Insights from a Simple Hotelling Model of the World Oil Market

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Abstract

This paper uses annual data on world oil price and consumption from 1965 to 2006 to calibrate a Hotelling model of optimal nonrenewable resource extraction. Numerical solutions are generated for various specifications of the elasticity of demand for both isoelastic demand and linear demand under each of two possible market structures: perfect competition and monopoly. Prior to the 1973 oil crisis, the model that best fits actual data is one of perfect competition with linear demand and a demand elasticity of -0.4. For the periods 1973-1981 and 1981-1990, the model that best fits actual data is one of monopoly with linear demand and demand elasticities of -0.8 and -0.7, respectively, suggesting that the market was strongly influenced by OPEC during this time. Under the model that best fits the most recent period (perfect competition with linear demand and demand elasticity -0.5), the real oil price (in 1982-1984 U.S. $) should fall in the range $60.87-$66.31/barrel over the years 2010-2030.

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1 INTRODUCTION

The problem of optimal nonrenewable resource extraction was first examined by Hotelling (1931), whose basic model predicted that the shadow price of the resource stock, which is an economic measure of the scarcity of the resource, should grow at the rate of interest. Since then, economists have expanded Hotelling's basic theoretical framework to allow for more realistic features such as increasing extraction costs (Hanson, 1980; Solow & Wan, 1976), unlimited potential reserves (Pindyck, 1978), exploration (Pesaran, 1990; Pindyck, 1978), market imperfections (see Cremer & Salehi-Isfahani, 1991 and references therein; Khalatbari, 1977; Stiglitz, 1976; Sweeney, 1977), Nash-Cournot behavior (Salant, 1976; Ulph & Folie, 1980), OPEC (Cremer & Weitzman, 1976; Hnyilicza & Pindyck, 1976; Pindyck, 1976; Lin, 2008a), technological progress (Farzin, 1992, 1995; Lin et al., 2008; Lin & Wagner, 2007), outward-shifting demand (Chapman, 1993; Chapman & Khanna, 2000), and uncertainty (Hoel, 1978; Pindyck, 1980).

Even with the addition of more realistic features, it is unclear if the Hotelling model explains the real world (Watkins, 1992). Like most classical economic theories, Hotelling's theory of optimal nonrenewable resource extraction assumes economically rational, wealth-maximizing behavior on the part of all agents involved. In a 1980-1981 study by Stanford University's Energy Modeling Forum of ten prominent models of the world oil market, the intertemporal optimization models, which assumed perfect foresight on the part of the agents, performed worse in predicting 1990 world oil market conditions than did the recursive simulation models (Huntington, 1994). Moreover, the one model that assumed wealth-maximizing OPEC behavior had difficulty explaining the actual initial price level: the model's price for 1980 was 28 percent above its actual level (Gately, 1984). Thus, the Hotelling-type models in the study did not appear to describe the real world behavior of the world oil market, even despite realistic features such as an inclusion of energy-saving trends in technology and a separation of the producers into those inside the cartel and those outside the cartel.

Would Hotelling's model still perform poorly in explaining the world oil market if we added more recent data from the quarter century that has passed since the first Energy Modeling Forum study? It is possible that, with a longer time series, the data can be better reconciled with the theory. For example, some empirical studies have found evidence for a U-shaped price path (see e.g., Slade, 1982; Moazzami & Anderson, 1994), therefore suggesting that resource prices will eventually increase in accordance with Hotelling's theory; to detect this phenomenon one would need a long time series that included recent data. It is also possible that the Hotelling model might explain the data over short intervals of time, when technology, discoveries of new reserves and changes in market power would not be expected to influence optimal extraction rates.

This paper uses data on world oil price and consumption from 1965 to 2006 to calibrate a basic Hotelling model of optimal oil extraction. The focus is on a very basic Hotelling model, without more realistic features, in order to obtain an upper bound on how poorly the Hotelling model performs. The calibrated model is used to simulate solution trajectories under both perfect competition and monopoly for various specifications of the elasticity of demand for both isoelastic demand and linear demand.

This paper follows most closely the work of Pindyck (1978), who first develops a theoretical model that allows for unlimited potential reserves and that requires resource producers to simultaneously determine optimal rates of exploration and production, and who then examines numerically the characteristics of the competitive and monopoly solutions to his model using data for oil in the Permian region of Texas over the period 1965-1974.

