Hotelling Revisited: Oil Prices and Endogenous Technological Progress[1]

C.-Y. Cynthia Lin,[2] Haoying Meng, Tsz Yan Ngai, Valeria Oscherov, Yan Hong Zhu

Abstract

This paper examines the Hotelling model of optimal nonrenewable resource extraction in light of empirical evidence that petroleum and minerals prices have been trendless despite resource scarcity. In particular, we examine how endogenous technology-induced shifts in the cost function would have evolved over time if they were to maintain a constant market price for nonrenewable resources. We calibrate our model using empirical data on world oil, and find that, depending on the estimate of the initial stock of reserve, oil reserves will likely be depleted some time between the years 2040 and 2075.

JEL Classification: O30, Q30, Q43, L16

Keywords: Hotelling, endogenous technological progress, world oil market

First draft: February 11, 2004

This draft: October 21, 2008

---

[1] University of California at Davis. This paper benefited from discussions with Martin Weitzman, Howard Stone, Gary Chamberlain, Partha Dasgupta, Aart de Zeeuw, Jerry Green, Anni Huhtala, Satoshi Kojima, Sjak Smulders, Anastasios Xepapadeas, and two anonymous referees, and from comments from participants at the 2004 EAERE-FEEM-VIU Summer School on Dynamic Models in Economics and the Environment in Venice. Lin received financial support from an EPA Science to Achieve Results graduate fellowship, a National Science Foundation graduate research fellowship, and a Repsol YPF - Harvard Kennedy School Pre-Doctoral Fellowship in energy policy. Meng, Ngai, Oscherov, and Zhu received funding from the National Science Foundation under the UC-Davis VIGRE Research Experiences for Undergraduates program. All errors are our own.

[2] Corresponding author: Agricultural and Resource Economics, University of California at Davis, One Shields Avenue, Davis, CA 95616; cclin@primal.ucdavis.edu.

1 Introduction

The basic Hotelling model of nonrenewable resource extraction predicts that the shadow price of the resource stock, which is an economic measure of the scarcity of the resource, should grow at the rate of interest (Hotelling, 1931). This prediction is known as the "Hotelling rule" (Krautkraemer, 1998). If the natural resource market is perfectly competitive, then the Hotelling rule implies that the market price minus marginal costs must grow at the rate of interest, and therefore that the natural resource price should be increasing over time if marginal costs are constant. In contrast to Hotelling's theoretical prediction, however, empirical studies have shown that mineral prices have been roughly trendless over time (see Krautkraemer, 1998, & references therein; Lin, 2008b; Lin & Wagner, 2007).[3] This paper develops a model of endogenous technological progress that reconciles Hotelling's theory with empirical evidence on world mineral prices.

There are several possible reasons why the Hotelling rule may not be a good guide to the actual behavior of mineral prices over time. In this paper we focus on two such reasons. First, the simple Hotelling model assumes that the costs of extraction do not depend on the stock of reserve remaining in the ground. However, it is plausible that extraction costs increase as more of the resource is extracted and fewer reserves remain. For instance, extraction costs may increase inversely with the remaining stock of reserves if the resource needed to be extracted from greater depths or if well pressure declined as more of the reserve was depleted. Another possible explanation is that since different grades of oil may differ in their extraction costs, and since the cheaper grades are likely to be mined to exhaustion before the more expensive grades are mined, the cost of extraction may increase as the cheaper grades are exhausted. We use the term "stock effect" to refer to the dependence of extraction cost on the stock of remaining reserve. With a stock effect, the shadow price rises less slowly than the rate of interest, but the market price still increases over time (Tietenberg,

[3] Some studies have shown evidence for a U-shaped price path (see e.g., Slade, 1982), but the coefficients on the quadratic trend are not robust to the period of estimation (Berck & Roberts, 1996).

1996).

In addition to stock effects, a second reason why the Hotelling rule does not adequately describe the actual behavior of world mineral prices is that technological progress may occur that enhances the ability of firms to extract ore. Such technological progress would cause the extraction cost schedule to decrease over time, and may result in a U-shaped market price trajectory in which the resource price declines over some initial interval and then increases as the effect of a finite reserve outweighs the effect of a declining extraction cost (Krautkraemer, 1998). Examples of technological progress that have decreased oil production costs include the advent of 3-D seismic technology, which helped oil producers avoid expensive dry holes; the advent of measurement while drilling and logging while drilling technology, which enabled producers to conduct tests while drilling; and the advent of deepwater technology, which enabled oil producers to produce in deepwater.

This paper thus focuses on two diametrically opposed factors that may cause real world prices to diverge from the basic Hotelling rule: stock effects which increase extraction costs and are consistent with rising resource prices, and technological progress which lowers extraction costs and may cause prices to initially decline.

Stock effects in extraction costs have previously been examined (see e.g., Farzin, 1992; Hanson, 1980; Solow & Wan, 1976); technological progress has been previously modeled as well (see e.g., Farzin, 1992, 1995). Rausser (1974) develops a comprehensive model of technological progress via learning by doing, a model which simultaneously optimizes the rate of resource use and investment. Other features examined have included Nash-Cournot behavior (Salant, 1976; Ulph & Folie, 1980), OPEC (Hnyilicza & Pindyck, 1976; Pindyck, 1976; Cremer & Weitzman, 1976), exploration (Pesaran, 1990; Pindyck, 1978), market imperfections (see Cremer & Salehi-Isfahani, 1991 and references therein; Khalatbari, 1977; Stiglitz, 1976; Sweeney, 1977), outward-shifting demand (Chapman, 1993; Chapman & Khanna, 2000), and uncertainty (Hoel, 1978; Pindyck, 1980).

We pursue the following novel research question: *if technological progress were an en-*

dogenous process that acted to stabilize resource prices, what would its time path look like? Given extraction costs as a function of extraction and of the remaining resource stock, we examine how cost-reducing technological progress would have evolved if they were to maintain a constant market price for the nonrenewable resource. We then calibrate our theoretical model with empirical data on world oil to project the time at which world oil reserves will be exhausted.

