\partial \epsilon^*}{\partial \gamma_K} \geq 0$ when:

1. We have large enough organic price premia $\frac{P_{org}-P_{con}}{P_{con}}$
2. or large enough $A_b + \frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields.

- We will have $\frac{\partial \epsilon^*}{\partial \gamma_K} \leq 0$ when both:

1. We have small enough organic price premia, $\frac{P_{org}-P_{con}}{P_{con}}$
2. and small enough $A_b + \frac{A_y}{\alpha_b}$, such that factors of production other than synthetic compounds and soil bacteria are sufficiently unimportant for determining yields.

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \frac{1}{\alpha_b^2} \cdot \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K} \left(\underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot A_y}_{\geq 0} - \underbrace{\left(\alpha_c - P_{con}^{-1} \right)}_{\geq 0} \cdot \underbrace{\mu \overline{C}}_{\geq 0} \right) \tag{B.24}$$

where $\alpha_c - P_{con}^{-1} \geq 0$ comes from $\hat{K}_{con} \leq 0$, which is satisfied in for a conventional OT1 farmer, since:

$$\hat{K}_{con} = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{\underbrace{(\rho+\mu)\,\gamma_{cc}\mu}_{\leq 0}} \leq 0 \tag{B.25}$$

$$\Rightarrow \underbrace{(\rho+\mu)}_{\geq 0} \left(\underbrace{\gamma_{cc}\mu\overline{C}}_{\leq 0} + \underbrace{\gamma_c}_{\leq 0} + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) - \underbrace{\gamma_K}_{\geq 0} \geq 0 \tag{B.26}$$

$$\Rightarrow \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \geq 0 \tag{B.27}$$

$$\Rightarrow \alpha_c - P_{con}^{-1} \geq 0 \tag{B.28}$$

So we have that:

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \underbrace{\frac{1}{\alpha_b^2} \cdot \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \left(\underbrace{\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot A_y}_{\geq 0} - \underbrace{\left(\alpha_c - P_{con}^{-1} \right)}_{\geq 0} \cdot \underbrace{\mu \overline{C}}_{\geq 0} \right) \tag{B.29}$$

We have $\frac{\partial \epsilon^*}{\partial \alpha_b} \geq 0$ when:

1. We have large enough organic price premia $\frac{P_{org} - P_{con}}{P_{con}}$
2. or large enough A_y, such that factors of production other than synthetic compounds and soil bacteria are sufficiently important in determining yields.

We will have $\frac{\partial \epsilon^*}{\partial \alpha_b} \leq 0$ when both:

1. We have small enough organic price premia $\frac{P_{org} - P_{con}}{P_{con}}$
2. and small enough A_y, such that factors of production other than synthetic compounds and soil bacteria are sufficiently unimportant for determining yields.

$$\frac{\partial \epsilon^*}{\partial \alpha_c} = \frac{\mu + \rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \frac{1}{\alpha_b} \cdot \mu \overline{C} \geq 0 \tag{B.30}$$

So that increasing the benefit of synthatic compounds (α_c) increases the value of (ϵ^*) and shrinks the set of initial capital stocks for which the farmer prefers to produce organically.

$$\frac{\partial \epsilon^*}{\partial P_{con}} = \frac{1}{P_{con}} \left(\underbrace{\frac{\mu + \rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho} P_{org} \left(A_b + \frac{A_y}{\alpha_b} \right)}_{\geq 0} + \underbrace{\frac{\mu + \rho}{P_{con}\gamma_K} \cdot \frac{1}{\rho} \left(\frac{1}{\alpha_b} \mu + P_{org}\gamma_K \right) \overline{C}}_{\geq 0} \right) \geq 0$$

So that increasing the price at which conventional farmers can sell their crops (P_{con}) increases the value of (ϵ^*) and shrinks the set of initial capital stocks for which the farmer prefers to produce organically.

Table B.2: Comparative Statics for ϵ^* When Conventional Farmer Adopts OT1

Parameter		Full Information: OT1
ρ	$-$	ρ and γ_K large enough, $\left(\frac{P_{\text{org}}}{P_{\text{con}}} - 1\right)$ and ϵ small enough.
	$+$	$\left(A_b + \frac{A_y}{\alpha_b}\right)$ sufficiently large
μ	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or $A_b + \frac{A_y}{\alpha_b}$ large enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and $(\mu+\rho)$ large enough.
P_{org}	-	
P_{con}	$+$	
α_b	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and A_y small enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or A_y large enough.
α_c	$+$	
γ_c	$+$	
γ_{cc}	$+$	
γ_K	$-$	$\frac{P_{org}-P_{con}}{P_{con}}$ small enough, and $A_b + \frac{A_y}{\alpha_b}$ small enough.
	$+$	$\frac{P_{org}-P_{con}}{P_{con}}$ large enough, or $A_b + \frac{A_y}{\alpha_b}$ large enough.
A_y	$+$	
A_b	$+$	
c_0	0	
C_0	0	
$\overline{C}$	$-$	$(P_{org} - P_{con})$ large enough
	$+$	$(P_{org} - P_{con})$ small enough

Notes: Table reports comparative statics for ϵ_+^* when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ_+^* lower than K_{org}.

B.1.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$

We also want to find how large the price premium needs to be in order to induce the fully informed farmer to prefer organic management. We derive this requirement for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ below.

The range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$, where:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho \cdot \epsilon\right)}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{B.31}$$

We now conduct a comparatic statics analysis for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$. The results are summarized in Table B.3.

Given that:

$$\frac{\partial \Delta(\epsilon)}{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)} \geq \underbrace{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \tag{B.32}$$

then any change in parameter values that increases the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ will shrink the set of organic price premia for which $\Delta(\epsilon) \geq 0$.

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \epsilon} = \frac{-\frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \rho}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \leq 0 \tag{B.33}$$

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \rho} = \frac{\mu \cdot \gamma_K}{(\mu+\rho)^2} \cdot \frac{\overline{C} - \epsilon}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.34}$$

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \mu} = \frac{\underbrace{\left(\gamma_{cc}\mu\hat{K}_{con}\right)}_{\geq 0} \cdot \overline{C} + \underbrace{\frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \overline{C} + \frac{\rho}{\mu+\rho} \cdot \epsilon\right)}_{\geq 0}}{\underbrace{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}}_{\geq 0}} \geq 0 \tag{B.35}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_K} = \underbrace{-\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}}_{\leq 0} \cdot\left(\underbrace{\left(\underbrace{\mu\gamma_{cc}\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)}_{\geq 0}+\frac{1}{\mu+\rho}\cdot\gamma_K\right)\cdot\mu\overline{C}^2}_{\geq 0}+\underbrace{\left(A_b+\frac{A_y}{\alpha_b}\right)\cdot\frac{\mu\overline{C}+\rho\cdot\epsilon}{\mu+\rho}}_{\geq 0}\right)\leq 0 \tag{B.36}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}} = \underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \cdot\left(\underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \cdot\left(\underbrace{\frac{1}{(\mu+\rho)}\cdot\gamma_K\cdot\frac{\rho}{\mu}\cdot\frac{\epsilon}{\overline{C}}}_{\geq 0}+\underbrace{(-\gamma_{cc})\,\mu\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)}_{\leq 0}\right)\cdot\underbrace{\gamma_K\mu\overline{C}}_{\geq 0}+\underbrace{\gamma_{cc}\mu^2\cdot\hat{K}_{con}}_{\geq 0}\right) \tag{B.37}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}} \leq 0$ for small enough ρ, small enough γ_K and small enough $\left(A_b+\frac{A_y}{\alpha_b}\right)$ (i.e synthetic compounds being relatively important, and the economic agent we caring enough about the future). We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\cdot\overline{C}} \geq 0$ for large enough ρ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} = \underbrace{\left(\gamma_K\cdot\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}\cdot\left(-\frac{1}{\alpha_b}\right)}_{\leq 0}\cdot\left(\underbrace{\gamma_{cc}\cdot\mu\left(\hat{K}_{con}-\frac{1}{2}\overline{C}\right)\cdot\mu\overline{C}}_{\geq 0}+\underbrace{\left(\frac{-\rho}{(\mu+\rho)}\right)\cdot\gamma_K\cdot\epsilon}_{\leq 0}\right) \tag{B.38}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \geq 0$ for large enough ρ and non-zero ϵ. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \leq 0$ for small enough ρ or ϵ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} = \underbrace{\left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)^{-2}}_{\geq 0} \cdot \left(\underbrace{\mu\left(-\gamma_{cc}\right) \cdot \left(\hat{K}_{con} - \frac{1}{2}\overline{C}\right) \cdot \mu\overline{C}}_{\leq 0} + \underbrace{\left(\frac{\rho}{\mu+\rho}\right) \cdot \gamma_K \cdot \epsilon}_{\geq 0}\right) \tag{B.39}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \geq 0$ for large enough ρ and non-zero ϵ. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \leq 0$ for small enough ρ or ϵ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} = \underbrace{\frac{A_y}{\left(\alpha_b \cdot \left(\gamma_K \cdot \overline{C} + A_b\right) + A_y\right)^2}}_{\geq 0} \cdot \left(\underbrace{\gamma_{cc}\mu \cdot \left(\hat{K}_{con} - \frac{1}{2}\overline{C}\right) \cdot \mu\overline{C}}_{\geq 0} + \underbrace{\frac{-\rho}{(\mu+\rho)} \cdot \gamma_K \cdot \epsilon}_{\leq 0}\right) \tag{B.40}$$

$$+ \underbrace{\frac{\left(-\left(\frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C}\right)}{\left(\alpha_b\left(\gamma_K \cdot \overline{C} + A_b\right) + A_y\right)}}_{\leq 0}$$