Though this paper also examines both competitive and monopoly solutions and allows for unlimited reserves, it differs from that of Pindyck (1978) in several ways. (1) The world oil data used to calibrate the model for 1965-2006 spans a wider geographic area and more years than does Pindyck's data. (2) To gauge whether or not Hotelling's simpler model can explain historical data, Hotelling's basic model allowing producers to choose extraction rates only is utilized, compared with Pindyck's more complex – and perhaps more realistic – model allowing producers to choose both exploration rates and extraction rates. (3) Both linear demand and isoelastic demand are utilized, whereas Pindyck utilizes only linear demand. (4) Analyses are performed for a range of demand price elasticities, while Pindyck uses just one price elasticity. (5) Price and extraction trajectories are compared with historical trajectories and model parameters are varied in an attempt to match the data, whereas Pindyck compares the optimal values of well drilling and price to historical time series only for the single set of model parameters used.[2]

Previous empirical studies of the Hotelling model, though relatively few, generally find the theory a poor depiction of the real world. As noted by Krautkraemer (1998, p. 2087): "There is strong empirical evidence that the basic Hotelling model of finite availability of nonrenewable resources does not adequately explain the observed behavior of nonrenewable prices, extraction and in situ values." These studies include empirical tests of the dynamic efficiency conditions of the Hotelling model under perfect competition that have been applied to data on a hard rock mining firm (Farrow, 1985), on Canadian metal mining firms (Halvorsen & Smith, 1991), and on Canadian copper mining firms (Young, 1992). Miller and Upton (1985) use data on U.S. domestic oil- and gas-producing companies to test another reduced-form implication of a Hotelling model. This reduced-form implication, which they term the "Hotelling Valuation Principle", is that the value of a unit of reserves in the ground is the same as its current value above the ground less the marginal costs of extracting it.

Although previous empirical studies of the Hotelling model have tended to find the theory lacking, and although very few empirical studies of the Hotelling model have been done in the context of the world oil industry, one exception is Lin and Wagner (2007), who combine stock effects and technological progress to

[2] Pindyck also compares the optimal and actual values to the myopic values that would occur if future depletion were ignored but the reserve-production ratio were maintained at its optimal level.

develop a theoretical Hotelling model consistent with the often-cited stylized fact that mineral prices have remained constant over a long period of time. Empirical tests of their model using data from 1970 to 2004 find the oil market consistent with their theoretical model.

The study in this paper differs from the previous studies of the Hotelling model because it employs an updated data set on world oil and because it allows for market structure and demand elasticities to differ among the subperiods 1965-1973 (the pre-1973 oil crisis period), 1973-1981 (the period between oil crises), 1981-1990 (the period immediately following the 1981 oil crisis), and 1990-2006 (the most recent period). This paper therefore examines whether using more recent data and examining short intervals of time can rescue Hotelling's theory. Does the simple Hotelling model yield any useful insights?

Prior to the 1973 oil crisis, the model that best fits actual data is one of perfect competition with linear demand and a demand elasticity of -0.4. For the periods 1973-1981 and 1981-1990, the model that best fits actual data is one of monopoly with linear demand and demand elasticities of -0.8 and -0.7, respectively, suggesting that the market was strongly influenced by OPEC during this time. Under the model that best fits the most recent period (perfect competition with linear demand and demand elasticity -0.5), the real oil price (in 1982-1984 U.S. $) should fall in the range $60.87-$66.31/barrel over the years 2010-2030.

The balance of this paper proceeds as follows. Section 2 presents the basic Hotelling model. Section 3 describes the data and explains the functional form assumptions and calibration methods used for the simulations. Results are presented in Section 4. Section 5 concludes.

2 THE BASIC HOTELLING MODEL

This section presents the theoretical model of optimal nonrenewable resource extraction under both perfect competition and monopoly. The notation follows closely that used by Weitzman (2003).