Characterizing technological progress has long been of interest to economists. Given the importance of the oil industry to the world economy, a study of the endogenous evolution of technological progress in the oil industry is especially valuable. This paper is also important because it develops a model that predicts the time at which oil reserves will be depleted. We find that, depending on the estimate of the initial stock of reserve, oil reserves will likely be depleted some time between the years 2040 and 2075.

The balance of this paper proceeds as follows. In Section 2, we present the basic Hotelling model. In Section 3, we derive and present the main results on endogenous technological progress. Section 4 concludes.

2 The Hotelling Model

In this section, we present a theoretical model of optimal nonrenewable extraction under perfect competition. We ignore inventories and assume competitive resource markets. We also ignore any common access problems that may arise in perfect competition.[4] Following Pindyck (1978), we assume a large number of identical firms that act as price takers. This is equivalent to assuming that a social planner or a state-owned monopoly has sole production rights and sets a competitive price.

The assumption of perfect competition may not be realistic. However, allowing for market power does not necessarily reconcile the theory with the data. For instance, according

[4] Sinn (1984) presents a model of exhaustible resource extraction in which storage facilities are used as a result of the common-pool problem.

to Salant's (1976) Nash-Cournot model with a cartel and a competitive fringe, the real price of oil increases monotonically for both constant and increasing marginal extraction costs. On the other hand, in Loury's (1986) Cournot model, the present value of price net of the constant extraction costs declines over time. Moreover, minerals markets that may seem to have market power may actually behave like perfectly competitive ones. For example, according to Agostini's (2006) empirical test of the competitive behavior of the U.S. copper industry before 1978, even though the copper industry was highly concentrated, the U.S. prices were close to the levels predicted by a competitive model of the industry.

In the case of the world oil market, perfect competition may be an appropriate characterization especially for more recent years. Results from Lin's (2008a) empirical dynamic model of the world oil market over the period 1970-2004 do not support either oligopoly among non-OPEC producers or collusion among OPEC producers in the production of oil in the last 15 years. Similarly, Lin (2008b) finds in her simulations of the basic Hotelling model that while a monopolistic market structure better explains the world oil market than perfect competition does prior to the 1973 Arab oil embargo, perfect competition fares better in the years following it.

A *resource* is defined as a naturally occurring substance that is considered valuable in its relatively unmodified (natural) form, and for which the primary activities associated with it are extraction and purification, as opposed to creation. *Nonrenewable resources* are resources for which there is a finite stock or supply; these include minerals such as oil, natural gas, tin, and copper. Let *reserve* be defined as the amount of the natural resource stock remaining in the ground that has yet to be extracted. *Depletion* occurs when the resource is being extracted; the resource is *depleted* when the entire stock of the resource has been extracted, leaving none remaining in the ground.

Let $t \in [0, \infty)$ index time. At time t, the supply of the mineral is given by $E(t)$, the total extraction flow in tons per unit time at time t. Let $S(t)$ denote the stock of oil remaining in the ground at time t:

$$S(t) = S(0) - \int_0^t E(\tau)d\tau \quad , \tag{1}$$

where the initial stock $S(0) = S_0$ is taken as given.

The market price of oil at time t is $P(t)$. The demand for oil at time t when the market price is P is given by the demand function $D(P, t)$. Markets are assumed to clear, which means that, at each time t, the price $P(t)$ acts to equate supply and demand:

$$E(t) = D(P(t), t) \ \forall t. \tag{2}$$

Assuming that the social and private discount rates are the same, that the initial stock S_0 is known, and that there are no externalities, the social planner's optimal control problem yields the same solution as would arise in perfect competition. In this case, under the additional assumption that the marginal utility of income is constant, the total benefits $U(\cdot, \cdot)$ that accrue from the consumption of the mineral at time t are given by the area under the demand curve,

$$U(E(t), t) = \int_0^{E(t)} D^{-1}(x; t)dx, \tag{3}$$

where $D^{-1}(\cdot; t)$ is the inverse of the demand curve with respect to price. This area measures the gross consumer surplus, and is a measure of the consumers' willingness-to-pay for the resource. Weitzman (2003) shows that using the area under the demand curve in place of revenue yields the same outcome as a perfectly competitive market.[5] Thus, in the absence

[5]This holds because, assuming constant marginal utility of income:

$$P(t) = \frac{\partial U(E(t))}{\partial E},$$

so that the first-order conditions for the social planner's problem are the same as those that arise in perfect competition.

of externalities, a perfectly competitive market maximizes total utility, or what Hotelling (1931) terms the "social value of the resource".

The cost of extracting E units of oil at time t when there are S units of reserve remaining is given by $C(S,E,t)$. We use the term *stock effect* to refer to the dependence $\frac{\partial C}{\partial S}(\cdot)$ of extraction cost on the stock S of reserve remaining, and this dependence is likely to be negative. We use the term *relative stock effect* to refer to the ratio $\frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}$ of the absolute value $\left|\frac{\partial C}{\partial S}(\cdot)\right|$ of the stock effect over the marginal extraction cost $\frac{\partial C}{\partial E}(\cdot)$. Similarly, we define the *marginal stock effect* as the dependence $\frac{\partial^2 C}{\partial S \partial E}(\cdot)$ of the marginal extraction cost on the stock S of reserve remaining.

Let $p(t)$ denote the non-negative current-value shadow price measuring the value of a unit of reserve at time t. This shadow price is known by a variety of terms, including "marginal user cost", because it measures the opportunity cost of extracting the resource; "in situ value", because it measures the marginal value of leaving an additional unit of resource in the ground; "scarcity rent", because it is an economic measure of scarcity, and "dynamic rent", to reflect the difference between price and marginal extraction cost (Krautkraemer, 1998; Weitzman, 2003).

The competitive interest rate is ρ.

