Tax policy can change the production path:
A model of optimal oil extraction in Alaska

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ABSTRACT

We model the economically optimal dynamic oil production decisions for seven production units (fields) on Alaska's North Slope. We use adjustment cost and discount rate to calibrate the model against historical production data, and use the calibrated model to simulate the impact of tax policy on production rate. We construct field-specific cost functions from average cost data and an estimated inverse production function, which incorporates engineering aspects of oil production into our economic modeling. Producers appear to have approximated dynamic optimality. Consistent with prior research, we find that changing the tax *rate* alone does not change the economically optimal oil production path, except for marginal fields that may cease production. Contrary to prior research, we find that the *structure* of tax policy can be designed to affect the economically optimal production path, but at a cost in net social benefit.

Keywords: oil production, taxation, Alaska

INTRODUCTION

Recent high oil prices have prompted oil-holding nations and states to revise their tax policies, including increasing tax rates and introducing credits and deductions meant to motivate exploration and development investment. Our research seeks to inform such policymaking by investigating the effect of government tax policy on firm behavior in oil production in Alaska. The main novelty of our paper is modeling the effects of a wide variety of tax *structures* (not just tax *rates*) on dynamically optimal oil production paths. To complete this modeling subject to data constraints, we develop a method for constructing field-specific cost functions that incorporate engineering aspects of oil production without direct observations of production cost.

We address the following questions: have oil producers approximated dynamically optimal production? Can tax policy affect the production path, for example by encouraging more rapid or more gradual oil production? Does government tax policy create inefficiency in the oil industry?

Our research approach was to simulate economically optimal production paths for units on the Alaska North Slope, compare them to actual production data to evaluate past producer behavior, and then use a model calibrated to historical data to simulate the effects of alternative tax policies on production paths and on the present discounted values of producer profits and state tax revenue. We present results for a range of tax policies, including the actual historical policies and an approximation of a new severance tax policy enacted in 2006 (revised in 2007). We also present empirical estimates for wellhead price, drilling cost, an inverse production function for producing wells, and field-specific production cost functions.

When a new oil field is found in Alaska, its extent is mapped and all lease-holders with a claim on the reserve must sign a unit operating agreement prior to production (Alaska Statute 31.05.110). Since unit agreements mitigate potential strategic interactions, we model the oil production decisions of the unit operator as the single owner of the resource, optimizing total production of the common resource. Hence, we model oil production for the seven individual units on Alaska's North Slope: Prudhoe Bay, Kuparuk River, Milne Point, Endicott, Badami, Colville River, and Northstar.[1] In general, we present data and results for the Prudhoe Bay unit in this paper; similar information for the other six units is available in the online Annex.

	Prudhoe	Kuparuk	Milne	Endicott	Badami	Colville	Northstar
Start Date	Jan.1978	Nov.1981	Oct.1985	Jun.1986	Jul.1998	Oct.2000	Sept.2001
Initial OIP	28,764	5,351	1,747	1,127	240	920	247
Initial Technically Recoverable Reserves (million bbl)							
	14,382	2,675	874	564	120	460	124
Technically Recoverable Reserves Remaining in 2006 (million bbl)							
	2,902	478	624	114	115	231	15
Historical Production (million bbl per month)							
Mean	33.02	7.29	0.98	1.82	0.05	3.11	1.72
Max.	51.85	10.52	1.83	3.70	0.22	4.18	2.44
Min.	6.00	1.09	0.00	0.00	0.00	0.53	0.00
Std. Dev.	13.03	2.04	0.59	1.19	0.04	0.65	0.45
Wellhead Value ($/bbl, 1982-84 dollars)							
Mean	12.19	11.95	10.69	10.59	13.66	15.86	16.47
Max.	27.90	27.90	27.90	27.90	27.90	27.90	27.90
Min.	5.05	5.05	5.05	5.05	5.05	9.43	9.43
Std. Dev.	5.61	5.52	4.99	5.04	6.45	5.97	6.26
Wells (count)							
Mean	701	378	86	50	5	37	13
Max.	961	552	142	64	7	59	19
Min.	113	1	1	1	2	13	1
Std. Dev.	264	138	45	14	1	13	5

Table 1: Summary statistics for historical data by unit. We scaled the original Oil in Place (OIP) data by 50% to estimate initial technically recoverable reserves. Note, maximum TAPS throughput is approximately two million barrels per day.

In practice, some strategic interactions may persist. For example, production shares for oil and gas may differ since individual leases are located above the oil reserve or gas cap. Since natural gas on the North Slope is stranded, without a pipeline to deliver it to market, the unit operator's decision to process associated gas into natural gas liquids (NGL) for shipment down the Trans-Alaska Pipeline System (TAPS) or for re-injection to boost oil recovery (flaring is not permitted) will depend on its relative oil and gas shares of production (Libecap and Smith, 1999). In this paper, however, we do not consider this type of potential strategic interaction and have omitted natural gas production decisions from our analysis since it is a stranded resource.

Our research builds on past work. The oil crises of 1973 and 1979/1980 motivated modeling designed to forecast future supply and demand. A dichotomy formed between models based on economic theory describing supply and demand interactions (Dasgupta and Heal, 1979; Pindyck, 1982; Horwich and Wimer, 1984; Griffin, 1985) and engineering-process models that simulate the exploration, development, and production processes (Davidsen et al., 1990). Neither approach accurately forecast future supply and demand (Kaufmann, 1991). Ruth and Cleveland (1993) extended this literature by using a nonlinear dynamic model of oil exploration, development, and production to simulate optimal depletion paths for the 48 contiguous United States in the period 1985 to 2020. They used the theoretical model of optimal depletion developed by Pindyck (1978). These integrated modeling efforts produced interest in more detailed consideration of producer-level decision making.