We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} \geq 0$ for small enough ρ and and large enough A_y. We will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_b} \leq 0$ for large enough ρ.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \gamma_{cc}} = \frac{\frac{1}{2}\left(\mu\overline{C}\right)^2}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.41}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \gamma_c} = \frac{\mu\overline{C}}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.42}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \alpha_c} = \frac{\left(\frac{1}{\alpha_b}\right)\cdot \mu\overline{C}}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.43}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial P_{con}} = \frac{\left(\frac{P_{con}^{-2}}{\alpha_b}\right)\cdot \mu\overline{C}}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \geq 0 \tag{B.44}$$

Table B.3: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ When Conventional Farmer Adopts OT1

Parameter	Full Information: OT1	
ϵ	-	
ρ	+	
μ	+	
γ_K	-	
$\overline{C}$	$-$	For small enough ρ, γ_K, $and\,(A_b + \frac{A_y}{a_b})$
	$+$	For large enough ρ
A_y	$-$	For small enough ρ or ϵ
	$+$	For large enough ρ and non-zero ϵ
A_b	$-$	For small enough ρ or ϵ
	$+$	For large enough ρ and non-zero ϵ
α_b	$-$	For large enough ρ
	$+$	For small enough ρ and large enough A_y
γ_{cc}	+	
γ_c	+	
α_c	+	
P_{con}	+	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ when the optimal solution for the conventional farmer is to disinvest as fast as possible until $K = 0$. A conventional farmer will prefer producing organically if $\frac{P_{org}-P_{con}}{P_{con}} > \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$.

B.2 Discrete Analysis for OT2/OT3/OT4

The sign of $\frac{\partial \Delta(\epsilon)}{\partial \epsilon}$ is given by:

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} = \frac{P_{con}\alpha_b\gamma_K}{\mu + \rho} \cdot \epsilon \geq 0 \tag{B.45}$$

Thus, $\Delta(\epsilon)$ is linear and weakly increasing in ϵ.

Let ϵ^* be the value of ϵ such that $\Delta(\epsilon^*) = 0$.

When $K_{org} = \overline{C}$, the range of ϵ yielding $\Delta(\epsilon) \geq 0$ is $\epsilon \geq \epsilon^*$ where:

$$\epsilon^* = \frac{1}{\gamma_K} \cdot \frac{\mu+\rho}{\rho} \cdot \left(\underbrace{\frac{1}{2} \cdot \frac{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{\mu+\rho}\right)^2}{(-\gamma_{cc})}}_{\geq 0} - \underbrace{\left(\frac{P_{org}}{P_{con}} - 1\right) \cdot \left(\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \right) \tag{B.46}$$

For a conventional OT2/OT3/OT4 farmer, there is a possibility that ϵ^* exceeds $\overline{C}$. When this happens there will be no feasible ϵ for which $\Delta(\epsilon) \geq 0$, and there will therefore be no feasible capital stock for which the fully informed farmer facing OT2/OT3/OT3 conditions will prefer to produce organically. ϵ^* will be more likely to exceed $\overline{C}$ when the farmer faces small organic price premia.

Under OT2/OT3/OT4 conditions, we have:

$$\hat{K}_j \leq \overline{C} \Rightarrow \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} - \frac{\gamma_K}{(\rho+\mu)} \geq 0, \tag{B.47}$$

and

$$\hat{K}_j \geq 0 \Rightarrow \gamma_c + \frac{\alpha_c - P_j^{-1}}{\alpha_b} \leq (-\gamma_{cc})\,\mu\overline{C} + \frac{\gamma_K}{(\rho+\mu)} \tag{B.48}$$

Also, given:

$$\hat{K}_{con} = \frac{(\rho+\mu)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu} = \overline{C} + \underbrace{\frac{(\rho+\mu)\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0} \leq \overline{C} \tag{B.49}$$

we know that $\left(\overline{C} - \hat{K}_{con}\right) \to 0$ implies:

$$\underbrace{\frac{(\rho+\mu)\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) - \gamma_K}{(\rho+\mu)\,\gamma_{cc}\mu}}_{\leq 0} \to 0 \tag{B.50}$$

or

$$\underbrace{\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\rho+\mu)}}_{\geq 0} \to 0 \tag{B.51}$$

$$\Rightarrow \underbrace{\frac{\alpha_c}{\alpha_b}}_{\geq 0} + \underbrace{\left(-\frac{1}{\alpha_b \cdot P_{con}} + \gamma_c - \frac{\gamma_K}{(\rho+\mu)}\right)}_{\leq 0} \to 0 \tag{B.52}$$

$$\Rightarrow \alpha_c \to \alpha_b \cdot \underbrace{\left(\frac{1}{\alpha_b \cdot P_{con}} - \gamma_c + \frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0} \tag{B.53}$$

That is, $\left(\overline{C} - \hat{K}_{con}\right) \to 0$, given constant $\overline{C}$, implies small enough α_c, such that $\underbrace{\alpha_c}_{\geq 0} - \alpha_b \cdot \underbrace{\left(\frac{1}{\alpha_b \cdot P_{con}} - \gamma_c + \frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0}$ approaches 0 from above.

B.2.1 Comparative statics for $\Delta(\epsilon)$

Now we discuss the signs of $\frac{\partial \Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org} = \overline{C}$.

The results are summarized in Table B.4.

$$\frac{\partial \Delta(\epsilon)}{\partial P_{\text{org}}} = \underbrace{\frac{1}{\rho} \cdot \left(A_y + \alpha_b \left(A_b + \overline{C}\gamma_K\right)\right)}_{\geq 0} \geq 0$$

$$\frac{\partial \Delta(\epsilon)}{\partial P_{\text{con}}} = \underbrace{\left(-\frac{1}{\rho} \cdot \alpha_b\right)}_{\leq 0} \cdot \left(\underbrace{A_b + \frac{A_y}{\alpha_b}}_{\geq 0} + \mu \cdot \left(\underbrace{\frac{P_{\text{con}}^{-1}}{\alpha_b} + \frac{1}{2} \cdot (-\gamma_{\text{cc}})\, \mu \cdot \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0}\right) \cdot \underbrace{\left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0} + \underbrace{\gamma_K \left(\overline{C} - \frac{\rho}{\mu+\rho}\epsilon\right)}_{\geq 0}\right) \leq 0 \tag{B.54}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \mu} = -\frac{1}{\rho} P_{\text{con}} \alpha_b \frac{\gamma_K}{(\mu+\rho)} \left(-\frac{\rho\overline{C} + \mu\hat{K}_{con}}{\rho+\mu} + \left(\overline{C} + \frac{\rho}{\mu+\rho} \cdot \epsilon\right)\right) \tag{B.55}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \mu} = -\frac{1}{\rho} \cdot \underbrace{\left(P_{\text{con}} \cdot \alpha_b \cdot \frac{\gamma_K}{\mu + \rho} \right)}_{\geq 0} \cdot \left(\underbrace{\overline{C} - \underbrace{\frac{\rho + \underbrace{\left(\frac{\hat{K}_{con}}{\overline{C}} \right)}_{\leq 1} \cdot \mu}{\rho + \mu}}_{\leq 1} \cdot \overline{C}}_{\geq 0} + \underbrace{\frac{\rho}{\mu + \rho}}_{\geq 0} \cdot \epsilon \right) \leq 0 \tag{B.56}$$

$$\begin{aligned} \frac{\partial \Delta(\epsilon)}{\partial \rho} = \underbrace{\rho^{-2} P_{\text{con}} \alpha_b}_{\geq 0} \cdot \Bigg(\Bigg(\underbrace{\frac{\rho}{(\mu + \rho)^2} \cdot (-\gamma_K)}_{\leq 0} + \underbrace{\frac{1}{2} \cdot (-\gamma_{\text{cc}}) \, \mu \left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} \Bigg) \underbrace{\cdot \mu \left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} \\ + \underbrace{\left(\frac{\rho}{\mu + \rho} \right)^2 \cdot (-\gamma_K) \cdot \epsilon}_{\leq 0} + \underbrace{\left(\frac{A_y}{\alpha_b} + A_b + \gamma_K \cdot \overline{C} \right) \cdot \left(-\frac{P_{\text{org}} - P_{con}}{P_{con}} \right)}_{\leq 0} \Bigg) \end{aligned} \tag{B.57}$$

So we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \geq 0$ for small enough γ_K and small enough organic price premium $\frac{P_{\text{org}} - P_{con}}{P_{con}}$.

On the other hand we will have $\frac{\partial \Delta(\epsilon)}{\partial \rho} \leq 0$ for large enough $\frac{P_{\text{org}} - P_{con}}{P_{con}}$.