The social planner's optimal control problem, which yields the same solution as would arise in perfect competition, is to choose her extraction profile $\{E(t)\}$ to maximize the present discounted value of her entire stream of net benefits, given her initial stock S_0 and given the relationship between her extraction $E(t)$ and the stock remaining $S(t)$, and subject to the constraints that both extraction and stock are nonnegative. Her problem is thus given by:

$$
\begin{aligned}
&\max_{\{E(t)\}} \int_0^\infty \left(U(E(t),t) - C(S(t),E(t),t)\right) e^{-\rho t} dt \\
&\text{subject to} \qquad \tfrac{dS(t)}{dt} = -E(t) \qquad : p(t) \\
&\qquad E(t) \geq 0 \\
&\qquad S(t) \geq 0 \\
&\qquad S(0) = S_0 \quad ,
\end{aligned} \tag{4}
$$

where the multiplier $p(t)$ associated with the equation of motion for the stock $S(t)$ of oil remaining is precisely the shadow price $p(t)$ of the reserve.

From the Maximum Principle, the first-order necessary conditions for a feasible trajectory $\{S^*(t), E^*(t)\}$ to be optimal are:

$$
[\#1]: \qquad p(t) = P(t) - \frac{\partial C\left(S(t), E(t), t\right)}{\partial E} \tag{5}
$$

$$
[\#2]: \qquad \frac{dp(t)}{dt} = \frac{\partial C\left(S(t), E(t), t\right)}{\partial S} + \rho p(t) \tag{6}
$$

$$
[\#3]: \qquad \lim_{t \to \infty} p(t) S(t) e^{-\rho t} = 0 \tag{7}
$$

Condition [#1] states that, at each time t, the shadow price $p(t)$ must equal the competitive market price $P(t)$ minus the marginal cost of extraction $\frac{\partial C(S(t),E(t),t)}{\partial E}$; this condition is needed to ensure static optimality at each point in time. Condition [#2] governs how the shadow price $p\left(t\right)$ must evolve over time; conditions [#1] and [#2] combined are needed to ensure intertemporal optimality over all finite subperiods. Condition [#3], the transversality condition, is required for the solution to be dynamically optimal over the entire infinite horizon (Weitzman, 2003).

In order to solve the Hotelling resource extraction problem (4) for extraction and market price trajectories, one needs to make functional form assumptions on both the demand function $D(P, t)$ and the cost function $C(S, E, t)$. In particular, we make the following assumptions on the cost function $C(S, E, t)$:

Assumption A1. (multiplicatively separable technological progress)

The cost function consists of two multiplicatively separable terms:

$$C(S, E, t) = \varphi(S, E) \cdot h(t) \tag{8}$$

where $\varphi(S, E) \geq 0$ is the time-independent component of the extraction cost, representing the costs in the absence of technological change; and $h(t)$ is the time-varying component, capturing how the cost schedule shifts over time due to changes in technology.

Assumption A2. (negative stock effects)

Both total cost and marginal extraction costs decrease with the amount of reserve S remaining in the ground:

$$\frac{\partial C}{\partial S}(S, E, t) \leq 0 \Rightarrow \frac{\partial \varphi}{\partial S}(S, E) \leq 0 \tag{9}$$

and

$$\frac{\partial^2 C}{\partial S \partial E}(S, E, t) \leq 0 \Rightarrow \frac{\partial^2 \varphi}{\partial S \partial E}(S, E) \leq 0. \tag{10}$$

We call $h(t)$ the *cost shifter*. It captures the effect of technology on the cost function. The lower the value of the cost shifter, the lower the costs of extraction and the higher the level of technology. If the cost function shifts down over time, then technological progress has occurred, and $\frac{dh(t)}{dt} \leq 0$.

Suppose endogenous technological progress acted to maintain prices at a constant price $\overline{P}$. We now ask: what would the trajectory of the technology-induced cost shifter $\{h(t)\}$ look like if it were to maintain the market price trajectory $\{P(t)$ constant at $\overline{P}$?

3 Endogenous Technological Progress

We now examine how endogenous technological progress would evolve so that resource prices could remain constant first in the benchmark case of no stock effects, then in the simplest case of a stock effect, and lastly in the most general case where only the assumptions A1 of a multiplicatively separable cost shifter and A2 of negative stock effects are made on the cost function and no assumptions are made on the demand function.

3.1 The benchmark case: No stock effects

As a benchmark, we first derive the rate at which the technology-induced cost shifter $h(t)$ must grow in order to maintain a constant market price for oil when costs do not depend on the remaining resource stock S.

Proposition 1 *Under A1, when extraction costs are nonzero, do not depend on the remaining resource stock (i.e.,* $\frac{\partial C}{\partial S}(\cdot) = 0$*) and are linear in the rate of extraction (i.e.,* $\frac{\partial^2 C}{\partial E^2}(\cdot) = 0$*), then, in order to maintain a constant market price (i.e.,* $P(t) = \overline{P}\ \forall t$*), the growth rate in the technology-induced cost shifter must be negative:*

$$\frac{\frac{dh(t)}{dt}}{h(t)} = \left(1 - \frac{\overline{P}}{\frac{\partial C}{\partial E}(S(t), E(t), t)}\right)\rho \leq 0. \tag{11}$$

Proofs are in the appendix. The intuition is as follows. As seen in Krautkraemer (1998), the growth rate of the resource price in the absence of stock effects is a weighted average of the discount rate and the growth rate in marginal extraction cost, where the weights $\theta(t)$ are given by the ratio of marginal extraction cost to price. The first term on the right-hand side of equation (22) measures the effect of scarcity while the second term measures the effect of changes in marginal extraction cost. If the marginal extraction cost $\frac{\partial C}{\partial E}(\cdot)$ is constant over time, then only the scarcity effect applies and market price grows at a rate less than the discount rate.[6] If technological change acts to decrease marginal extraction costs, then it is possible for the declining cost effect to dominate the scarcity effect, especially at the beginning of an extraction horizon, and therefore for prices to decline.[7]

Thus, when marginal costs are constant over time, the effect of scarcity would cause the market price to rise. In order to cancel out this effect so that market prices are constant, technology must evolve so that the marginal cost schedule shifts downwards over time.

3.2 The basic case: Simple stock effect

To examine how endogenous technological progress would evolve if it were to keep the market price for oil constant in the presence of stock effects, we first use a simple cost function that exhibits a stock effect. In particular, we assume that the cost function takes the following form:

Assumption B1. (simple stock effect)

Extraction costs are linear in extraction E and exponential in the remaining reserve stock S:

$$\varphi(S, E) = F(S) \cdot E \tag{12}$$

where the exponential stock-dependent component $F(S)$ of the cost function is given by:

$$F(S) = \Psi e^{-\sigma S}, \tag{13}$$

with $\Psi \geq 0$ and $\sigma \geq 0$.