Our focus on the impact of severance tax policy on oil production is most directly related to a study by Kunce (2003), who also considered severance tax incentives in the U.S. oil industry.[2] However, Kunce continued with integrated modeling of exploration, development and production by extending previous research by Deacon et al. (1990) and Moroney (1997) and embedding tax policy into Pindyck's (1978) theoretical model of exhaustible resource supply.[3] As a result, Kunce was limited in the complexity of tax policies that could be modeled and, consequently, found that changes in severance tax *rates* had little effect on oil field activity.

We model producer behavior at the unit level, taking known fields as given (i.e., the exploration stage is complete) and modeling production decisions only. Less complexity in our model structure enabled consideration of more complex tax policies (i.e., with credits for investment expenditures), which produced the finding that changes in tax policy *structure* can affect the optimal time path of oil production while changes in tax *rate* do not. Thus, our findings confirm results found by Helmi-Oskoui et al. (1992) and Kunce (2003) but contribute the additional insight regarding tax structure that implies a different interpretation for public policy than the conclusion offered by Kunce.

METHODS

Assuming Alaskan unit operators are price takers selling into the world oil market at market price, their optimization problem is to choose the production profile $\{Q(t)\}$ to maximize the present discounted value of the entire stream of future profits from oil production.

The state of Alaska collects four types of tax related to oil production: royalty, severance tax, corporate income tax, and property tax. We focus on the two largest: royalty and severance tax. Royalty refers to payments made to a landowner – the state government in the case of Alaska – for the rights to produce oil. Severance tax is imposed on the extraction of a natural resource, for its severance from the state in which it originated. Until recently, the severance tax in Alaska was adjusted by the economic limit factor (ELF), which was a fraction between zero and one.[4]

Unit operators are constrained by four physical realities of non-renewable resource extraction: the change in reserve is equal to the rate of production, the rate of production is nonnegative (i.e., producers do not re-inject oil), the stock is nonnegative (i.e., no oil can be produced when the stock is depleted), and the stock available in the first period equals the initial reserve of the resource. Consequently, the unit operator's optimal control problem can be written as follows:

$$(1) \quad Max_{\{Q_i(t)\}} \sum_{t=0}^{\infty} \beta^t \{P(t)Q_i(t)\left(1 - R_{it} - T_{it}F_i\big(Q_i(t)\big)\right) - C_i\big(Q_i(t), S_i(t)\big)\}$$

$$\text{s.t.} \quad \begin{aligned} & S_i(t+1) - S_i(t) = Q_i(t) \\ & Q_i(t) \geq 0 \\ & S_i(t) \geq 0 \\ & S_i(0) = S_o \end{aligned}$$

where i indexes units and t indexes time, $P(t)$ is the wellhead value (market price less shipping cost) for Alaska North Slope crude, $S_i(t)$ is reserves remaining, $Q_i(t)$ is the oil production per period, $C_i(Q_i(t),S_i(t))$ is the total cost of production given by the Composite Cost Function, R_{it} are the lease royalty percentages, T_{it} are the severance tax percentages modified by the economic limit factor $F_i(Q_i(t))$, β is the discount factor $1/(1+\rho)$, and ρ is the discount rate. Since oil producers in Alaska are price takers whose production does not influence the world oil price, the price function is exogenous.

By the principle of optimality, we can re-write the optimal control problem in Equation 1 using the following Bellman Equation.

$$(2) \qquad V(S_i(t)) = \max_{Q_i(t)}[P(t)Q_i(t)(1-R_{it}-T_{it}F_i(Q_i(t)))-C_i(Q_i(t),S_i(t))]+\beta V(S_i(t+1))$$

Maximizing the first-period value function - subject to the equation of motion, initial reserves, and the restrictions that production and reserves remaining are nonnegative - solves for the entire optimal time path of production that maximizes the present discounted value of profits. The data used are listed in Table 2.

Variable	Units	Definition of Original Data	Source	Sample Mean
θ_i	Billion barrel	Original Oil in Place (OIP) for unit *i*	AOGCC, 2008	5.5
S_{it}	Billion barrel	Reserves remaining for unit *i* in month *t*, where S(0) = 50% of OIP	calculated	2.4
Q_{it}	Million bbl/mo.	Quantity oil produced from unit *i* in month *t*	McMains, 2007	10.6
α_t	$/barrel	Alaska wellhead value, weighted average for all destinations, annual 1978–2006	ADR, 2007	$12.19
μ_t	$/barrel	USA spot price, FOB, average weighted by volume, weekly, 1997–2004	EIA, 2007	$13.46
ϵ_t	$/barrel	Forecast USA wellhead value, 2004–2030, reference, low- and high-price cases, annual	EIA, 2007	$24.42 $17.91 $36.57
η_s	$/barrel	Total facilities investment cost of production (capital cost) in 2003 by field size (13 categories)	Attanasi and Freeman, 2005	$1.64[i] $1.35[ii]
Ш$_{it}$	Count	Number of active wells by field for each month of production	McMains, 2007	270
Π_t	$ mil./well $/ft.	Well drilling cost data for Alaska, per well and per foot	API, 1969-2004	$3.6 $341

[i] Average of the 13 categories defined by Attanasi and Freeman (2005). [ii] Average of facilities investment cost of production for monthly observations for all seven fields on the Alaska North Slope.