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_c} = -\frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \mu \cdot \underbrace{\left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} \leq 0 \tag{B.58}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_{\text{cc}}} = -\frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \cdot \frac{1}{2} \cdot \left(\mu \left(\overline{C} - \hat{K}_{con} \right) \right)^2 \leq 0 \tag{B.59}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \gamma_K} = \frac{1}{\rho} \cdot P_{\text{con}} \alpha_b \cdot \underbrace{\left(\underbrace{\left(\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}} \right)}_{\geq 0} \overline{C} + \frac{\mu}{\mu + \rho} \cdot \underbrace{\left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} + \frac{\rho}{\mu + \rho} \cdot \underbrace{\epsilon}_{\geq 0} \right)}_{\geq 0} \geq 0 \tag{B.60}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \overline{C}} = \frac{1}{\rho} \cdot \underbrace{(P_{\text{org}} - P_{\text{con}})}_{\geq 0} \cdot \alpha_b \cdot \gamma_K \geq 0 \tag{B.61}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} = \frac{1}{\rho} \cdot P_{\text{con}} \cdot \Bigg(\underbrace{\left(\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}}\right) \cdot \left(A_b + \gamma_K \cdot \overline{C}\right)}_{\geq 0} + \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon}_{\geq 0}$$
$$+ \underbrace{\frac{1}{2}\gamma_{cc} \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b} \cdot \underbrace{\mu \left(\overline{C} - \hat{K}_{con}\right)}_{\geq 0}}_{?} \Bigg) \tag{B.62}$$

So we will have $\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} \geq 0$ for large enough $\frac{P_{\text{org}} - P_{\text{con}}}{P_{\text{con}}}$, and we will have $\frac{\partial \Delta(\epsilon)}{\partial \alpha_b} \leq 0$ for large enough $\hat{c} = \mu \left(\overline{C} - \hat{K}_{con}\right)$, given $\gamma_{cc} \neq 0$. Note that $\hat{c} = \mu \left(\overline{C} - \hat{K}_{con}\right)$ will be larger for larger $\overline{C}$ or μ, or smaller $\hat{K}_{con}$. Given

$$\hat{K}_{con} = \frac{(\mu + \rho)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b}\right) - \gamma_K}{(\mu + \rho)\,\gamma_{cc}\mu} \geq 0$$

$$\Rightarrow (\mu + \rho)\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{\text{con}}^{-1}}{\alpha_b}\right) - \gamma_K \leq 0$$

we know that $\hat{K}_{con}$ will be smaller for smaller $-\gamma_{cc}$, $-\gamma_c$, or γ_K, or bigger α_c.

$$\frac{\partial \Delta(\epsilon)}{\partial A_y} = \underbrace{\frac{1}{\rho}\left(P_{\text{org}} - P_{\text{con}}\right)}_{\geq 0} \geq 0 \tag{B.63}$$

$$\frac{\partial \Delta(\epsilon)}{\partial A_b} = \underbrace{\frac{1}{\rho} \cdot \alpha_b \cdot \left(P_{\text{org}} - P_{\text{con}}\right)}_{\geq 0} \geq 0 \tag{B.64}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \alpha_c} = -\frac{1}{\rho} \cdot P_{\text{con}} \cdot \underbrace{\mu \left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} \leq 0 \tag{B.65}$$

$$\frac{\partial \Delta(\epsilon)}{\partial \epsilon} = \frac{1}{\mu + \rho} \cdot P_{con} \alpha_b \gamma_K \geq 0 \tag{B.66}$$

Table B.4: Comparative Statics for $\Delta(\epsilon)$ When Conventional Farmer Adopts OT2/OT3/OT4 and $K_{org} = \overline{C}$

Parameter i	**Sign**	**Condition**
A_b	$+$	
A_y	$+$	
P_{con}	$-$	
P_{org}	$+$	
$\overline{C}$	$+$	
μ	$-$	
ρ	$-$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$.
	$+$	Small γ_K and $\frac{P_{org}-P_{con}}{P_{con}}$.
γ_{cc}	$-$	
γ_c	$-$	
γ_K	$+$	
α_b	$-$	Large enough $\hat{c} = \mu\left(\overline{C} - \hat{K}_{con}\right)$, given $\gamma_{cc} \neq 0$. (Note that $\uparrow \hat{c} \Rightarrow \uparrow \overline{C}$, μ, α_c, $\downarrow \lvert\gamma_{cc}\rvert$, $\lvert\gamma_c\rvert$, γ_K).
	$+$	Large enough $\frac{P_{org}-P_{con}}{P_{con}}$.
α_c	$-$	
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon)$ when $\hat{K}_j \in [0, K_{0,j}]\,\forall j \in \{con, org\}$, assuming $K_{org} = \overline{C}$ (not responsive to assumption that $\bar{c} = \mu\overline{C}$ (looks the same either way)). A conventional farmer is said to prefer producing organically when $\Delta(\epsilon) > 0$.

B.2.2 Comparative statics for ϵ^*

We now calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$.

The results are summarized in Table B.5.

$$\frac{\partial \epsilon^*}{\partial \mu} = \frac{1}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{\frac{1}{2} \cdot \mu \cdot \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} + \frac{\gamma_K}{\rho + \mu} \right)}_{\geq 0} \underbrace{\left(\overline{C} - \hat{K}_{con} \right)}_{\geq 0} + \underbrace{\left(-\gamma_K \cdot \left(\frac{P_{org}}{P_{con}} + 1 \right) \overline{C} - \left(\frac{P_{org}}{P_{con}} - 1 \right) \cdot \left(A_b + \frac{A_y}{\alpha_b} \right) \right)}_{\leq 0} \right) \tag{B.67}$$

The sign of $\frac{\partial \epsilon^*}{\partial \mu}$ is still ambiguous without futher restrictions to our parameter values. However, we can see that for large enough organic premia we will have $\frac{\partial \epsilon^*}{\partial \mu} \geq 0$. On the other hand, for small enough organic premia we will have

$$\frac{\partial \epsilon^*}{\partial \mu} \to \frac{1}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{\frac{1}{2} \cdot \mu^2 \left(-\gamma_{cc} \right) \left(\overline{C} - \hat{K}_{con} \right)^2}_{\geq 0} - \underbrace{\gamma_K \left(\overline{C} + \frac{\rho}{\rho + \mu} \cdot \overline{C} + \frac{\mu}{\rho + \mu} \cdot \hat{K}_{con} \right)}_{\geq 0} \right) \tag{B.68}$$

Therefore, given small enough organic price premia (such that the value of $\frac{\partial \epsilon^*}{\partial \mu}$ is close enough to the value of the expression we have above), and large enough $\hat{K}_{con}$ (s.t. $\left(\overline{C} - \hat{K}_{con} \right) \to 0$), we will have $\frac{\partial \epsilon^*}{\partial \mu} \leq 0$.

Since $\left(\overline{C} - \hat{K}_{con} \right) \to 0$ implies that $\underbrace{\alpha_c}_{\geq 0} - \alpha_b \cdot \underbrace{\left(\frac{1}{\alpha_b \cdot P_{con}} - \gamma_c + \frac{\gamma_K}{(\rho + \mu)} \right)}_{\geq 0}$ approaches 0 from above, we will have $\frac{\partial \epsilon^*}{\partial \mu} \leq 0$ for small enough organic price premia, and and small enough α_c.

$$\frac{\partial \epsilon^*}{\partial \rho} = - \left(\frac{\mu}{\rho} \right)^2 \cdot \left(\frac{\overline{C} - \hat{K}_{con}}{(\rho + \mu)^2} + K_{org} - \frac{1}{\mu} \overline{C} \right) \tag{B.69}$$

Given $K_{org} = \overline{C}$, this yields:

$$\frac{\partial \epsilon^*}{\partial \rho} = -\left(\frac{\mu}{\rho}\right)^2 \cdot \left(\underbrace{\frac{\overline{C} - \hat{K}_{con}}{(\rho+\mu)^2}}_{\geq 0} + \frac{1}{\mu} \underbrace{(\mu - 1)}_{\leq 0} \overline{C} \right) \tag{B.70}$$

The sign of $\frac{\partial \epsilon^*}{\partial \rho}$ is ambiguous, but we can see that for large enough μ (such that $\frac{1}{\mu} \underbrace{(\mu-1)}_{\leq 0} \overline{C} \to 0$) we will have $\frac{\partial \epsilon^*}{\partial \rho} \leq 0$. On the other hand, for small enough α_c such that $\frac{\overline{C}-\hat{K}_{con}}{(\rho+\mu)^2} \to 0$, we will have $\frac{\partial \epsilon^*}{\partial \rho} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K^2} \cdot \left(\frac{P_{org}}{P_{con}} \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b} \right) \mu \left(\overline{C} - K_{org}\right) + \underbrace{\left(\frac{P_{org}}{P_{con}} - 1\right) \cdot \left(A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \right.$$
$$\left. + \left(\underbrace{\frac{1}{2} \cdot \gamma_{cc} \cdot \left(\frac{\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K}{\gamma_{cc}} \right)^2}_{\leq 0} + \underbrace{\frac{\gamma_K}{(\rho+\mu)} \cdot \frac{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)}{\gamma_{cc}}}_{\leq 0} \right) \right) \tag{B.71}$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_K}$ is still ambiguous, but we can see that as $\left(A_b + \frac{A_y}{\alpha_b}\right)$ increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_K}$ will become non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_K} \geq 0$). On the other hand, as the organic price premia shrinks (such that $\left(\frac{P_{org}}{P_{con}} - 1\right) \to 0$), and the certification criteria becomes stricter (such that $\left(\overline{C} - K_{org}\right) \to 0$), we will eventually get $\frac{\partial \epsilon^*}{\partial \gamma_K} \leq 0$.

$$\frac{\partial \epsilon^*}{\partial \gamma_{cc}} = \underbrace{\frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \left(\underbrace{-\frac{P_{org}}{P_{con}} \cdot \frac{1}{2}\left(\mu\left(\overline{C} - K_{org}\right)\right)^2}_{\leq 0} + \underbrace{\frac{1}{2} \cdot \frac{1}{\gamma_{cc}^2} \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)^2}_{\geq 0} \right) \tag{B.72}$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ is still ambiguous, but we can see that as $\frac{P_{org}}{P_{con}}$ (which reflects the organic premium) increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ will become non-positive ($\frac{\partial \epsilon^*}{\partial \gamma_{cc}} \leq 0$). On the other hand, as the organic certification becomes stricter $\left(K_{org} \to \overline{C}\right)$, $\frac{\partial \epsilon^*}{\partial \gamma_{cc}}$ eventually becomes non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_{cc}} \geq 0$).