[6] In the absence of extraction costs altogether, the market price would grow at the rate of discount.

[7] As the marginal extraction cost decreases over time, however, the scarcity effect is given greater weight and may eventually dominate, causing prices to increase. Thus, cost-decreasing technological progress may result in a U-shaped price path (Krautkraemer, 1998).

The *stock effect parameter* σ measures the extent of the dependence of cost on the remaining stock of reserve; the greater the σ, the more severe the dependence.

There is some empirical support for B1: using world data on proven and estimated reserves and on extraction costs compiled by the East-West Center Energy Program to test a variety of function forms for the oil extraction costs, Chakravorty, Roumasset and Tse (1997) found that the cost function that best fit the data was of the form:

$$C(S,E) = \Psi e^{\sigma(S_0 - S)} E$$

where, when S is in units of 10^{15} British thermal units (Btu) and costs are in units of dollars per million Btu, the parameter values are given by $\Psi = 0.1774$ and $\sigma = 0.000217$.[8]

Let the *stock effect growth rate* g to be defined as the growth rate of the stock-dependent component $F(S)$ of the cost function:

$$g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))}. \tag{14}$$

It turns out that the stock effect growth rate g is a measure of the stock effect, as seen in the following Lemma:

Lemma 2 *(i) The stock effect growth rate g is equal to the relative stock effect:*

$$g(t) = \frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}. \tag{15}$$

(ii) The stock effect growth rate g is also proportional to the stock effects parameter σ*:*

$$g(t) = \sigma E(t). \tag{16}$$

[8] The cost function estimated by Chakravorty et al. (1997) differs from the one implied by A1 and B1, however, because it does not include the time-dependent term $h(t)$.

In addition to assumption B1 on the cost function, for this basic case we also make the following assumption on demand:

Assumption B2. (stationary demand)

Demand is stationary:

$$D(P,t) = D(P) \ \ \forall t.$$

If endogenous technological progress acted to maintain prices at a constant price $\overline{P}$, then under B2, extraction rates $E(t)$ would be constant at $\overline{E} \equiv D(\overline{P})$ and the reserve remaining at time t would be given by $S(t) = S_0 - \overline{E}t \equiv \overline{S}(t)$. The resource is exhausted at time $T = \frac{S_0}{\overline{E}}$. By Lemma 2, when market prices are constant, the stock effect growth rate $g(t)$ is constant at $\overline{g} = \sigma \overline{E}$. Technological progress must then evolve to shift the cost function downwards over time, as seen in the following Proposition.

Proposition 3 *Under A1, A2, B1, and B2, the growth rate in the technology-induced cost shifter that maintains a constant market price is negative:*

$$\frac{\frac{dh(t)}{dt}}{h(t)} = \left(1 - \frac{\overline{P}}{\frac{\partial C}{\partial E}\left(S(t), E(t), t\right)}\right)\rho \leq 0. \tag{17}$$

This result is similar to the result in Proposition 1 for the case of no stock effects and the intuition is the same as well. The optimal trajectories for $E(t)$ and $S(t)$ are different in the presence of stock effects, however, so the value for the cost shifter at any point in time is likely to be different as well. This value is given by the following expression.

Corollary 4 *Under A1, A2, B1, and B2, the technology-induced cost shifter $h(t)$ that would act so as to maintain the market resource price $P(t)$ constant at $\overline{P}$ is given by:*

$$h(t) = h(0)e^{\rho t} + \frac{\rho}{\rho + g}\frac{\overline{P}}{\Psi}e^{\sigma S_o}\left(e^{-gt} - e^{\rho t}\right), \tag{18}$$

where $h(0) \in \left[\frac{\rho}{\rho+g}\frac{\overline{P}}{\Psi}e^{\sigma S_o}\left(1-e^{-(\rho+g)\frac{S_0}{\overline{E}}}\right), \frac{\overline{P}}{\Psi}e^{\sigma S_o}\right]$.

We calibrate our theoretical model using statistics on world oil price and world oil consumption over 1965-2006 reported in Lin (2008b). In particular, we set $\overline{P}$ equal to the mean real world oil price over 1965-2006, which was 16.52 dollars (1982-1984 U.S. dollars) per barrel, and set $\overline{E}$ equal to the mean world oil consumption over 1965-2006, which was 61.92 million barrels per day. For the cost function, we use the parameter estimates for Ψ and σ reported in Chakravorty, Roumasset and Tse (1997). We set the the discount rate at $\rho = 0.05$. We vary the initial stock S_0 between 1.7 and 2.5 trillion barrels. One study estimates the ultimate world recovery of oil reserves to be 1.8 trillion barrels; another estimates it to be 2.1 trillion barrels (see Deffeyes, 2001, & references therein).

Figure 1 presents a simulation of the trajectory for the cost shifter $h(t)$ that maintains a constant market price, as given by Corollary 4, under various specifications for the initial stock S_0 of reserves. Corresponding trajectories for marginal cost and for the shadow price are presented in Figures 2 and 3, respectively. The time at which oil will be depleted varies with the initial stock, as shown in Table 1. If the initial stock is between 1.7 and 2.5 trillion barrels, then oil reserves will be depleted some time between the years 2040 and 2075.[9]

3.3 The general case of fully flexible cost and demand

In this section we examine endogenous technological progress with minimal restrictions on the cost function and no restrictions on demand. In particular, the only assumptions that we make on the cost function $C(S, E, t)$ are assumption A1 of multiplicatively separable technological progress and assumption A2 of negative stock effects. The demand function $D(P, t)$ can be of any form; in particular, unlike in the previous section, demand can be non-stationary.

When demand is non-stationary, then under a constant market price $\overline{P}$, the extraction

[9] In actuality, oil might be depleted earlier than we estimate if demand increases over time; oil might be depleted later if new discoveries are found.

rate at time t would be given by $E(t) = D(\overline{P}, t) \equiv \overline{E(t)}$ and the remaining resource stock at time t would be given by $S(t) = S_0 - \overline{E(t)}t \equiv \overline{S(t)}$. The non-negative stock effect growth rate $g(t)$, which is a measure of relative stock effect, is now given by:

$$g(t) = \frac{\left| \frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial S} \right|}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}}.$$

Our general result is as follows.