Let $t \in [0, T]$ index time. At time t, the supply of oil is given by $E(t)$, the total extraction flow in units of oil per unit time at time t.[3] Let $X(t)$ denote the total cumulative stock of oil extracted at time t:

$$X(t) = X(0) + \int_0^t E(\tau) d\tau \quad , \tag{1}$$

where the initial stock $X(0)$ is taken as given. No fixed quantity is assumed for the total availability of the resource, although, as shown in Farzin (1992), only a limited total amount will be economically recoverable at any time. Unlimited potential reserves is a realistic depiction of the real world, since technological progress

[3] I assume that, at any given time t, all the oil extracted at time t is sold on the market at time t.

and new discoveries will always make more reserves available and feasible for extraction. As Adelman (1992, p. 50) writes: "We will never get to the end of our oil resources. We will stop impounding them into reserves when it no longer pays."

The market price of oil at time t is $P(t)$. The demand for oil when the market price is P is given by the demand function $D(P)$. At each time t, the price $P(t)$ adjusts to equate supply and demand:

$$E(t) = D(P(t)) \;\; \forall t. \tag{2}$$

The cost of extracting E units of oil when the total stock already extracted is X is given by $C(X, E)$. The term "stock effects" is used to refer to the dependence of extraction cost on the stock X of reserve extracted. There are several possible reasons why this dependence is positive. First, extraction costs may increase with the cumulative stock extracted if the resource needed to be extracted from greater depths as it was being depleted. Second, costs may increase if well pressure declined as more of the reserve was depleted. Third, since different grades of oil may differ in their extraction costs, and since the cheaper grades are likely to be mined to exhaustion before the more expensive grades are mined, the cost of extraction may increase as the cheaper grades are exhausted, and therefore as the total stock already extracted increased. As the stock of oil is being depleted, the industry moves towards increasingly costly sources of oil, including those in remote locations and deep water, and towards expensive unconventional sources such as oil shale and tar sands.

Let $p(t)$ denote the non-negative current-value shadow price measuring the value of a unit of reserve at time t. This shadow price is known by a variety of terms, including "marginal user cost", because it measures the opportunity cost of extracting the resource; "in situ value", because it measures the marginal value of leaving an additional unit of resource in the ground; "scarcity rent", because it is an economic measure of scarcity, and "dynamic rent", to reflect the difference between price and marginal extraction cost (Devarajan & Fisher, 1982; Krautkraemer, 1998; Weitzman, 2003).[4] The competitive interest rate is ρ.

The producer's optimal nonrenewable resource extraction problem is to choose the extraction profile $\{E(t)\}$ to maximize the present discounted value of the entire stream of per-period net benefits $G(X, E)$, given her initial stock $X(0)$ and given the relationship between her extraction $E(t)$ and the cumulative stock extracted $X(t)$, and subject to the constraints that both extraction and stock are nonnegative. Her problem is thus given by:

[4] Much work has also been done in attempt to measure the shadow price of the resource (see e.g., Devarajan & Fisher, 1982; Halvorsen & Smith, 1984; Lasserre, 1985).

$$\max_{\{E(t)\}} \int_0^\infty G(X(t), E(t)) e^{-\rho t} dt$$

$$\begin{aligned} \text{subject to} \qquad & \tfrac{dX}{dt}(t) = E(t) \qquad : q(t) \\ & E(t) \geq 0 \\ & X(t) \geq 0 \\ & X(0) = X_0, \end{aligned} \tag{3}$$

where $q(t) \leq 0$ is the multiplier associated with the equation of motion for the total stock $X(t)$ of oil extracted. The absolute value of this multiplier is precisely the shadow price $p(t)$ of the reserve:

$$p(t) = |q(t)|\,. \tag{4}$$

When the oil market is perfectly competitive, extraction $E(t)$ represents the total amount of oil extracted by all the firms in the market at a given point in time.[5] Under perfect competition, the per-period net benefits $G(X, E)$ from extracting E units of oil when the total stock already extracted is X is given by total benefits minus total costs:

$$G(X, E) = U(E) - C(X, E). \tag{5}$$

Assuming that the social and private discount rates are the same and that there are no externalities, the social planner's optimal control problem yields the same solution as would arise in perfect competition. In this case, under the additional assumption that the marginal utility of income is constant, the total benefits $U(E(t))$ from oil at time t is given by the area under the demand curve:

$$U(E(t)) = \int_0^{E(t)} D^{-1}(x) dx. \tag{6}$$

where $D^{-1}(E)$ is the inverse of the demand curve with respect to price. This area measures the gross consumer surplus, and is a measure of the consumers' willingness-to-pay for the resource. As shown in Weitzman (2003), using the area under the demand curve in place of revenue yields the same outcome as a perfectly competitive market.[6] Thus, in the absence of externalities, a perfectly competitive market maximizes total utility, or what

[5] Any common access problems that may arise in perfect competition are ignored. In other words, I assume, as does Pindyck (1978), that there is a large number of identical firms that all ignore each other, or, equivalently, that a social planner or a state-owned company has sole production rights and sets a competitive price.