$$\frac{\partial \epsilon^*}{\partial \gamma_c} = \frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(\underbrace{-\frac{P_{org}}{P_{con}} \cdot \mu \left(\overline{C} - K_{org}\right)}_{\leq 0} + \underbrace{\frac{1}{(-\gamma_{cc})} \underbrace{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\rho+\mu)}\right)}_{\geq 0}}_{\geq 0} \right) \tag{B.73}$$

The sign of $\frac{\partial \epsilon^*}{\partial \gamma_c}$ is still ambiguous, but we can see that as $\frac{P_{org}}{P_{con}}$ (which reflects the organic premium) increases in magnitude eventually $\frac{\partial \epsilon^*}{\partial \gamma_c}$ will become non-positive ($\frac{\partial \epsilon^*}{\partial \gamma_c} \leq 0$). On the other hand, as the organic certification becomes stricter $\left(K_{org} \to \overline{C}\right)$, $\frac{\partial \epsilon^*}{\partial \gamma_c}$ eventually becomes non-negative ($\frac{\partial \epsilon^*}{\partial \gamma_c} \geq 0$).

$$\frac{\partial \epsilon^*}{\partial P_{con}} = \underbrace{\frac{\mu+\rho}{\rho} \cdot \frac{1}{\gamma_K}}_{\geq 0} \cdot \left(\underbrace{\frac{P_{org}}{P_{con}^2} \mu \left(\overline{C} - K_{org}\right)}_{\geq 0} \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right) \right. \tag{B.74}$$

$$\left. + \underbrace{\frac{1}{(-\gamma_{cc})} \cdot \left(\frac{P_{con}^{-2}}{\alpha_b}\right)}_{\geq 0} \cdot \underbrace{\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)}_{\geq 0} + \underbrace{\frac{P_{org}}{P_{con}^2}\left(\gamma_K \cdot K_{org} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0} \right)$$

The sign of $\frac{\partial \epsilon^*}{\partial P_{con}}$ is still ambiguous. However, we can see that for relatively large $\overline{C}$ and organic certification criteria is weak enough (such that $\left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right) \leq 0$ and also such that $\left(\overline{C} - K_{org}\right)$ is relatively large, so that $\frac{P_{org}}{P_{con}^2}\mu\left(\overline{C} - K_{org}\right) \cdot \left(\frac{1}{2}\gamma_{cc}\mu\left(\overline{C} - K_{org}\right) + \gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right)$ is negative and has a large magnitude), then we will have $\frac{\partial \epsilon^*}{\partial P_{con}} \leq 0$. On the other hand, when the certification criteria is strict enough (so that $\left(\overline{C} - K_{org}\right)$ is close enough to zero), we will have $\frac{\partial \epsilon^*}{\partial P_{con}} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial \overline{C}} = \underbrace{\frac{\mu}{\rho}}_{\geq 0} \left(\underbrace{\frac{(\mu+\rho)}{\gamma_K} \cdot \frac{P_{org}}{P_{con}}}_{\geq 0} \cdot \left(\underbrace{(-\gamma_{cc})\,\mu\left(\overline{C} - K_{org}\right)}_{\geq 0} - \underbrace{\left(\gamma_c + \frac{\alpha_c - P_{org}^{-1}}{\alpha_b}\right)}_{\geq 0} \right) - 1 \right) \tag{B.75}$$

The sign of $\frac{\partial \epsilon^*}{\partial \overline{C}}$ is still ambiguous. However, we can see that for relatively large $\overline{C}$ (such that

$\left(\overline{C}-K_{org}\right)$ is relatively large), we will have $\frac{\partial\epsilon^*}{\partial\overline{C}}\geq 0$. On the other hand, for sufficiently strict certification criteria (such that $\left(\overline{C}-K_{org}\right)\to 0$) we will have $\frac{\partial\epsilon^*}{\partial\overline{C}}\leq 0$.

$$\frac{\partial\epsilon^*}{\partial K_{org}}=\underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\frac{P_{org}}{P_{con}}\cdot\mu}_{\geq 0}\left(\underbrace{\gamma_{cc}\mu\left(\overline{C}-K_{org}\right)}_{\leq 0}+\underbrace{\gamma_c+\frac{\alpha_c-P_{org}^{-1}}{\alpha_b}}_{\geq 0}\right)+\underbrace{\left(1-\frac{\mu+\rho}{\rho}\cdot\frac{P_{org}}{P_{con}}\right)}_{\leq 0} \quad \text{(B.76)}$$

We see that the sign of $\frac{\partial\epsilon^*}{\partial K_{org}}$ is ambiguous. We have that for large enough α_c, small enough γ_K , and $K_{org}\neq 0$, we will have $\frac{\partial\epsilon^*}{\partial K_{org}}\geq 0$. On the other hand, for weak enough initial certification criteria (such that $\left(\overline{C}-K_{org}\right)$ is sufficiently large), $|\gamma_{cc}|$ large enough, and $\mu\neq 0$, we will have $\frac{\partial\epsilon^*}{\partial K_{org}}\leq 0$.

$$\frac{\partial\epsilon^*}{\partial\alpha_c}=\underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}\cdot\frac{1}{\alpha_b}}_{\geq 0}\cdot\left(\underbrace{\frac{P_{org}}{P_{con}}\mu\left(K_{org}-\overline{C}\right)}_{\leq 0}+\underbrace{\frac{1}{(-\gamma_{cc})}\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{1}{(\rho+\mu)}\gamma_K\right)}_{\geq 0}\right) \quad \text{(B.77)}$$

The sign of $\frac{\partial\epsilon^*}{\partial\alpha_c}$ is ambiguous, but we see that if $K_{org}\neq\overline{C}$ and the organic premium is high enough, then we will have $\frac{\partial\epsilon^*}{\partial\alpha_c}\leq 0$. On the other hand, if the certification criteria is strict enough (such that $\left(K_{org}-\overline{C}\right)$ is close enough to zero), then we will have $\frac{\partial\epsilon^*}{\partial\alpha_c}\geq 0$.

$$\begin{aligned}\frac{\partial\epsilon^*}{\partial\alpha_b}=\underbrace{\frac{\mu+\rho}{\rho}\cdot\frac{1}{\gamma_K}}_{\geq 0}\cdot\frac{1}{\alpha_b^2}\cdot&\left(\underbrace{\frac{P_{org}}{P_{con}}\mu\left(\overline{C}-K_{org}\right)}_{\geq 0}\cdot\underbrace{\left(\alpha_c-P_{org}^{-1}\right)}_{\geq 0}+\underbrace{\left(\frac{P_{org}}{P_{con}}-1\right)\cdot A_y}_{\geq 0}\right. \quad \text{(B.78)}\\&\left.+\underbrace{\frac{1}{\gamma_{cc}}\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{1}{(\rho+\mu)}\gamma_K\right)\left(\alpha_c-P_{con}^{-1}\right)}_{\leq 0}\right)\geq 0\end{aligned}$$

The sign of $\frac{\partial\epsilon^*}{\partial\alpha_b}$ is ambiguous. However, we can see that if factors of production other than soil bacteria and synthetic compounds are relatively unimportant for production (so that $A_y\to 0$), and the certification criteria is sufficiently strict (such that $\left(\overline{C}-K_{org}\right)\to 0$), then we will have

$\frac{\partial \epsilon^*}{\partial \alpha_b} \leq 0$. On the other hand, if the organic price premium is high enough and also either $A_y \neq 0$ or $K_{org} \neq \overline{C}$, then we will have $\frac{\partial \epsilon^*}{\partial \alpha_b} \geq 0$.

$$\frac{\partial \epsilon^*}{\partial A_b} = \frac{\mu + \rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(1 - \frac{P_{org}}{P_{con}}\right) \leq 0 \tag{B.79}$$

$$\frac{\partial \epsilon^*}{\partial A_y} = \frac{\mu + \rho}{\rho} \cdot \frac{1}{\gamma_K} \cdot \left(1 - \frac{P_{org}}{P_{con}}\right) \cdot \frac{1}{\alpha_b} \leq 0 \tag{B.80}$$

Table B.5: Comparative Statics for ϵ^* When Conventional Farmer Adopts OT2/OT3/OT4 and $K_{org} = \overline{C}$

Parameter i	**Sign**	**Condition**
A_b	$-$	
A_y	$-$	
P_{con}	$+$	
P_{org}	$-$	
$\overline{C}$	$-$	
μ	$-$ $+$	Small enough $(P_{org} - P_{con})$ and $\left(\overline{C} - \hat{K}_{con}\right)$. Given constant $\overline{C}$, the latter implies small enough α_c. Large $(P_{org} - P_{con})$.
ρ	$-$ $+$ 0	Large enough μ. Small enough α_c (so that $\hat{K}_j \uparrow$and $\hat{K}_j \rightarrow \overline{C}^-$). For: (1) small enough α_c and also large enough μ; or (2) small enough μ.
γ_{cc}	$+$	
γ_c	$+$	
γ_K	$-$ $+$	For $\frac{P_{org}}{P_{con}}$ small enough. For $\frac{P_{org}}{P_{con}}$ large enough, and $(A_b+A_y) \neq 0$.
α_b	$-$ $+$	For small enough A_y or $\frac{P_{org}}{P_{con}}$. For large enough A_y and $\frac{P_{org}}{P_{con}}$.
α_c	$+$	

Notes: Table reports comparative statics for ϵ^* when $\hat{K}_j \in [0, \overline{C}]\, \forall j \in \{con, org\}$, assuming $K_{org} = \overline{C}$ (not responsive to assumption that $\bar{c} = \mu\overline{C}$ (looks the same either way)). A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ^* lower than K_{org}.