Table 2: Variable definitions, data sources, and sample means. Free on board (FOB) price is equivalent to wellhead value since the buyer pays the transportation cost from origin to the final destination. We used the urban consumer price index to adjust all monetary data to 1982-84 constant US dollars.

Field-specific cost functions

A function to define the cost of oil production is necessary for modeling under the assumption of profit-maximizing behavior. Information on the cost of oil production, however, is guarded as proprietary. Although Chakravorty et al. (1997) used cost data compiled by the East-West Center Energy Program to estimate extraction cost functions econometrically and Dismukes et al. (2003) compiled information on per-unit costs for oil and gas activities by water depth in the Gulf of Mexico, the distinct environment (arctic) and location (remote on-shore) of Alaska's North Slope suggest production costs differ from these other oil production operations. Consequently, we develop an estimation of Alaska-specific costs and field-specific cost functions for modeling the seven unique North Slope units.

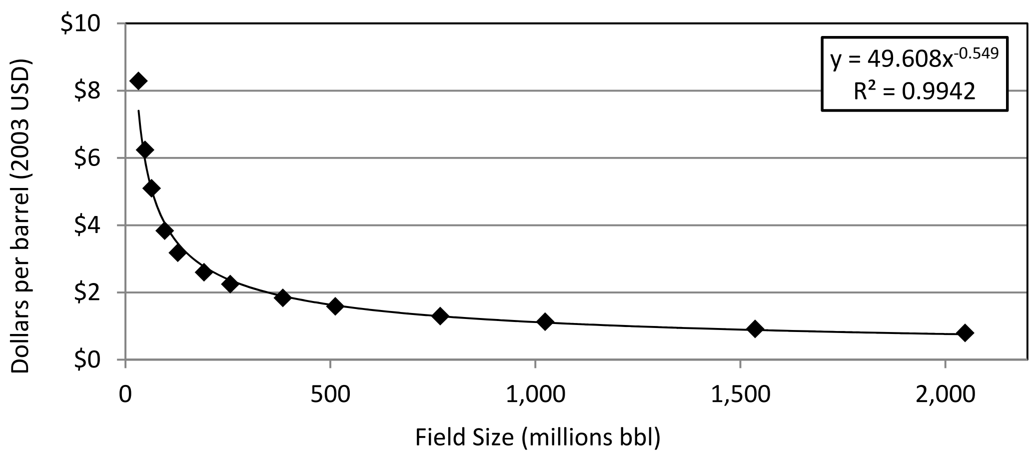

Figure 1: Average facilities investment cost (capital cost) of production showing economies of scale for increasing field size (Attanasi and Freeman, 2005). These estimates provide a reasonable approximation of total production costs since the Alaska oil industry is capital-dominated, meaning labor and other costs of production are relatively small (personal communication, Neal Fried, Alaska Department of Labor, July, 2007).

Economic theory and reservoir geology suggest a production cost function with the following three attributes: 1) economies of scale for increasing field size (evident in Figure 1);[5] 2) a time trend as the North Slope industry developed, technology improved and adapted to the arctic environment, rigs and labor became less limiting, and learning occurred for arctic operations (evident in Figure 2); and 3) diseconomies of scale for high production rate due to physical constraints on oil flow rate in the reservoir (evident in Figure 3). Since each field is unique in its geology, oil properties, and context of development, we estimate field-specific cost functions.

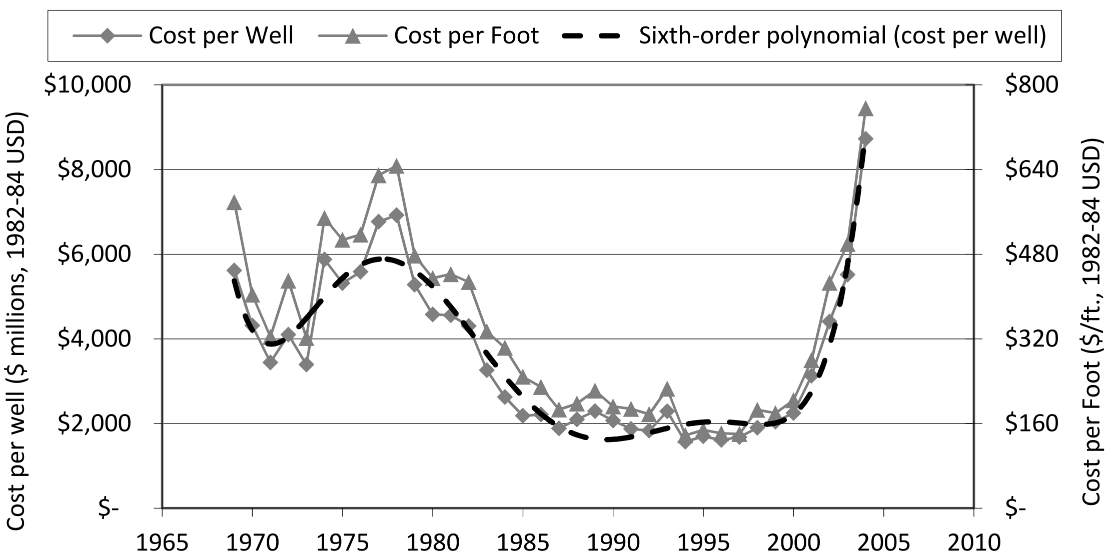

Figure 2: Well drilling costs in Alaska over time, with a sixth-order polynomial regression shown. Data on the drilling cost per well and per foot were compiled from the American Petroleum Institute's Joint Association Survey of the U.S. Oil and Gas Industry from the years 1969 through 2004 (API, 1969-2004). These costs are Alaska-specific; we used the cost of onshore oil wells and dry holes (i.e., we did not use cost data for offshore or gas wells).