Proposition 5 *Under A1 and A2, the growth rate of the cost shifter that maintains a constant market price is given by:*

$$\frac{\frac{dh(t)}{dt}}{h(t)} = \left(1 - \frac{\overline{P}}{\frac{\partial C}{\partial E}\left(\overline{S(t)}, \overline{E(t)}, t\right)}\right)\rho + g(t) + \frac{\frac{\partial^2 \varphi(\overline{S(t)}, \overline{E(t)})}{\partial S \partial E}\overline{E(t)}}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}} - \frac{\frac{\partial^2 \varphi(\overline{S(t)}, \overline{E(t)})}{\partial E^2}\frac{\partial \overline{E(t)}}{\partial t}}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}}. \quad (19)$$

The growth rate in the cost shifter is now composed of four terms. The first term is the same as the expression for the growth rate for the case of no stock effect and the case of a simple stock effect, and is negative. The second term is the stock effect growth rate, and is positive. The third term is negative. The sign of the last term depends of the convexity or concavity of the cost function with respect to extraction and the growth rate in demand. Thus, unlike in the previous two scenarios, the growth rate in the cost shifter is no longer unambiguously negative, but instead depends on the values of the parameters and on the functional form assumptions.

In this general case, technological progress does not necessarily have to evolve to continually shift the cost function downwards in order for market prices to remain constant. For example, if there were increasing returns to scale in extraction such that the cost function were concave with respect to extraction $\left(\frac{\partial^2 \varphi(\overline{S(t)}, \overline{E(t)})}{\partial E^2} < 0\right)$ and if demand were increasing over time $\left(\frac{\partial \overline{E(t)}}{\partial t} > 0\right)$, then the fourth term in the expression in (19) would be positive and the rate at which technological progress would need to shift the cost function downwards would be reduced. Another scenario that would mitigate the need for technological progress on the supply side would be the case of decreasing returns to scale in extraction $\left(\frac{\partial^2 \varphi(\overline{S(t)},\overline{E(t)})}{\partial E^2} > 0\right)$ and decreasing demand over time $\left(\frac{\partial \overline{E(t)}}{\partial t} < 0\right)$, for example due to technological progress in energy efficiency and energy-saving technology on the demand side. Thus, demand-side technological progress can at least partially offset the need for supply-side technological progress.

4 Conclusion

This paper examines the Hotelling model of optimal nonrenewable resource extraction in light of empirical evidence that petroleum and minerals prices been trendless despite resource scarcity. In particular, we examine how endogenous technology-induced shifts in the cost function would have evolved over time if they were to maintain a constant market price for nonrenewable resources. We calibrate our model using data on the world oil market.

According to the results, for the cases of no stock effect and a simple stock effect, endogenous technology must shift costs downward over time in order to maintain constant market prices. For general cost and demand functions, however, the growth rate in the cost shifter may be positive or negative.

Our research is important for several reasons. First, characterizing technological progress has long been of interest to economists. Given the importance of the oil industry to the world economy, a study of the endogenous evolution of technological progress in the oil industry is especially valuable. Second, a projection of the time at which oil reserves will be depleted is of interest to academic economists, industry practitioners and policy-makers alike. Unlike Lin and Wagner (2007), whose theoretical model of constant prices requires steady-state growth, our model allows the resource to be eventually exhausted. We find that, depending on the estimate of the initial stock of reserve, oil reserves will likely be depleted some time between the years 2040 and 2075. The results of this paper reconcile Hotelling's theoretical model with empirical evidence on the trendless nature of world mineral prices and may have important implications for future examinations of endogenous technological progress. Such future examinations may include, for example, adding capital investment to the model of technological progress, along the lines of Rausser (1974).

5 Appendix: Proofs

Proof of Proposition 1. When there are no stock effects (i.e., $\frac{\partial C}{\partial S}(\cdot,\cdot,\cdot) = 0$), then condition [#2] yields the Hotelling rule that the shadow price rises at the rate of interest:

$$\frac{\frac{dp(t)}{dt}}{p(t)} = \rho. \tag{20}$$

When combined with condition [#1], this means that the market price minus marginal costs must increase at the rate of interest:

$$\frac{\frac{d}{dt}\left(P(t) - \frac{\partial C}{\partial E}(S(t), E(t), t)\right)}{P(t) - \frac{\partial C}{\partial E}(S(t), E(t), t)} = \rho, \tag{21}$$

which yields, after rearranging terms, the following equation for the growth rate of market price in the absence of stock effects:

$$\frac{\frac{dP(t)}{dt}}{P(t)} = (1-\theta(t))\rho + \theta(t)\frac{\frac{d}{dt}\left(\frac{\partial C}{\partial E}(S(t), E(t), t)\right)}{\frac{\partial C}{\partial E}(S(t), E(t), t)} \tag{22}$$

where the weight $\theta(t)$ is defined as:

$$\theta(t) \equiv \frac{\frac{\partial C}{\partial E}(S(t), E(t), t)}{P(t)}. \tag{23}$$

When marginal extraction costs are nonzero, $\theta(t) \geq 0$. Moreover, from [#1] and the non-negativity of the shadow price, $\theta(t) \leq 1$. Under A1, when there are no stock effects and costs are linear in extraction, equation (22) reduces to:

$$\frac{\frac{dP(t)}{dt}}{P(t)} = (1-\theta(t))\rho + \theta(t)\frac{\frac{dh(t)}{dt}}{h(t)}. \tag{24}$$

In order for market price to be constant (i.e., $\frac{dP(t)}{dt} = 0$), we need $h(t)$ to rise at rate $\left(1 - \frac{1}{\theta(t)}\right)\rho$, which is non-positive since $\theta(t) \leq 1$.

Proof of Lemma 2.(i) $g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))} = \frac{-F'(S(t))E(t)}{F(S(t))} = \frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}$. (ii) $g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))} = \sigma E(t)$.