B.2.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing OT2/OT3/OT4 conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below.

Given the assumption that $K_{org} = \overline{C}$, and assuming conventional crop prices are not zero, the threshold organic price premium is given by:

$$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* = \frac{\left(\frac{1}{2}\cdot(-\gamma_{cc})\,\mu^2\left(\overline{C}-\hat{K}_{con}\right)^2 - \frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)} \tag{B.81}$$

Then we can determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters by examining the signs of the partials below. The results are summarized in Table B.6.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\epsilon} = -\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1} \leq 0 \tag{B.82}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\rho} = \frac{\underbrace{\gamma_K\cdot\mu}_{\geq 0}\cdot\left(K_0-\hat{K}_{con}\right)}{\underbrace{(\mu+\rho)^2}_{\geq 0}\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}} \tag{B.83}$$

where under OT2/OT3/OT4 conditions we have that $\hat{K}_{con}\in[0,\overline{C}]$. So we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\rho}\geq 0$ for large enough initial capital stock, K_0, and we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\rho}\leq 0$ for small enough initial capital stock, K_0.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\mu} = \underbrace{\frac{\gamma_K}{\mu+\rho}}_{\geq 0}\cdot\left(\overline{C}-\frac{\mu\cdot\hat{K}_{con}+\rho\cdot K_0}{\mu+\rho}\right)\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \tag{B.84}$$

Under OT2/OT3/OT4 conditions we have that $\hat{K}_{con}\in[0,\overline{C}]$. In all cases we also have that $K_0\in[0,\overline{C}]$. Therefore, it must be the case that

$$\left(\overline{C} - \frac{\mu \cdot \hat{K}_{con} + \rho \cdot K_0}{\mu + \rho}\right) \geq 0. \tag{B.85}$$

So we have that

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \mu} = \underbrace{\frac{\gamma_K}{\mu + \rho}}_{\geq 0} \cdot \underbrace{\left(\overline{C} - \frac{\mu \cdot \hat{K}_{con} + \rho \cdot K_0}{\mu + \rho}\right)}_{\geq 0} \cdot \underbrace{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \geq 0 \tag{B.86}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \gamma_K} = \left(\underbrace{\frac{\mu \cdot \hat{K}_{con} + \rho \cdot K_0}{\mu + \rho} - \overline{C}}_{\leq 0} + \frac{\underbrace{\frac{1}{2} \cdot \gamma_{cc} \cdot \left(\mu \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \left(\overline{C} - K_0\right)}_{\geq 0}}{\underbrace{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0}} \cdot \overline{C} \right) \cdot \underbrace{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \tag{B.87}$$

Here we will have that $\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \gamma_K} \leq 0$ if K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand we will have $\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \gamma_K} \geq 0$ if both $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$) and also $\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)$ is sufficiently small.

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \overline{C}} = \left(\underbrace{\frac{1}{2} \cdot \gamma_{cc} \cdot \left(\mu \cdot \left(\overline{C} - \hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \left(\overline{C} - K_0\right)}_{\geq 0} \right) \cdot \underbrace{\left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)^{-2} \cdot \gamma_K}_{\geq 0} \tag{B.88}$$

The farmer will face $\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*}{\partial \overline{C}} \geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \overline{C}} \leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} = \left(\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot(\overline{C}-K_0)}_{\geq 0}\right) \cdot \underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}\cdot\left(\frac{1}{\alpha_b}\right)}_{\geq 0} \tag{B.89}$$

The farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_y} \leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} = \left(\underbrace{\frac{1}{2}\cdot\gamma_{cc}\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\leq 0} + \underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot(\overline{C}-K_0)}_{\geq 0}\right) \cdot \underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-2}}_{\geq 0} \tag{B.90}$$

The farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \geq 0$ when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand the farmer will face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial A_b} \leq 0$ when K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$).

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_b} = \left(\underbrace{\mu\left(\hat{K}_{con}-\overline{C}\right)}_{\leq 0}\cdot\left(\frac{P_{con}\alpha_c-1}{P_{con}\alpha_b^2}\right)\right.$$

$$\left. + \frac{\underbrace{\frac{1}{2}\cdot(-\gamma_{cc})\cdot\left(\mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\right)^2}_{\geq 0} + \underbrace{\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(K_0-\overline{C}\right)}_{\leq 0}}{\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}_{\geq 0}}\cdot\underbrace{\frac{A_y}{\alpha_b^2}}_{\geq 0}\right)$$

$$\cdot\underbrace{\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1}}_{\geq 0} \tag{B.91}$$

The farmer will therefore face $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_b} \leq 0$, for example, when $\hat{K}_{con}$ is sufficiently large (and therefore sufficiently close to $\overline{C}$). On the other hand we will have $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_b} \geq 0$ when both K_0 is sufficiently large (and therefore sufficiently close to $\overline{C}$), and the marginal revenue associated with conventional synthetic compound use is sufficiently close to the marginal cost of synthetic compound use such that $P_{con}\alpha_c \to 1$.

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_{cc}} = \frac{1}{2}\cdot\left(\mu\left(\overline{C}-\hat{K}_{con}\right)\right)^2\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1} \geq 0 \tag{B.92}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\gamma_c} = \mu\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1} \geq 0 \tag{B.93}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial\alpha_c} = \mu\cdot\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\frac{1}{\alpha_b}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1} \geq 0 \tag{B.94}$$

$$\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial P_{con}} = \mu\left(\overline{C}-\hat{K}_{con}\right)\cdot\left(\frac{P_{con}^{-2}}{\alpha_b}\right)\cdot\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)^{-1} \geq 0 \tag{B.95}$$

Table B.6: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ When Conventional Farmer Adopts OT2/OT3/OT4 and $K_{org} = \overline{C}$

Parameter	Full Information: OT2/OT3/OT4	
ϵ	-	
ρ	$-$	For small enough K_0.
	$+$	For large enough K_0.
μ	$+$	
γ_K	$-$	Sufficiently large K_0.
	$+$	Sufficiently large $\hat{K}_{con}$ *and* small enough $\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)$.
$\overline{C}$	$-$	Sufficiently large K_0.
	$+$	Sufficiently large $\hat{K}_{con}$.
A_y	$-$	Sufficiently large K_0.
	$+$	Sufficiently large $\hat{K}_{con}$.
A_b	$-$	Sufficiently large K_0.
	$+$	Sufficiently large $\hat{K}_{con}$.
α_b	$-$	Sufficiently large $\hat{K}_{con}$.
	$+$	Sufficiently large K_0 *and* $P_{con}\alpha_c \to 1^+$.
γ_{cc}	$+$	
γ_c	$+$	
α_c	$+$	
P_{con}	$+$	
c_0	0	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ when $\hat{K}_j \in [0, \overline{C}]\, \forall j \in \{con, org\}$, assuming $K_{org} = \overline{C}$ (not responsive to assumption that $\overline{c} = \mu\overline{C}$ (looks the same either way)).

B.3 Discrete Analysis for OT3'

When $\gamma_{cc} = 0$ and $\mu \neq 0$, $R_{con}(K) = \rho \, \forall K$ implies:

$$-\mu + \frac{\gamma_K}{\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}} = \rho \tag{B.96}$$

$$\Rightarrow \frac{\gamma_K}{\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}} = \mu + \rho \tag{B.97}$$

$$\Rightarrow \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} = \frac{\gamma_K}{\mu + \rho} \tag{B.98}$$

B.3.1 Comparative statics for $\Delta(\epsilon)$

Now we discuss the signs of $\frac{\partial \Delta(\epsilon)}{\partial i}$, imposing the assumption that $K_{org} = \overline{C}$.

The results are summarized in Table B.7.

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \rho} = -\frac{1}{\rho^2} \cdot P_{con} \cdot \alpha_b \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right) + \left((\rho - 1) \cdot \frac{\left(\frac{\mu + \rho - 1}{\rho - 1} \right) \cdot \mu + \rho}{(\mu + \rho)^2} \right) \cdot \gamma_K \cdot \epsilon \right) \tag{B.99}$$

So we have the following *sufficient* conditions

- $\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \rho} \leq 0$ for
  - Sufficiently large $\frac{P_{org} - P_{con}}{P_{con}}$
- $\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \rho} \geq 0$ for
  - Sufficiently small $\frac{P_{org} - P_{con}}{P_{con}}$ and μ, and $\rho < 1$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \mu} = -P_{con} \cdot \alpha_b \cdot \frac{1}{(\mu + \rho)^2} \cdot \gamma_K \cdot \epsilon \leq 0 \tag{B.100}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial P_{org}} = \frac{1}{\rho} \cdot \alpha_b \gamma_K \overline{C} + \frac{1}{\rho} \alpha_b \left(A_b + \frac{A_y}{\alpha_b} \right) \geq 0 \tag{B.101}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial P_{con}} = -\frac{1}{\rho} \cdot \alpha_b \cdot \left(A_b + \frac{A_y}{\alpha_b} + \gamma_K \cdot \left(\frac{\mu}{\mu + \rho} + \underbrace{\frac{\overline{C}}{\epsilon}}_{\geq 1} - 1 \right) \cdot \epsilon \right) \leq 0 \tag{B.102}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \alpha_b} = \frac{1}{\rho} \cdot P_{con} \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \left(\gamma_K \overline{C} + A_b \right) + \frac{\rho}{\mu + \rho} \cdot \gamma_K \cdot \epsilon \right) \geq 0 \tag{B.103}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \gamma_K} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \overline{C} + \frac{\rho}{\mu + \rho} \cdot \epsilon \right) \geq 0 \tag{B.104}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \overline{C}} = \frac{1}{\rho} \left(P_{org} - P_{con} \right) \cdot \alpha_b \gamma_K \geq 0 \tag{B.105}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial A_b} = \frac{1}{\rho} \alpha_b \left(P_{org} - P_{con} \right) \geq 0 \tag{B.106}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial A_y} = \frac{1}{\rho} \left(P_{org} - P_{con} \right) \geq 0 \tag{B.107}$$

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \epsilon} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \left(1 - \underbrace{\frac{\mu}{\mu + \rho}}_{\leq 1} \right) \geq 0 \tag{B.108}$$

B.3.2 Comparative statics for ϵ^*

We now calculate the partials of ϵ^* with respect to our model parameters. We assume organic certification requires having pristine soils, such that $K_{org} = \overline{C}$.