[6] This holds because, assuming constant marginal utility of income:

Hotelling (1931) terms the "social value of the resource".

When oil is produced by a single monopolist or by a group of colluding joint profit maximizing producers, rather than by a multitude of perfectly competitive producers, the per-period net benefits $G(X, E)$ are given by the monopolist's per-period profit, which equals total revenue minus total costs. Total revenue $\Phi(E(t))$ at time t is given by:

$$\Phi(E(t)) = D^{-1}(E(t)) \cdot E(t). \tag{7}$$

As a consequence, the monopolist's per-period profit $G(X, E)$ is given by:

$$G(X, E) = \Phi(E) - C(X, E). \tag{8}$$

From the Maximum Principle, the first-order necessary conditions for a feasible trajectory $\{X^*(t), E^*(t)\}$ to be optimal are:[7]

$$[\#1]: \qquad p(t) = \frac{\partial G\left(X(t), E(t)\right)}{\partial E} \tag{9}$$

$$[\#2] \quad : \qquad \frac{dp}{dt}(t) = -\frac{\partial C\left(X(t), E(t)\right)}{\partial X} + \rho p(t) \tag{10}$$

$$[\#3]: \qquad \lim_{t \to \infty} p(t) X(t) e^{-\rho t} = 0 \tag{11}$$

Condition [#1] states that, at each time t, the shadow price $p(t)$ must equal the competitive market price $P(t)$ minus the marginal cost of extraction $\frac{\partial C(X(t),E(t))}{\partial E}$ under perfect competition and marginal revenue $\Phi'(E(t))$ minus marginal cost $\frac{\partial C(X(t),E(t))}{\partial E}$ under monopoly; this condition is needed to ensure static optimality at each point in time. Condition [#2] governs how the shadow price $p\left(t\right)$ must evolve over time; conditions [#1] and [#2] combined are needed to ensure intertemporal optimality over all finite subperiods. Condition [#3], the

$$P(t) = \frac{\partial U(E(t))}{\partial E},$$

so that the first-order conditions for the social planner's problem are the same as those that arise in perfect competition.

[7] If then per-period net benefit function $G(X, E)$ is concave in both X and E, then, since the control set $\{E \mid E \geq 0\}$ is convex, the first-order conditions are both necessary and sufficient for an optimum (Weitzman, 2003).

transversality condition, is required for the solution to be dynamically optimal over the entire infinite horizon (Weitzman, 2003).

One can use the constraints, market clearing condition and first-order conditions to reformulate the Hotelling problem into the following ordinary differential equation boundary value problem:

$$\begin{aligned}
\text{differential equations} \quad &: \quad \frac{d}{dt}\frac{\partial G(X(t),D(P(t)))}{\partial E} = -\frac{\partial C(X(t),D(P(t)))}{\partial X} + \rho\left(\frac{\partial G(X(t),D(P(t)))}{\partial E}\right) \\
& \qquad \frac{d}{dt}X(t) = D(P(t)) \\
\text{boundary conditions} \quad &: \quad X(0) = X_0 \\
& \qquad \lim_{t\to\infty}\left(P(t) - \frac{\partial G(X(t),D(P(t)))}{\partial E}\right)X(t)e^{-\rho t} = 0
\end{aligned} \tag{12}$$

The solution to the boundary value problem (12) is equivalent to that of the optimal control problem (3); I thus derive solutions to the latter problem by solving the former.