Proof of Proposition 3. Under A1, B1 and B2, [#1] can be written as $p(t) = \overline{P} - F(S(t))h(t)$, which implies $h(t) = \frac{\overline{P}-p(t)}{F(S(t))}$. Taking the derivative with respect to time yields:

$$\begin{aligned}\frac{dh(t)}{dt} &= \frac{-\frac{dp(t)}{dt}}{F(S(t))} - \frac{\overline{P}-p(t)}{F(S(t))}\frac{\frac{d}{dt}F(S(t))}{F(S(t))} \\ &= \frac{-\frac{dp(t)}{dt}}{F(S(t))} - g(t)\frac{\overline{P}-p(t)}{F(S(t))}.\end{aligned}$$

Thus,

$$\begin{aligned}\frac{\frac{dh(t)}{dt}}{h(t)} &= \frac{\frac{dp(t)}{dt}}{p(t)-\overline{P}} - g(t) \\ &= \frac{g(t)\left(p(t)-\overline{P}\right)+\rho p(t)}{p(t)-\overline{P}} - g(t),\end{aligned}$$

where the second line comes from [#2]. Further simplification yields the desired result.

Proof of Corollary 4. The closed-form equation (18) for $h(t)$ is the solution to the linear first-order differential equation (17). Condition [#1] and the non-negativity of $p(t)$ implies $h(t) \leq \frac{\overline{P}}{\Psi}e^{\sigma S_o - gt}$ $\forall t$, so $h(0) \leq \frac{\overline{P}}{\Psi}e^{\sigma S_o}$. Non-negativity of costs implies $h(t) \geq 0$ $\forall t$, which then implies that $h(0) \geq \frac{\rho}{\rho+g}\frac{\overline{P}}{\Psi}e^{\sigma S_o}\left(1 - e^{-(\rho+g)\frac{S_0}{\overline{E}}}\right)$.

Proof of Proposition 5. Similar to proof of Proposition 3.

References

[1] Agostini, C.A. (2006). Estimating market power in the US copper industry. Review of Industrial Organization, 28 (1), 17-39.

[2] Berck, P., & Roberts, M. (1996). Natural resource prices: Will they ever turn up? Journal of Environmental Economics and Management, 31 (1), 65-78.

[3] Chakravorty, U., Roumasset, J., & Tse, K. (1997). Endogenous substitution among energy resources and global warming. The Journal of Political Economy, 105 (6), 1201-1234.

[4] Chapman, D. (1993). World oil: Hotelling depletion or accelerating use? Natural Resources Research, 2 (4), 331-339.

[5] Chapman, D., and Khanna, N. (2000). World oil: The growing case for international policy. Contemporary Economic Policy, 18 (1), 1-13.

[6] Cremer, J., and Salehi-Isfahani, D. (1991). Models of the oil market. Harwood Academic Publishers, New York.

[7] Cremer, J., and Weitzman, M.L. (1976). OPEC and the monopoly price of world oil. European Economic Review 8, 155-164.

[8] Deffeyes, K.S. (2001). Hubbert's peak: the impending world oil shortage. Princeton, NJ: Princeton University Press.

[9] Farrow, S. (1985). Testing the efficiency of extraction from a stock resource. The Journal of Political Economy, 93 (3), 452-487.

[10] Farzin, Y.H. (1992). The time path of scarcity rent in the theory of exhaustible resources. Economic Journal 102, 841-851.

[11] Farzin, Y.H. (1995). Technological change and the dynamics of resource scarcity measures. Journal of Environmental Economics and Management 29, 105-120.

[12] Hanson, D.A. (1980). Increasing extraction costs and resource prices: Some further results. The Bell Journal of Economics 11(1), 335–342.

[13] Hnyilicza, E. and Pindyck, R.S. (1976). Pricing policies for a two-part exhaustible resource cartel: The case of OPEC. European Economic Review 8, 139-154.

[14] Hoel, M. (1978). Resource extraction, uncertainty, and learning. The Bell Journal of Economics 9(2), 642–645.

[15] Hotelling, H. (1931). The economics of exhaustible resources. The Journal of Political Economy, 39 (2), 137-175.

[16] Khalatbari, F. (1977). Market imperfections and the optimum rate of depletion of natural resources. Economica 44(176), 409–414.

[17] Krautkraemer, J.A. (1998). Nonrenewable resource scarcity. Journal of Economic Literature, 36 (4), 2065-2107.

[18] Lin, C.-Y.C. (2008a). An empirical dynamic model of OPEC and non-OPEC. Working paper. University of California at Davis.

[19] Lin, C.-Y.C. (2008b). Insights from a simple Hotelling model of the world oil market. Working paper. University of California at Davis.

[20] Lin, C.-Y.C. & Wagner, G. (2007). Steady-state growth in a Hotelling model of resource extraction. Journal of Environmental Economics and Management, 54 (1), 68-83.

[21] Loury, G.C. (1986). A theory of 'oil'igopoly: Cournot equilibrium in exhaustible resource markets with fixed supplies. International Economic Review, 27 (2), 285-301.

[22] Pesaran, M.H. (1990). An econometric analysis of exploration and extraction of oil in the U.K. Continental Shelf. The Economic Journal 100 (401), 367-390.

[23] Pindyck, R.S. (1976). Gains to producers from the cartelization of exhaustible resources. MIT Energy Lab working paper (No. MIT-EL 76-012WP).

[24] Pindyck, R.S. (1978). The optimal exploration and production of nonrenewable resources. J. of Political Economy, 86 (5), 841-861.

[25] Pindyck, R.S. (1980). Uncertainty and exhaustible resource markets. The Journal of Political Economy 88(6), 1203–1225.

[26] Rausser, G.C. (1974). Technological change, production, and investment in natural resource industries. The American Economic Review, 64(6), 1049-1059.

[27] Salant, S.W. (1976). Exhaustible resources and industrial structure: A Nash-Cournot approach to the world oil market. The Journal of Political Economy, 84 (5) 1079-1094.

[28] Sinn, H.-W. (1984). Common property resources, storage facilities and ownership structures: A Cournot model of the oil market. Economica, 51 (203), 235-252.