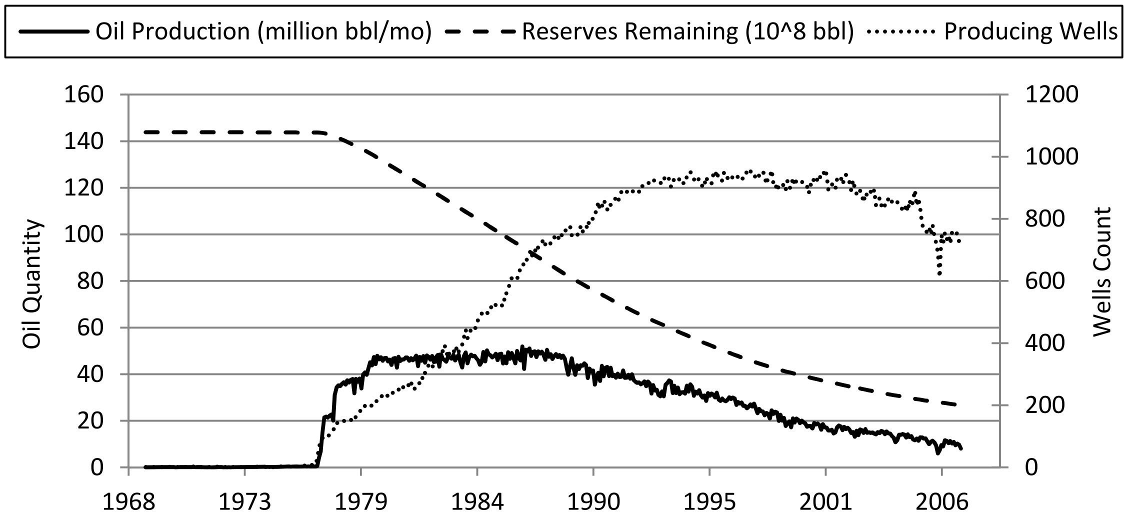

Figure 3: The historical number of producing wells, production rate, and reserves remaining for Prudhoe Bay over time. The number of wells increases in order to maintain a certain production rate while reserves remaining declines. In fact, the increased number of wells is often insufficient to maintain the production rate, causing the typical tailing-off of production for the field. Similar data for the other six North Slope units we modeled are shown in the online Annex.

We build a Composite Cost Function – $C_i(Q_i(t),S_i(t))$ – with these attributes by scaling an average Alaska North Slope cost function (incorporating economies of scale) by a constructed Alaska-specific Drilling Cost Scalar (incorporating time trends in technology and learning) and a field-specific Wells Scalar (incorporating physical constraints of reservoir and oil properties). We use the data from Attanasi and Freeman (2005) in Figure 1 to estimate a Base Average Cost fFunction that describes the average cost of production for a particular field size on the Alaska North Slope in 2003. We use the Alaska-specific well drilling cost data in Figure 2 to construct a scalar for the time trend in production cost. We use the field-specific well data in Figure 3 to construct a scalar for production rate. The Composite Cost Function is then defined as the product of the Base Average Cost Function and one or more of the two scalars, depending on conditions in the modeling (Equation 8). This produces field-specific production cost surfaces with marginal cost increasing as reserves are depleted and as production rate exceeds limits to reservoir flow rates. We now describe the estimation of the Composite Cost Function in detail, taking each of its three components in turn.

The Base Average Cost Function

We fit a continuous function for average cost ($/bbl) for oil production to the total facilities investment cost data estimated by Attanasi and Freeman (Figure 1). The reserves remaining in each field over time are calculated as initial reserves less cumulative monthly production. We then use the facilities investment cost function to estimate the average cost of production ($/bbl) associated with each field's remaining size in each month.[6] Multiplying this average cost of production by the quantity of production in a particular month yields the average total cost of production. We thus construct data on production rate (*Q*), reserves remaining (*S*), and total cost of production for each field in each month of 2003.

We use these data to estimate a total cost function, $cost_i = c_1 Q_i^{c_2} S_i^{c_3}$, which is similar in form to previous studies of oil production and incorporates both production and stock effects (Lin & Wagner, 2007; Lin, 2009; Lin et al., 2009; Allan et al., 2009). A log-linear form was used to estimate parameters by ordinary least squares, where *S* is reserves remaining (millions of barrels), *Q* is production rate (millions of barrels per month), and cost is measured in constant 1982-84 US dollars. The resulting Base Average Cost Function is given in Equation 3, and shown for Prudhoe Bay in Figure 4 (similar figures for the other six units are given in the online Annex).