3 DATA AND CALIBRATION

Annual data spanning the years 1965-2006 is used to calibrate the theoretical model and assess its validity. For oil price P, I use the real annual spot price for crude oil obtained from the World Bank, averaged over the Brent, Dubai, and West Texas Intermediate (WTI) prices, for 1965-2001, and add to the series an annual average of the weekly world crude oil price from the Energy Information Administration for 2002-2006. I deflate the price to 1982-1984 U.S. dollars using the consumer price index (CPI).[8] Oil quantity E is world oil consumption as reported by BP. Table 1 provides summary statistics for my price and quantity data.

TABLE 1. Summary statistics

Variable	mean	s.d.	min	max	trend
real world oil price (1982-1984$/barrel)	16.52	10.67	3.12	44.75	0.16 (0.14)
world oil consumption (million barrels/day)	61.92	12.98	31.23	83.72	1.00 * (0.05)

Notes: The data covers the years 1965-2006.
The trend is the coefficient on year when the variable is regressed on year and a constant.
Significance codes: * 0.001 % level.

For the cost function $C(X, E)$, the cost function for oil estimated by Chakravorty, Roumasset and Tse (1997) is used. Their estimate was derived from world data on proven and estimated reserves and on extraction

[8] A U.S. deflator rather than a world deflator is used because the original nominal time series was in current U.S. dollars.

costs compiled by the East-West Center Energy Program. After trying a variety of functional forms, the marginal cost function they found to fit the data was of the form:

$$\frac{\partial C(X,E)}{\partial E} = c_1 e^{c_2 X} \tag{13}$$

where, when X is in units of 10^{15} British thermal units (Btu) and costs are in units of dollars per million Btu, the parameter values are given by $c_1 = 0.1774$ and $c_2 = 0.000217$.[9] This paper therefore uses the following cost function:[10]

$$C(X,E) = c_1 e^{c_2 X} E + c_0. \tag{14}$$

For demand, a crucial parameter is the elasticity of demand ε. Possible values for the elasticity, according to various empirical studies and surveys, are shown in Table 2; they range from -0.5 to 0.0.

TABLE 2. Estimates of the demand elasticity ε

Estimate of ε	Source
-0.49 to -0.45	Berndt & Wood (1975, p. 265)
-0.3	Dahl (1993)
-0.12 (short run); -0.33 (long run)	Dahl (1994b)
>-0.3	Dahl (1994a)
-0.5	Edmonson (1975, p. 172)
0.0	Lin (2008b)[11]
-0.3	Nordhaus (1980, p. 347)
-0.5 to -0.1	Pindyck (1978, p. 857)

In this paper, the model is run under different values of ε, mostly in the range $\varepsilon \in [-2, 0]$, except as noted below.

Two different forms for the demand function $D(P)$ are used. In the first form, demand is isoelastic:

$$D(P) = d_1 P^{-d_2}. \tag{15}$$

where the parameter d_2 is the absolute value of the elasticity of demand:

[9] Because my data is in terms of barrels rather than mmBtu, these parameters are converted using the conversion factor: 5.8004 mmBtu = 1 barrel (USGS, 2004).

[10] The fixed cost to extraction c_0 is assumed to be constant with respect to the stock. As a consequence, the magnitude of the fixed costs to extraction does not affect the solution to the optimal control problem.

[11] These are from the regressions of world oil demand on either OPEC or non-OPEC price.

$$d_2 = |\varepsilon| \, . \tag{16}$$

Isoelastic demand functions are common in theoretical Hotelling models, as they can lend themselves to analytic solutions (see e.g., Lin & Wagner, 2007; Stiglitz, 1978). For the monopoly case, the shadow price is positive only if $\varepsilon < -1$, which is more elastic than the range of the estimates in Table 2. Thus, in order for any extraction to occur under monopoly when demand is isoelastic, the demand function needs to exhibit a price elasticity that is more elastic than reported previously. Since d_1 satisfies:

$$d_1 = E \cdot P^{d_2}, \tag{17}$$

The parameter d_1 is calibrated by using data on world oil consumption and world oil price for a given base year τ for E and P, respectively.

The second form of the demand function used is linear demand:

$$D(P) = d_1 - d_2 P, \tag{18}$$

when the slope d_2 as a function of the elasticity of demand is given by:

$$d_2 = |\varepsilon| \cdot \frac{E}{P} \tag{19}$$

and the intercept d_1 as a function of the slope is given by:

$$d_1 = E + d_2 P \ . \tag{20}$$

Pindyck (1978) uses a linear demand function in his simulations. The parameters d_1 and d_2 are calibrated by using data on world oil consumption and world oil price for a given base year τ for E and P, respectively.