[29] Slade, M.E. (1982). Trends in natural resource commodity prices: An analysis of the time domain. Journal of Environmental Economics and Management, 9 (2), 122-137.

[30] Solow, R.M. and Wan, F.Y. (1976). Extraction costs in the theory of exhaustible resources. The Bell Journal of Economics 7(2), 359–370.

[31] Stiglitz, J.E. (1976). Monopoly and the rate of extraction of exhaustible resources. The American Economic Review 66(4), 655–661.

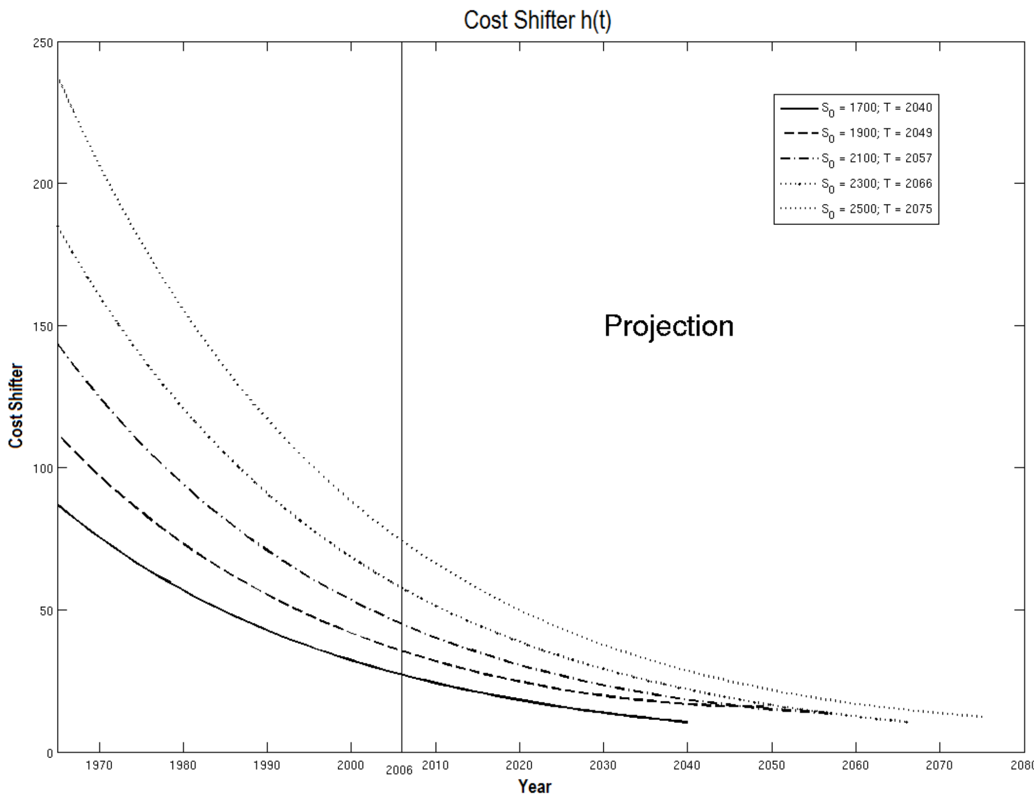
[32] Sweeney, J.L. (1977). Economics of depletable resources: Market forces and intertemporal bias. The Review of Economic Studies 44(1), 124–141.

[33] Tietenberg, T.H. (1996). Environmental and natural resource economics (4th ed.). New York: HarperCollins.

[34] Ulph, A.M., and Folie, G.M. (1980). Exhaustible resources and cartels: An intertemporal Nash-Cournot model. The Canadian Journal of Economics, 13 (4), 645-658.

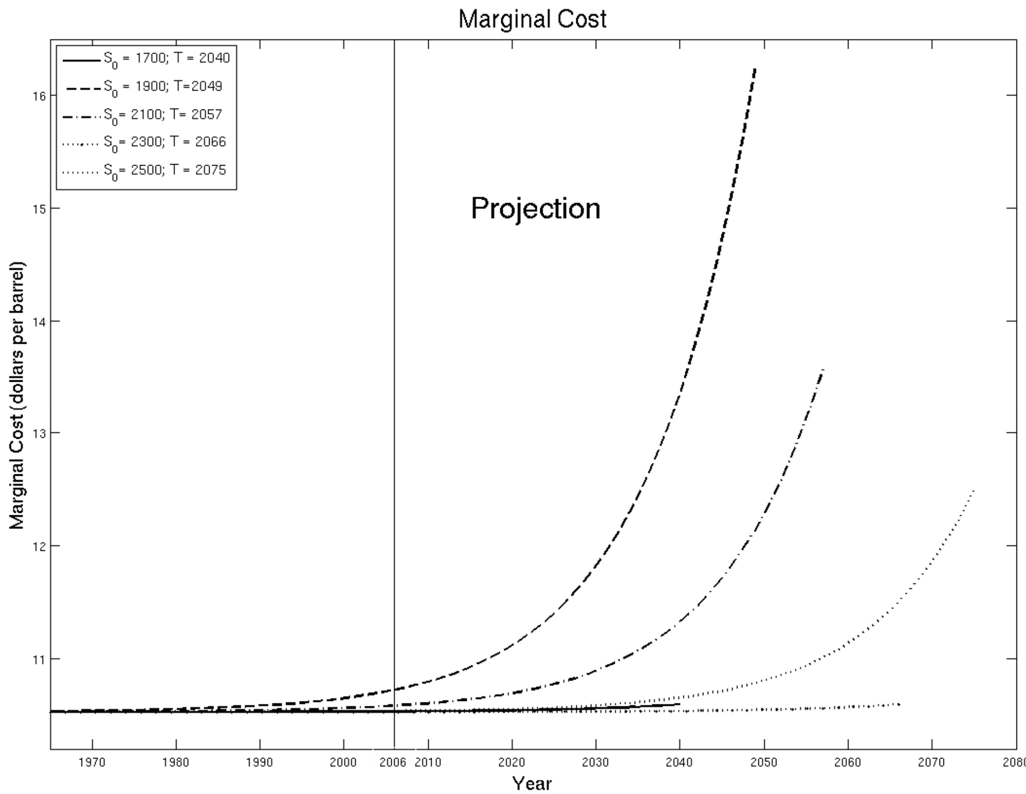
[35] Weitzman, M.L. (2003). Income, wealth, and the maximum principle. Cambridge, MA: Harvard University Press.