The results are summarized in Table B.8.

Next we consider the partials of ϵ^* w.r.t. our model parameters.

Assuming $P_{con} \cdot \alpha_b \cdot \gamma_K \cdot \frac{\rho}{\mu+\rho} \cdot \neq 0$, we can then write:

$$\epsilon^* = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right) \leq 0 \tag{B.109}$$

Given

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \epsilon} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \gamma_K \left(1 - \frac{\mu}{\mu + \rho} \right) \geq 0, \tag{B.110}$$

$\epsilon^* \leq 0$ implies that the OT3' farmer prefers organic given any initial capital stock. Still, we may at some point be interested in how the value of ϵ^* responds to changes in our parameter values in this case, so we will calculate the partials of ϵ^* wrt to these model parameters.

$$\frac{\partial \epsilon^*}{\partial \rho} = \frac{1}{\rho} \cdot \underbrace{\left(\frac{\mu + \rho}{\rho} - 1 \right)}_{\geq 0} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right) \geq 0 \tag{B.111}$$

$$\frac{\partial \epsilon^*}{\partial \mu} = -\left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right) \leq 0 \tag{B.112}$$

$$\frac{\partial \epsilon^*}{\partial P_{org}} = -\frac{\mu + \rho}{\rho} \cdot \left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right) \leq 0 \tag{B.113}$$

$$\frac{\partial \epsilon^*}{\partial P_{con}} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{1}{P_{con}} + \frac{P_{org} - P_{con}}{P_{con}^2} \right) \underbrace{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)}_{\geq 0} \geq 0 \tag{B.114}$$

$$\frac{\partial \epsilon^*}{\partial \alpha_b} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\frac{A_y}{\alpha_b^2} \cdot \gamma_K^{-1} \right) \geq 0 \tag{B.115}$$

$$\frac{\partial \epsilon^*}{\partial \gamma_K} = \frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \cdot \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-2} \geq 0 \tag{B.116}$$

$$\frac{\partial \epsilon^*}{\partial \overline{C}} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \leq 0 \tag{B.117}$$

$$\frac{\partial \epsilon^*}{\partial A_b} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\gamma_K^{-1} \right) \leq 0 \tag{B.118}$$

$$\frac{\partial \epsilon^*}{\partial A_y} = -\frac{\mu + \rho}{\rho} \cdot \left(\frac{P_{org} - P_{con}}{P_{con}} \right) \left(\frac{1}{\alpha_b} \cdot \gamma_K^{-1} \right) \leq 0 \tag{B.119}$$

B.3.3 Comparative statics for threshold organic price premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$

Next we are interested in describing how large the organic price premium needs to be in order to induce a fully informed farmer facing OT3' conditions to prefer to produce organically. We derive an inequality describing the necessary conditions below.

Assuming that $P_{con} \neq 0$, and assuming that $\frac{1}{\rho}\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can write:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^* = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{B.120}$$

Given

$$\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)} = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \geq 0, \tag{B.121}$$

$\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^* \leq 0$ implies that the OT3' farmer prefers organic given any non-negative price premium. Still, we may at some point be interested in how the value of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ responds to changes in our parameter values in this case, so we will calculate the partials of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ wrt to these model parameters.

Then we can determine how $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ changes in response to changes in our model parameters by examining the signs of the partials below. The results are summarized in Table B.9.

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \epsilon} = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \leq 0 \tag{B.122}$$

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \rho} = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\mu}{(\mu + \rho)^2} \cdot \epsilon \leq 0 \tag{B.123}$$

$$\frac{\partial \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial \mu} = \frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{(\mu + \rho)^2} \cdot \epsilon \geq 0 \tag{B.124}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial P_{con}} = 0 \tag{B.125}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial \alpha_b} = \frac{1}{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)^2} \cdot \left(-\frac{A_y}{\alpha_b^2} \cdot \gamma_K^{-1} \right) \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{B.126}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial \gamma_K} = \frac{1}{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)^2} \cdot \left(- \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-2} \right) \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{B.127}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial A_y} = \frac{1}{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)^2} \cdot \left(\frac{1}{\alpha_b} \cdot \gamma_K^{-1} \right) \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \geq 0 \tag{B.128}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial A_b} = \frac{1}{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)^2} \cdot \gamma_K^{-1} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \geq 0 \tag{B.129}$$

$$\frac{\partial \left(\frac{P_{org} - P_{con}}{P_{con}} \right)^*}{\partial \overline{C}} = \frac{1}{\left(\overline{C} + \left(A_b + \frac{A_y}{\alpha_b} \right) \cdot \gamma_K^{-1} \right)^2} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \geq 0 \tag{B.130}$$

Table B.7: Comparative Statics for $\Delta^{OT3'}(\epsilon)$ For Conventional OT3' Farmer When $K_{org} = \overline{C}$

Summary of $\frac{\partial \Delta^{OT3'}(\epsilon)}{\partial i}$		
i	Sign	Condition
ρ	$-$	Large $\frac{P_{org}-P_{con}}{P_{con}}$.
	$+$	Small $\frac{P_{org}-P_{con}}{P_{con}}$ and μ, and ρ¡1.
μ	-	
P_{org}	$+$	
P_{con}	-	
α_b	$+$	
α_c	N/A	N/A
γ_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	$+$	
A_y	$+$	
A_b	$+$	
$\overline{C}$	$+$	
ϵ	$+$	

Notes: Table reports comparative statics for $\Delta(\epsilon)$, assuming that $K_{org} = \overline{C}$ and $R_{con}(K) = \rho \forall K$, and assuming $K_{org} = \overline{C}$ (Does not depend on whether or not $\overline{c} = \mu\overline{C}$). A conventional farmer is said to prefer producing organically when $\Delta(\epsilon) > 0$.

* Does not depend on whether or not $\overline{c} = \mu\overline{C}$

Table B.8: Comparative Statics for ϵ^* For Conventional OT3' Farmer When $K_{org} = \overline{C}$

Summary of $\frac{\partial \epsilon^*}{\partial i}$		
i	Sign	Condition
ρ	+	
μ	-	
P_{org}	-	
P_{con}	+	
α_b	+	
α_c	N/A	N/A
α_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	+	
A_y	-	
A_b	-	
c_0	N/A	N/A
C_0	N/A	N/A
$\overline{C}$	-	

Notes: Table reports comparative statics for ϵ^* assuming that $K_{org} = \overline{C}$ and $R_{con}(K) = \rho \forall K$, and assuming $K_{org} = \overline{C}$ (Does not depend on whether or not $\overline{c} = \mu\overline{C}$). A conventional farmer will prefer producing organically for all $K_{0,con} = K_{org} - \epsilon$ at least ϵ^* lower than K_{org}.

Table B.9: Comparative Statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ For Conventional OT3' Farmer When $K_{org}=\overline{C}$

Summary of $\frac{\partial\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*}{\partial i}$		
i	Sign	Condition
ϵ	-	
ρ	-	
μ	+	
P_{con}	0	
α_b	-	
α_c	N/A	N/A
γ_c	N/A	N/A
γ_{cc}	N/A	N/A
γ_K	-	
A_y	+	
A_b	+	
c_0	N/A	N/A
C_0	N/A	N/A
$\overline{C}$	+	

Notes: Table reports comparative statics for $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*$ assuming that $K_{org}=\overline{C}$ and $R_{con}(K)=\rho\forall K$, and assuming $K_{org}=\overline{C}$ (Does not depend on whether or not $\overline{c}=\mu\overline{C}$).

* Does not depend on whether or not $\overline{c}=\mu\overline{C}$

C Investment Under Uncertainty

C.1 Threshold organic premium under no uncertainty

When there is no uncertainty about P_{org} and P_{org} is not stochastic but instead known and fixed, the threshold $P^*_{org,det}(K)$ is given by:

$$P^*_{org,det}(K) = \frac{V_{con}(K)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{C.1}$$

where $V_{con}(K)$ comes from our solutions to the conventional problem.

We now confirm, for each of OT1, OT2/OT3/OT4, and OT3', that when $K_{org} = \overline{C}$, $P^*_{org,det}(K)$ yields the same threshold organic premium $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ that we previously derived in our local discrete analysis when there is no uncertainty and when $K_{org} = \overline{C}$.