The world oil industry first began shortly after George Bissel visited oil springs in western Pennsylvania in 1853 (Yergin, 1992). According to Zimmermann (1951, p. 512), the first year in which world petroleum production data is available is 1857, when 2000 barrels of oil were produced. The initial year is therefore set at 1857, corresponding to $t = 0$, and the cumulative stock of extracted resource, X_o, is set at 2000 barrels.[12] The base year τ used for calibrating the demand function is 1985.[13] The market interest rate is set at $\rho = 0.05$.

[12] I also run simulations in which I instead pin down the initial price $P(0)$ to equal to actual real world oil price at time t. Since the first year of price data in my data set in 1965, for these simulations $t = 0$ corresponds to 1965, when the real world oil price was \$4.51/barrel. However, because pinning down the initial price rather than initial stock often leads to solutions with negative stocks, and because the qualitative features of the results are robust to the type of initial condition chosen, the results of the simulations in which an initial condition was imposed on price are not reported here.

[13] As a robustness check, a base year of 1990 was also used, and the qualitative results were unchanged.

4 SIMULATION RESULTS

This section presents results under the two market structures of perfect competition and monopoly for both isoelastic demand and linear demand, for a total of four scenarios, and compares these results to the actual data. For each of the four scenarios, the demand elasticity is varied to discern which elasticity generates trajectories for market price and for extraction that best fit the data. For both perfect competition scenarios, the elasticity is varied from -2 to 0. For monopoly under isoelastic demand, because extraction would only occur when $\varepsilon < -1$, the demand elasticity is varied from -2 to -1.1. For monopoly under linear demand, the elasticity is varied from -2 to -0.1.[14]

I use the model's simulated solution for the years 1965-2006, the period spanned by my actual data, as well as for the subperiods 1965-1973 (the pre-1973 oil crisis period), 1973-1981 (the period between oil crises), 1981-1990 (the period immediately following the 1981 oil crisis), and 1990-2006 (the most recent period), when comparing the model to data.

To examine the fit of the model for each possible combination of market structure, demand and demand elasticity, three different measures are used. The first measure of fit is based on the summary statistics. The summary statistics examined are the means, standard deviations, maxima, and minima of the market price $P(t)$ and extraction $E(t)$ over the years in each period examined. A solution fits the data well if its summary statistics are similar to those of the actual data. The second measure of fit is the mean squared error (MSE). In particular, for each period examined, the MSE between the model's solution and the actual data is examined for both market price and extraction. A lower MSE indicates a better fit. The third measure of fit is correlation. The correlation between the model's predicted trajectory and the actual trajectory is calculated for both market price and extraction. A higher correlation indicates a better fit. Summary statistics, MSE and correlation are used to choose the market structure - demand specification - elasticity combination that best fit the data.[15]

Prior to the 1973 oil crisis, the model that best fits actual data is one of perfect competition with linear demand and a demand elasticity of -0.4. For the periods 1973-1981 and 1981-1990, the model that best fits actual data is one of monopoly with linear demand and demand elasticities of -0.8 and -0.7, respectively, suggesting that the market was strongly influenced by OPEC during this time. Under the model that best fits the most recent period (perfect competition with linear demand and demand elasticity -0.5), the real oil price (in 1982-1984 U.S. $) should fall in the range $60.87-$66.31/barrel over the years 2010-2030.

Figure 1 plots the trajectories for market price and extraction over 1965-2006 for the scenarios that best fit each of the subperiods, respectively. For each subperiod, the solid line highlights the scenario that best fits

[14] No solution exists for monopoly under linear demand when demand is perfectly inelastic.
[15] Graphs for all scenarios using data from 1965-2001 are presented in Lin (2005).

the data. Several insights can be gleaned from the results.

In the first decade after its inception, OPEC engaged in activities that "were generally of a low-profile nature, as OPEC set out its objectives, established its Secretariat, which moved from Geneva to Vienna in 1965, adopted resolutions and engaged in negotiations with the companies" (OPEC, 2007). The result that OPEC did not collude prior to the 1973 oil crisis is consistent with these activities.