Figure 1: The cost shifter for oil under a simple stock effect.



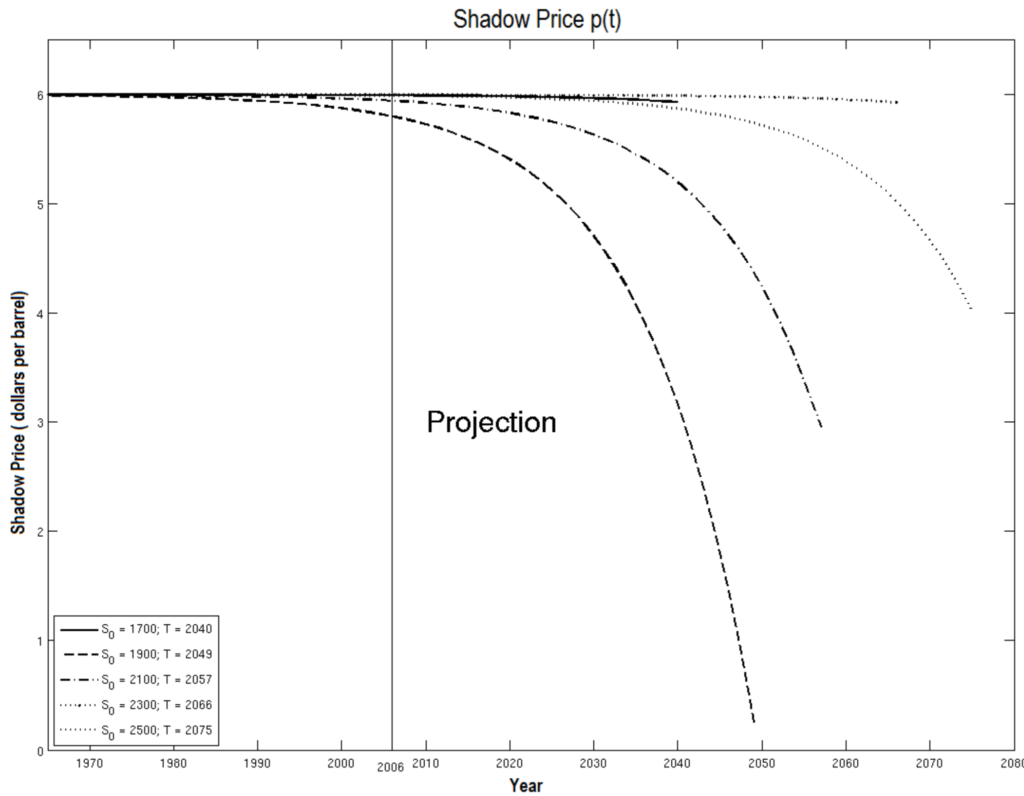
Notes: This figure presents a simulation of the trajectory for the cost shifter *h(t)* that maintains a constant market price for oil in the model of the simple stock effect, as given by Corollary 4, under various specifications for the initial stock S_0 of reserves. S_0 is initial stock of oil in billion barrels. T is the time at which oil will be depleted. We set P equal to the mean real world oil price over 1965-2006, which was 16.52 dollars (1982-1984 U.S. dollars) per barrel, and set E equal to the mean world oil consumption over 1965-2006, which was 61.92 million barrels per day. For the cost function, we use the parameter estimates for Ψ and σ reported in Chakravorty, Roumasset and Tse (1997). We set the the discount rate at ρ=0.05.

Figure 2: Marginal cost for oil under a simple stock effect.



Notes: This figure presents a simulation of the trajectory for the marginal cost when market price for oil is constant in the model of the simple stock effect, as given by Corollary 4, under various specifications for the initial stock S_0 of reserves. S_0 is initial stock of oil in billion barrels. T is the time at which oil will be depleted. We set P equal to the mean real world oil price over 1965-2006, which was 16.52 dollars (1982-1984 U.S. dollars) per barrel, and set E equal to the mean world oil consumption over 1965-2006, which was 61.92 million barrels per day. For the cost function, we use the parameter estimates for Ψ and σ reported in Chakravorty, Roumasset and Tse (1997). We set the the discount rate at ρ=0.05.

Figure 3: The shadow price for oil under a simple stock effect.



Notes: This figure presents a simulation of the trajectory for the shadow price when market price for oil is constant in the model of the simple stock effect, as given by Corollary 4, under various specifications for the initial stock S_0 of reserves. S_0 is initial stock of oil in billion barrels. *T* is the time at which oil will be depleted. We set P equal to the mean real world oil price over 1965-2006, which was 16.52 dollars (1982-1984 U.S. dollars) per barrel, and set E equal to the mean world oil consumption over 1965-2006, which was 61.92 million barrels per day. For the cost function, we use the parameter estimates for Ψ and σ reported in Chakravorty, Roumasset and Tse (1997). We set the the discount rate at ρ=0.05.

Table 1. Simulation scenarios

S_0 (Initial Stock)	**T (End Year)**	**h_0 (Cost shifter)**
1700 billion barrels	2040	86.933
1900 billion barrels	2049	111.9
2100 billion barrels	2057	143.855
2300 billion barrels	2066	184.996
2500 billion barrels	2075	237.96

Note: This table presents the values for the time T at which oil will be depleted and the initial value h_o of the cost shifter used for each value of the initial stock S_0 analyzed in the simulations presented in Figures 1-3 of the oil market when market price for oil is constant and there is a simple stock effect. S_0 is initial stock of oil in billion barrels. T is the time at which oil will be depleted. We set P equal to the mean real world oil price over 1965-2006, which was 16.52 dollars (1982-1984 U.S. dollars) per barrel, and set E equal to the mean world oil consumption over 1965-2006, which was 61.92 million barrels per day. For the cost function, we use the parameter estimates for Ψ and σ reported in Chakravorty, Roumasset and Tse (1997). We set the the discount rate at ρ=0.05.