C.1.1 Conventional OT1 farmer

Previously we found that when there is no uncertainty and $K_{org} = \overline{C}$, the range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ for a convetional OT1 farmer is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho \cdot \epsilon\right)}{\gamma_K \cdot \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.2}$$

From our OT1 analysis we know that:

$$V_{con}(K) = \frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu+\rho)}\gamma_K K_0 + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right) \tag{C.3}$$

and so $P^*_{org}(K)$ can be written as

$$P^*_{org,det}(K) = \frac{\frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu+\rho)}\gamma_K K_0 + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{C.4}$$

$$P^*_{org,det}(K) = \frac{P_{con} \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.5}$$

From this equation we can derive an organic price premium needed to induce a preference for organic management:

$$P^*_{org,det}(K) = \frac{P_{con}\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.6}$$

$$\frac{P^*_{org,det}(K)}{P_{con}} = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.7}$$

$$\frac{P^*_{org,det}(K)}{P_{con}} - 1 = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 \tag{C.8}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} - \gamma_K\overline{C} - A_b - \frac{A_y}{\alpha_b}}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.9}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K\overline{C} + \frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot K}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.10}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\overline{C} - \frac{\rho}{(\mu+\rho)} \cdot K\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.11}$$

If we define K in terms of its distance from $\overline{C}$, ϵ, such that $K = \overline{C} - \epsilon$ we then have:

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\overline{C} - \frac{\rho}{\mu+\rho} \cdot \left(\overline{C} - \epsilon\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.12}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\frac{\mu+\rho}{\mu+\rho} \cdot \overline{C} - \frac{\rho}{\mu+\rho} \cdot \left(\overline{C} - \epsilon\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.13}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \overline{C} + \frac{\rho}{\mu+\rho} \cdot \epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.14}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} - \frac{1}{\mu+\rho}\cdot\gamma_K\cdot\left(\mu\cdot\overline{C} + \rho\cdot\epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.15}$$

Note that this organic price premium is the same as the $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ that we derived in our OT1 local discrete analysis when $K_{org} = \overline{C}$.

C.1.2 Conventional OT2/OT3/OT4 farmer

Previously we found that when there is no uncertainty and $K_{org} = \overline{C}$, the range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ for OT2/OT3/OT4 is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\frac{1}{2}\cdot\frac{1}{(-\gamma_{cc})}\cdot\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\mu+\rho)}\right)^2 - \frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.16}$$

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\frac{1}{2}\cdot\frac{\alpha_b}{(-\gamma_{cc})}\cdot\frac{1}{\rho}\cdot\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{\gamma_K}{(\mu+\rho)}\right)^2 - \frac{1}{\mu+\rho}\cdot\alpha_b\gamma_K\cdot\epsilon}{\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.17}$$

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = \frac{\frac{1}{\rho}\cdot\alpha_b\cdot\frac{1}{2}\cdot(-\gamma_{cc})\cdot\mu^2\cdot\left(\hat{K}_{con} - \overline{C}\right)^2 - \frac{1}{\mu+\rho}\cdot\alpha_b\gamma_K\cdot\epsilon}{\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.18}$$

From our OT2/OT3/OT4 analysis we know that:

$$\begin{aligned} V_{con}(K) = P_{con}\alpha_b\Bigg(&\frac{1}{\rho}\cdot\left(A_b + \frac{A_y}{\alpha_b}\right) + \frac{\mu}{\rho}\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\overline{C} \\ &+\frac{1}{2}\cdot\frac{1}{\rho}\left(\gamma_{cc}\left(\mu\overline{C}\right)^2 - \frac{1}{\gamma_{cc}}\left(\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} - \frac{1}{(\rho+\mu)}\gamma_K\right)^2\right) + \frac{\gamma_K}{\mu+\rho}\cdot K_{org} - \frac{\gamma_K}{\mu+\rho}\cdot\epsilon\Bigg) \end{aligned}$$

or

$$\begin{aligned} &V_{con}(K) = \\ &\quad\frac{1}{\rho}\cdot P_{con}\alpha_b\left(\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right) \end{aligned} \tag{C.19}$$

and so $P^*_{org}(K)$ can be written as

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con}\alpha_b \left(\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right) \right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b} \right)} \tag{C.20}$$

$$P^*_{org}(K) = \frac{P_{con} \left(\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right) \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.21}$$

From this equation we can derive an organic price premium needed to induce a preference for organic management:

$$P^*_{org}(K) = \frac{P_{con} \left(\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right) \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.22}$$

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.23}$$

$$\begin{aligned} \frac{P^*_{org}(K)}{P_{con}} - 1 = & \frac{\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K_0 + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \\ & - \frac{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \end{aligned} \tag{C.24}$$

If we define K_0 in terms of its distance from $\overline{C}$, ϵ, such that $K_0 = \overline{C} - \epsilon$ we then have:

$$\begin{aligned} & \frac{P^*_{org}(K) - P_{con}}{P_{con}} = \\ & \frac{\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon \right) + \frac{\mu}{\mu+\rho} \cdot \gamma_K \cdot \overline{C} - \frac{\mu}{\mu+\rho} \cdot \gamma_K \cdot \overline{C} + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b} \right) \cdot \mu \overline{C} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con} \right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \\ & - \frac{\gamma_K \overline{C}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \end{aligned} \tag{C.25}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon-\frac{\mu}{\mu+\rho}\cdot\gamma_K\cdot\overline{C}+\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.26}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}-\frac{1}{\mu+\rho}\cdot\gamma_K\right)\cdot\mu\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.27}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\left(\frac{(\mu+\rho)\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\mu+\rho)}\right)\cdot\mu\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.28}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\left(\frac{(\mu+\rho)\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\mu+\rho)\mu\gamma_{cc}}\right)\cdot\gamma_{cc}\mu^2\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.29}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\left(\frac{(\mu+\rho)\left(\gamma_{cc}\mu\overline{C}+\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\mu+\rho)\mu\gamma_{cc}}-\overline{C}\right)\cdot\gamma_{cc}\mu^2\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.30}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\left(\hat{K}_{con}-\overline{C}\right)\cdot\gamma_{cc}\mu^2\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.31}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\frac{1}{2}\cdot(-\gamma_{cc})\,\mu^2\left(2\left(\overline{C}-\hat{K}_{con}\right)\cdot\overline{C}-\left(\overline{C}^2-\hat{K}_{con}^2\right)\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.32}$$

$$\frac{P_{org}^{*}(K)-P_{con}}{P_{con}}=\frac{\frac{1}{2}\cdot(-\gamma_{cc})\,\mu^2\left(\overline{C}^2-2\hat{K}_{con}\cdot\overline{C}+\hat{K}_{con}^2\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.33}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\frac{1}{2} \cdot (-\gamma_{cc})\, \mu^2 \cdot \left(\hat{K}_{con} - \overline{C}\right)^2 - \frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \epsilon}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.34}$$

Note that this organic price premium is the same as the $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ that we derived in our OT2/OT3/OT4 local discrete analysis when $K_{org} = \overline{C}$.

C.1.3 Conventional OT3' farmer

Previously we found that when there is no uncertainty and $K_{org} = \overline{C}$, the range of $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)$ yielding $\Delta(\epsilon) \geq 0$ for OT3' is $\frac{P_{org}-P_{con}}{P_{con}} \geq \left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$, where:

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \leq 0 \tag{C.35}$$

From our OT3' analysis we know that:

$$G_{con}(K) = P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu + \rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right) \tag{C.36}$$

$$V_{con}(K) = \frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu + \rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right) \tag{C.37}$$

and so $P^*_{org,det}(K)$ can be written as

$$P^*_{org,det}(K) = \frac{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.38}$$

$$P^*_{org,det}(K) = \frac{P_{con} \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.39}$$

From this equation we can derive an organic price premium needed to induce a preference for organic management:

$$P^*_{org,det}(K) = \frac{P_{con} \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.40}$$

$$\frac{P^*_{org,det}(K)}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.41}$$

$$\frac{P^*_{org,det}(K)}{P_{con}} - 1 = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.42}$$

If we define K_0 in terms of its distance from $\overline{C}$, ϵ, such that $K_0 = \overline{C} - \epsilon$ we then have:

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - \overline{C} + \epsilon) + K\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K \overline{C}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.43}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \epsilon + K - \overline{C}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} \tag{C.44}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\frac{\mu}{\mu+\rho} \cdot \epsilon + K - \overline{C}}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}}. \tag{C.45}$$

Since in case OT3', when K follows the farmer's optimal trajectory we will have $K(t) = K_0 \equiv \overline{C} - \epsilon \, \forall t$, the equation above can be expressed as

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\frac{\mu}{\mu+\rho} \cdot \epsilon + \overline{C} - \epsilon - \overline{C}}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}}, \tag{C.46}$$

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{\frac{\mu}{\mu+\rho} \cdot \epsilon - \frac{\mu+\rho}{\mu+\rho} \cdot \epsilon}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}}, \tag{C.47}$$

or

$$\frac{P^*_{org,det}(K) - P_{con}}{P_{con}} = \frac{-\frac{\rho}{\mu+\rho} \cdot \epsilon}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \tag{C.48}$$

$$\left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu + \rho} \cdot \epsilon \tag{C.49}$$

Note that this organic price premium is the same as the $\left(\frac{P_{org}-P_{con}}{P_{con}}\right)^*_{deterministic}$ that we derived in our OT3' local discrete analysis when $K_{org} = \overline{C}$.

C.2 Threshold organic premium under uncertainty

Now we will analyze the uncertainty case where P_{org} is stochastic. Let's assume that P_{org} evolves as a first-order Markov process $P'_{org} \overset{iid}{\sim} F_{P_{org}}(\cdot|P_{org})$.

For simplicity we also assume that $a_t = 0$ does not give the farmer information about the distribution of P_{org} because, for example, the distribution of P_{org} is known to the farmer ahead of the starting period and is not affected by their adoption decision in the current period.