(3) Base Average Cost Function:

$$c_i(Q_i(t),S_i(t)) = c_1 Q^{c2} S^{c3} = 9.15e7(Q^{1.00065})(S^{-0.549262})$$

Standard error: (3.40e5) (4.75e-4) (6.51e-4)

Adjusted R^2: 0.9999

All coefficients are statistically significant at the 0.1% level.

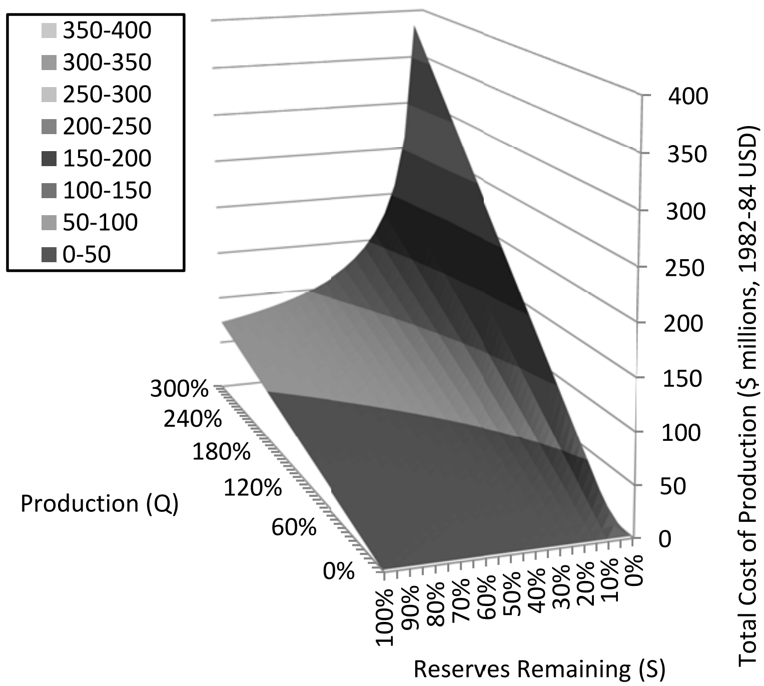

Figure 4: The base average total cost of production from Prudhoe Bay for combinations of reserves remaining and production rate (given in percentage terms, from zero to 100 percent of original technically recoverable reserves in the field and from 0 to 300 percent of historical maximum production rate).

Time Trend in Drilling (and Production) Costs

Drilling costs in Alaska have fluctuated over time (Figure 2). One explanation is quasi-rents from drilling equipment scarcity, materials costs, technological change, and improvement in operational knowledge. In this light, it is reasonable to think of a scalar for oil production cost based on drilling cost that is an approximation of similar fluctuations in the factors affecting the cost of oil production.[7]

Consequently, we use changes in drilling cost over time as a reasonable indicator for changes in total facilities and equipment costs of oil production. A drilling cost scalar (*d*) for multiplication of the base average cost function to account for the evolution of drilling costs over time (a proxy for changes in oil production costs) is defined as the ratio of drilling cost in a particular year relative to the reference cost in 2003 (Equation 4). The result is a multiplier for scaling the base cost function in years other than 2003 appropriately for changes in oil production costs. This drilling cost scalar ranges from 0.28 to 1.6.

(4) Drilling Cost Scalar: $d = c_4 + c_5Y + c_6Y^2 + c_7Y^3 + c_8Y^4 + c_9Y^5 + c_{10}Y^6$

	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
Coefficient	1.414	- 0.5840	0.1610	-0.01758	0.0008877	-0.0000211	1.92E-07
std. error	.1563	.1086	.02428	.002400	.0001165	2.72e-06	2.44e-08

Adjusted $R^2 = 0.9233$; all coefficients are statistically significant at the 0.1% level. The variable Y is indexed to the year 1969 to avoid overflow errors and adjusted for a two-year average lag between changes in drilling costs and oil production costs (e.g., for the year 1985, Y=15).

Since the rapid trend of increase in well drilling costs in the period 2000 to 2004 (Figure 2) is unlikely to continue indefinitely, we applied the drilling cost scalar in the composite cost function only for the historical period 1969 to 2004. The implicit assumption is that drilling costs remain constant at 2004 levels, aside from inflationary change, after 2004.

Inverse Production Function

Oil fields are generally characterized by the quantity of oil in place, the fraction that is technically recoverable, and the anticipated maximum production rate (and thus lifetime of the field). Since physical properties of the reservoir and oil determine maximum flow rate, economic modeling of optimal production should incorporate this physical reality. We choose to incorporate it in our cost function.

Physical limits to oil flow rate are unique for each particular field and imply decreasing returns to production rate. That is, the number of additional wells needed for an increment in production rate increases as production exceeds the reservoir flow rate

(Bedrikovetsky, 1993; Allain, 1979). We estimate field-specific inverse production functions from data on the number of operating wells to capture this effect in the Composite Cost Function. In particular, we regress the number of producing wells on oil production rate and reserves remaining. We estimate two inverse production functions, one presuming constant returns to scale (i.e., a plane, Equation 5) and a second presuming decreasing returns to scale (i.e., a convex surface, Equation 6). Since the number of wells required for oil production is reservoir specific, we estimate field-specific coefficients and allow the functional specification for the latter estimation to vary across fields. We use a stepwise variable selection process to define the decreasing returns model, with some iteration for the Prudhoe Bay and Kuparuk fields.[8]