Collusion among OPEC producers in the 1970s and 1980s is consistent with the results of Griffin (1985), who finds that over 1971-1983, the partial market-sharing cartel model could not be rejected for all 11 countries. OPEC itself claims that "OPEC rose to international prominence during this decade, as its Member Countries took control of their domestic petroleum industries and acquired a major say in the pricing of crude oil on world markets" (2007).

The result that OPEC producers have not succeeded in colluding in recent years is consistent with the studies of Marcel and Mitchell (2006) and Sperling and Gordon (2007), who argue that while OPEC producers may have succeeded in colluding when OPEC was first formed, they have failed to behave as a cartel in the past two decades in part because the state-owned companies that comprise OPEC have juggled multiple objectives, economic and otherwise, and thus have not acted so as to maximize joint profits. This result is also consistent with the empirical results of Lin (2008a), who estimates a dynamic model of the world oil market and tests whether OPEC countries colluded and whether non-OPEC countries behaved oligopolistically over the period 1970-2004, and who finds support for oligopolistic behavior among non-OPEC producers and collusion among OPEC producers in the earlier period but not the later period.

In each of the subperiods, linear demand fits the data better than isoelastic demand does, suggesting that there is a choke price, perhaps due to a renewable backstop technology, above which consumers are not willing to pay for oil. Demand for oil, which was relatively inelastic prior to the 1973 oil crisis, became more elastic in the period immediately following the oil crisis, as consumers responded to high oil prices by curtailing energy consumption and developing energy efficiency. Since then, however, oil demand has become increasingly price inelastic, as society has become more oil dependent. This result is consistent with the study of Hughes, Knittel and Sperling (2008), who find that gasoline demand in the United States was signficiantly more price inelastic during 2001-2006 than during 1975-1980.

5 CONCLUSION

This paper uses data on world oil price and consumption to calibrate a Hotelling model of optimal resource extraction with unlimited potential reserves when costs exhibit stock effects. Prior to the 1973 oil crisis, the model that best fits actual data is one of perfect competition with linear demand and a demand elasticity of -0.4. For the periods 1973-1981 and 1981-1990, the model that best fits actual data is one of monopoly with linear demand and demand elasticities of -0.8 and -0.7, respectively, suggesting that the market was strongly influenced by OPEC during this time. Under the model that best fits the most recent period (perfect competition with linear demand and demand elasticity -0.5), the real oil price (in 1982-1984 U.S. $) should fall in the range $60.87-$66.31/barrel over the years 2010-2030.

So, how well does the simple Hotelling model presented appear to explain historical data? As seen from Figure 1, the best-fitting models still do not fit the data very well. However, although the best-fitting theoretical models fail to capture the upward trend in extraction and the short-run volatility in price and quantity, the basic Hotelling model does yield several realistic results that are worth noting. First, the market structure results suggest that OPEC producers exerted substantial market power over 1973-1990, but neither in the initial years of its formation nor in more recent years. Second, demand for oil was inelastic before the 1973 oil crisis, became more elastic during the crisis and its immediate aftermath, and has become more inelastic since then. Third, the linearity of demand suggests that, despite their inelasticity, if the price of oil were to rise high enough, consumers would switch to alternative sources of energy.

The focus of this paper was on a very basic Hotelling model, without more realistic features, in order to obtain an upper bound on how poorly the Hotelling model performs, and results suggest that even the most basic Hotelling model does yield insights. In order to further reconcile the theory with data, especially for the more recent period, one should augment the basic model with such modifications as a different estimate of the cost function, technological progress, oligopoly, a model of OPEC and non-OPEC producers, time-varying demand, uncertainty, capacity constraints, crude oil quality differentials, institutional reshaping of petroleum markets, lags in consumer and producer responses to oil price changes, precautionary purchases on the demand side, the timing of investment in capital-intensive exploration and development, undersupply, and myopia. For example, because it assumes that markets always clear, the basic Hotelling model ignores a salient feature of mineral industries: the investment timing problem. Because producers can only add capacity through capital intensive exploration and development, supply is to often unable to stay in step with demand. Models that incorporate more realistic features will be the subject of future work.

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FIGURE 1.

Price and Extraction Trajectories for 1965-2006