Assuming that $\alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right) \neq 0$, we can find a value of P_{org} for each K at which the value of continuing to produce conventionally is equal to the value of producing organically. We will denote this value as $P^*_{org}(K)$.

$$\begin{aligned} P^*_{org}(K) = & \frac{V_{con}(K)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \\ & + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \end{aligned} \tag{C.50}$$

where $K^*(t)$ is the solution for a conventional OT1 farmer; where $T_{org}(P_{org}, K)$ is the time at which the farmer adopts organic (i.e., the first time t when $a(P_{org}, K) = 1$, and therefore the first time t when $V_{org}(P_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]$); and where a conventional farmer will adopt organic ($a = 1$) the first time when:

$$a(P_{org}, K) = \mathbf{1}\{V_{org}(P_{org}) > G^*_{con}(K) + \beta \cdot \mathbb{E}\left[v(P'_{org}, K') \mid P_{org}, K\right]\} \tag{C.51}$$

C.2.1 Conventional OT1 farmer

Based on our OT1 solution, we know that we can write:

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con}\alpha_b \cdot \left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.52}$$

$$P^*_{org}(K) = \frac{P_{con}\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.53}$$

From this equation we can write an expression for the organic price premium required to induce adoption of organic management when the farmer faces uncertainty in the value of P_{org}:

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.54}$$

$$\frac{P^*_{org}(K)}{P_{con}} - 1 = \frac{\left(\frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot (K) + \left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.55}$$

$$\frac{P^*_{org}(K)}{P_{con}} - \frac{P_{con}}{P_{con}} = \frac{\left(\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + \frac{\rho}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\overline{C} - \epsilon\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.56}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \gamma_K\overline{C} + \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\rho\overline{C} - \rho\epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.57}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \frac{\left(\frac{1}{2}\gamma_{cc}\mu\overline{C} + \gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} - \frac{1}{(\mu+\rho)} \cdot \gamma_K \cdot \left(\mu\overline{C} + \rho\epsilon\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.58}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.59}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \underbrace{\frac{\underbrace{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}_{\geq 0}}{\underbrace{P_{con} \cdot \dfrac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \dfrac{A_y}{\alpha_b}\right)}_{\geq 0}}}_{\geq 0} \tag{C.60}$$

C.2.2 Conventional OT2/OT3/OT4 farmer

Based on our OT2/OT3/OT4 solution, we know that we can write:

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con}\alpha_b \left(\frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot K + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right) \cdot \mu\overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2} \cdot \gamma_{cc}\mu^2 \left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.61}$$

$$P^*_{org}(K) = \frac{P_{con}\left(\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot K + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \quad (C.62)$$

From this equation we can write an expression for the organic price premium required to induce adoption of organic management when the farmer faces uncertainty in the value of P_{org}:

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\left(\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot K + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \quad (C.63)$$

$$\frac{P^*_{org}(K)}{P_{con}} - 1 = \frac{\left(\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot K + \left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \quad (C.64)$$

$$\frac{P^*_{org}(K)}{P_{con}} - \frac{P_{con}}{P_{con}} = \frac{\left(\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} + \frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\left(\overline{C} - \epsilon\right) + A_b + \frac{A_y}{\alpha_b} + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right.}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \quad (C.65)$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\left(\left(\gamma_c + \frac{\alpha_c - P_{con}^{-1}}{\alpha_b}\right)\cdot\mu\overline{C} - \frac{\mu}{\mu+\rho}\cdot\gamma_K\cdot\overline{C} - \frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon + \frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2 - \hat{K}^2_{con}\right)\right)}{\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \quad (C.66)$$

$$\frac{P^*_{org}(K)-P_{con}}{P_{con}} = \frac{\left(\left(\frac{(\mu+\rho)\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\mu+\rho)}\right)\cdot\mu\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}^2_{con}\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.67}$$

$$+\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t),K^*(t))))-V_{con}(K^*(T_{org}(P_{org}(t),K^*(t))))\mid P_{org},K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}$$

$$\frac{P^*_{org}(K)-P_{con}}{P_{con}} = \tag{C.68}$$

$$\frac{\left(\left(\frac{(\mu+\rho)\left(\gamma_c+\frac{\alpha_c-P_{con}^{-1}}{\alpha_b}\right)-\gamma_K}{(\mu+\rho)\mu\gamma_{cc}}+\overline{C}-\overline{C}\right)\cdot\gamma_{cc}\mu^2\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}^2_{con}\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}}$$

$$+\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t),K^*(t))))-V_{con}(K^*(T_{org}(P_{org}(t),K^*(t))))\mid P_{org},K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}$$

$$\frac{P^*_{org}(K)-P_{con}}{P_{con}} = \frac{\left(\left(\hat{K}_{con}-\overline{C}\right)\cdot\gamma_{cc}\mu^2\overline{C}+\frac{1}{2}\cdot\gamma_{cc}\mu^2\left(\overline{C}^2-\hat{K}^2_{con}\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.69}$$

$$+\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t),K^*(t))))-V_{con}(K^*(T_{org}(P_{org}(t),K^*(t))))\mid P_{org},K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}$$

$$\frac{P^*_{org}(K)-P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\cdot(-\gamma_{cc})\,\mu^2\cdot\left(2\cdot\left(\overline{C}-\hat{K}_{con}\right)\cdot\overline{C}-\left(\overline{C}^2-\hat{K}^2_{con}\right)\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.70}$$

$$+\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t),K^*(t))))-V_{con}(K^*(T_{org}(P_{org}(t),K^*(t))))\mid P_{org},K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}$$

$$\frac{P^*_{org}(K)-P_{con}}{P_{con}} = \frac{\left(\frac{1}{2}\cdot(-\gamma_{cc})\,\mu^2\cdot\left(\overline{C}^2-2\hat{K}_{con}\overline{C}+\hat{K}^2_{con}\right)-\frac{\rho}{\mu+\rho}\cdot\gamma_K\cdot\epsilon\right)}{\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}} \tag{C.71}$$

$$+\frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t),K^*(t))))-V_{con}(K^*(T_{org}(P_{org}(t),K^*(t))))\mid P_{org},K\right]}{P_{con}\cdot\frac{1}{\rho}\cdot\alpha_b\left(\gamma_K\overline{C}+A_b+\frac{A_y}{\alpha_b}\right)}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\frac{1}{2} \cdot (-\gamma_{cc})\, \mu^2 \cdot \left(\hat{K}_{con} - \overline{C}\right)^2 - \frac{\rho}{\mu+\rho} \cdot \gamma_K \cdot \epsilon}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.72}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.73}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \underbrace{\frac{\underbrace{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}_{\geq 0}}{\underbrace{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0}}}_{\geq 0} \tag{C.74}$$

C.2.3 Conventional OT3' farmer

Based on our OT3' solution, we know that we can write:

$$P^*_{org}(K) = \frac{\frac{1}{\rho} \cdot P_{con} \cdot \alpha_b \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.75}$$

$$P^*_{org}(K) = \frac{P_{con} \cdot \left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{\frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.76}$$

From this equation we can write an expression for the organic price premium required to induce adoption of organic management when the farmer faces uncertainty in the value of P_{org}:

$$\frac{P^*_{org}(K)}{P_{con}} = \frac{\left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \left(\overline{C} - K_0\right) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.77}$$

$$\frac{P^*_{org}(K)}{P_{con}} - 1 = \frac{\left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \left(\overline{C} - K_0\right) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} - 1 + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.78}$$

$$\frac{P^*_{org}(K)}{P_{con}} - \frac{P_{con}}{P_{con}} = \frac{\left(\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \left(\overline{C} - K_0\right) + K\right) + A_b + \frac{A_y}{\alpha_b}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} - \frac{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.79}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \left(\overline{C} - K_0\right) + K - \overline{C}\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}. \tag{C.80}$$

Given that for the OT3' solution $K(t) = K_0 \, \forall t$ the equation above can be expressed as:

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot \left(\overline{C} - K_0\right) - \left(\overline{C} - K_0\right)\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}, \tag{C.81}$$

or

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(\frac{\mu}{\mu+\rho} \cdot (\overline{C} - K_0) - \frac{\mu+\rho}{\mu+\rho} \cdot (\overline{C} - K_0)\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}, \tag{C.82}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = \frac{\gamma_K \cdot \left(-\frac{\rho}{\mu+\rho} \cdot \epsilon\right)}{\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}, \tag{C.83}$$

$$\frac{P^*_{org}(K) - P_{con}}{P_{con}} = -\frac{1}{\overline{C} + \left(A_b + \frac{A_y}{\alpha_b}\right) \cdot \gamma_K^{-1}} \cdot \frac{\rho}{\mu+\rho} \cdot \epsilon + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}, \tag{C.84}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \frac{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)} \tag{C.85}$$

$$\left(\frac{P_{org}(K) - P_{con}}{P_{con}}\right)^*_{uncertainty} = \left(\frac{P_{org} - P_{con}}{P_{con}}\right)^*_{deterministic} + \underbrace{\frac{\underbrace{\mathbb{E}\left[V_{org}(P_{org}(T_{org}(P_{org}(t), K^*(t)))) - V_{con}(K^*(T_{org}(P_{org}(t), K^*(t)))) \mid P_{org}, K\right]}_{\geq 0}}{\underbrace{P_{con} \cdot \frac{1}{\rho} \cdot \alpha_b \left(\gamma_K \overline{C} + A_b + \frac{A_y}{\alpha_b}\right)}_{\geq 0}}}_{\geq 0} \tag{C.86}$$