(5) Constant returns wells plane: $w_i = c_{11i} + c_{12i}Q + c_{13i}S$

(6) Decreasing returns wells surfaces:

Prudhoe Bay: $W_P = c_{14P} + c_{15P}Q + c_{16P}Q^2 + c_{17P}Q^3 + c_{18P}S + c_{19P}S^2 + c_{20P}S^3$

Kuparuk River: $W_K = c_{14K} + c_{15K}Q + c_{16K}Q^2 + c_{17K}Q^3 + c_{18K}S$

Milne Point: $W_M = c_{14M} + c_{15M}QS + c_{16M}Q^2S + c_{17M}Q^3$

Endicott: $W_E = c_{14E} + c_{15E}QS + c_{16E}QS^2 + c_{17E}Q + c_{18E}Q^2 + c_{19E}Q^3 + c_{20E}S$

Colville: $W_C = c_{14C} + c_{15C}Q + c_{16C}Q^2 + c_{17C}Q^3 + c_{18C}S + c_{19C}S^2$

Northstar: $W_N = c_{14N} + c_{15N}Q + c_{16N}Q^2 + c_{17N}Q^3 + c_{18N}QS$

In Equations 5 and 6, w_i and W_i are the number of wells, for the constant returns and decreasing returns cases respectively. Regression results are presented in Tables 3 and 4, and example plots for Prudhoe Bay are shown in Figure 5. The Durbin-Watson statistics presented include a correction for first order serial autocorrelation using a Cochrane-Orcutt procedure (Ramanathan, 2002).

	Coefficient on:				
	Constant	Q	S	Adj. R^2	DW stat.
Colville	88.74***	0.4024	-0.1558***	0.9788	2.063
std. error	3.331	0.5406	0.006456		
Endicott	63.34***	3.680***	-0.06773***	0.9640	2.555
std. error	4.199	0.6659	0.01486		
Kuparuk	567.2***	2.759**	-0.1182***	0.9966	2.007
std. error	32.33	0.9377	0.02143		
Milne	341.3***	18.80***	-0.3549***	0.9909	2.647
std. error	42.42	2.013	0.05567		
Northstar	16.87***	2.482***	-0.1251***	0.9468	1.356
std. error	0.5411	0.2916	0.003566		
Prudhoe	1067***	1.100**	-0.06161**	0.9982	2.613
std. error	147.6	0.3379	0.02028		

Table 3: Parameter estimates for the constant returns wells plane, for Q and S in millions of barrels. Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).

	Coefficient on:								
Colville	Constant	Q	Q^2	Q^3	S	S^2		Adj. R^2	DW Stat.
Coefficient.	68.29***	8.077	-2.957	0.3580	-0.06314	-0.0001420		0.9789	2.031
std. error	12.46	7.738	2.828	0.3359	0.05522	8.367E-05			
Endicott	Constant	QS	QS^2	Q	Q^2	Q^3	S	Adj. R^2	DW Stat.
Coefficient	65.16***	0.1717***	-0.0002076***	-7.467	-8.061***	0.8540**	-0.1112***	0.9718	2.200
std. error	2.313	0.01754	0.00002135	4.036	1.968	0.3229	0.005755		
Kuparuk	Constant	Q	Q^2	Q^3	S			Adj. R^2	DW Stat.
Coefficient	513.4***	18.25**	-1.626	0.05035	-0.1069***			0.9967	2.107
std. error	42.99	6.259	1.022	0.05388	0.02986				
Milne	Constant	QS	Q^2S	Q^3				Adj. R^2	DW Stat.
Coefficient	144.0**	0.07291***	-0.03961**	5.551				0.9928	2.359
std. error	48.01	0.009908	0.01323	3.332					
Northstar	Constant	Q	Q^2	Q^3	QS			Adj. R^2	DW Stat.
Coefficient	0.8448	21.47***	-7.330*	1.265	-0.08180***			0.8904	0.7714
std. error	1.569	4.063	3.456	0.8765	0.003816				
Prudhoe	Constant	Q	Q^2	Q^3	S	S^2	S^3	Adj. R^2	DW Stat.
Coefficient	68.04	10.76***	-0.3047***	0.002887***	0.3434***	-4.594E-05***	1.512E-09***	0.9985	2.382
std. error	40.95	1.497	0.06473	0.0007389	0.01654	2.163E-06	8.711E-11		

Table 4: Parameter estimates for the decreasing returns wells surface. Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).

We discontinue analysis of the Badami unit at this point because historical data on wells and production revealed sporadic activity as production repeatedly failed to meet expectations. This can be seen in the historical production statistics in Table 1 and in the online Annex.

To incorporate the inverse production functions into our composite cost function, we again define a scalar (the "Wells Scalar") for multiplying the Base Average Cost Function to increase production cost when production rate exceeds physical limits to flow rate. We define this scalar as the ratio of the decreasing returns wells surface to the constant returns wells plane and invoked it in the Composite Cost Function only when the ratio was greater than one (i.e., when production rate exceeded the range of constant returns). Thus the Base Average Cost Function can be scaled up by the Wells Scalar. Figure 5 illustrates this concept graphically for Prudhoe Bay (similar figures for the other six fields are shown in the online Annex).

(7) Wells Scalar: $WS_i = W_i / w_i$

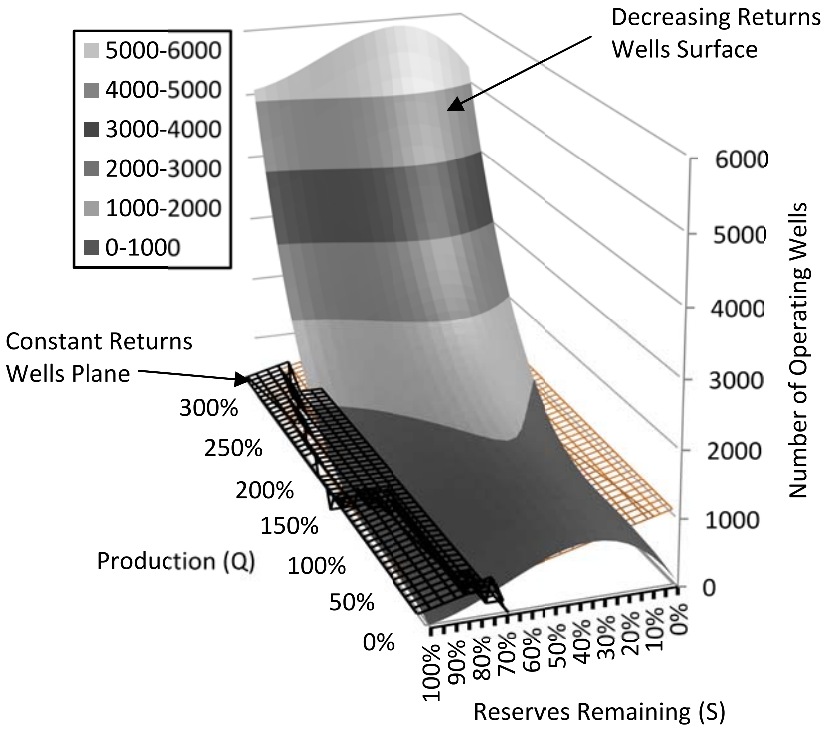

Figure 5: Prudhoe Bay wells as a function of production (Q) and reserves remaining (S), with the constant returns plane and the decreasing returns surface shown. The horizontal and depth axes are given in percentage terms, from zero to 100% of original technically recoverable reserves and from 0% to 300% of historical maximum production rate. The wells scalar is invoked when it is greater than one, which is true for Q, S combinations that cause the decreasing returns surface to climb higher than the constant returns plane. Note, this generally occurs when production rate is approximately 150% or more of the historical maximum, implying rational historical producer behavior in choosing production rate less than the level at which diminishing returns occur.

The Composite Cost Function

The Base Average Cost Function is estimated assuming rational behavior of producing in the realm of constant-returns (i.e., not pushing the production rate past physical limits to flow rate) in the year 2003. For our application of a cost function in a dynamic optimization model that does not constrain production rate and covers the period 1978 to 2170, we incorporate both decreasing returns in production rate and time trends in production cost into the Base Average Cost Function to create the Composite

Cost Function. Figure 6 shows an example of the Composite Cost Function for Prudhoe Bay (similar figures for the other six units are shown in the online Annex).

(8) Composite Cost Function: $C_i(Q_i(t),S_i(t)) =$

$$\begin{array}{lll} c_i(Q_i(t),S_i(t)) & \text{if} & w_i > W_i \text{ and Year} > 2004 \\ c_i(Q_i(t),S_i(t)) \bullet d & \text{if} & w_i > W_i \text{ and Year} \leq 2004 \\ c_i(Q_i(t),S_i(t)) \bullet WS_i & \text{if} & w_i \leq W_i \text{ and Year} > 2004 \\ c_i(Q_i(t),S_i(t)) \bullet WS_i \bullet d & \text{if} & w_i \leq W_i \text{ and Year} \leq 2004 \end{array}$$

In Equation 8, $c_i(Q_i(t),S_i(t))$ is the Base Average Cost Function defined by Equation 3, d is the Drilling Cost Scalar defined in Equation 4, WS_i is the Wells Scalar defined in Equation 7, and w_i and W_i are the number of wells (for the constant returns and decreasing returns cases, respectively).

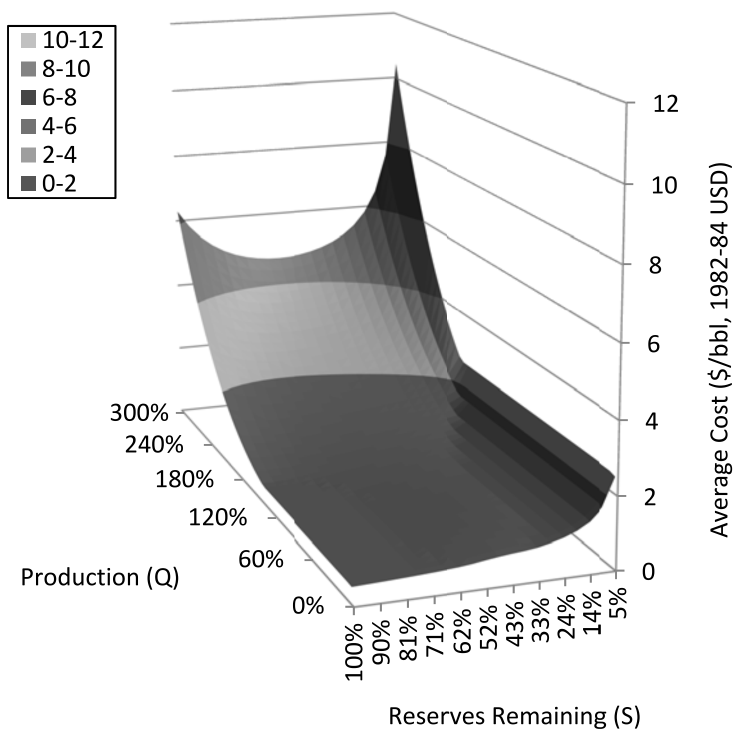

Figure 6: The composite cost function for Prudhoe Bay in 2003. In the range of historical production rates, production cost is approximately $0.5 to $1 per barrel in 1982-84 dollars. But getting the last few barrels of technically recoverable oil might cost $2.5 or more, and producing at twice the historical maximum rate might cost $2 per barrel.

Price Functions

We estimate price functions exogenously via regression analysis of historic Alaska wellhead value data and EIA price forecast data. We append the EIA oil price projections, which are for wellhead value (a.k.a. FOB price) in the contiguous United States through 2030 (EIA, 2007), to historical data on Alaska North Slope wellhead value to incorporate the EIA forecast modeling into our estimates of future price behavior (Figure 7). We fit functions to these data for the EIA reference, high price, and low price scenarios with multiple linear regression for a second degree polynomial functional form, where P(t) is wellhead value in 1982-84 dollars per barrel and Month is indexed to January, 1978 (Table 5).

(9) Price at the Wellhead: $P(t) = c_{21} + c_{22}\text{Month} + c_{23}\text{Month}^2$

	constant	Month	Month^2	Adj. R^2
Low Price	11.90***	0.01325	- 0.0000101	0.0207
std. error	2.507	0.01836	0.0000283	
Reference	12.11***	0.002013	0.0000404	0.4427
std. error	2.378	0.01742	0.0000268	
High Price	12.31***	- 0.01691	0.0001313***	0.8221
std. error	2.426	0.01777	0.0000274	

Table 5: Wellhead price function parameter estimates (1982-84 dollars per barrel; Month indexed to January, 1978). Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).

We include a fourth scenario for fixed price since conversations with industry suggested that long-range planning is often done with a single price estimate rather than a functional form of price projection (personal communication, Simon Harrison, BP Exploration Alaska, July 2, 2007). We examine the impact of all four price scenarios in our sensitivity analysis of the dynamic optimization model. We choose to use price scenarios rather than stochastic prices in this modeling because the former more accurately represents the decision-making processes used for investment timing and production decisions in major oil companies.

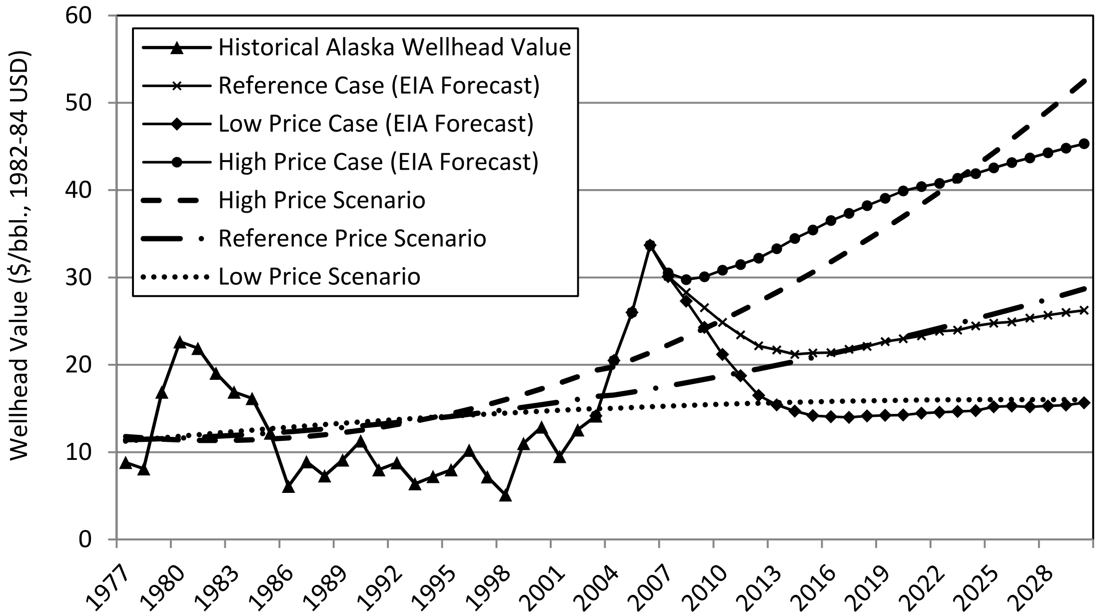

Figure 7: Alaska wellhead value with historical data (ADR, 2007) and projections (EIA forecasts, adjusted for Alaska's fiscal year and shipping costs). Three price scenarios were defined from these data by fitting Equation 10 to the historical data with low, reference, and high price EIA cases. A fourth price scenario was defined as a fixed price of $20 per barrel.

Modeling Economically Optimal Dynamic Oil Production

Having compiled data and estimated functions for the components of the unit operator's optimal control problem (Equation 1), we construct a model of the economically optimal dynamic oil production using the corresponding Bellman Equation (Equation 2). Maximizing the first period value function – subject to the equation of motion, initial reserves, and the constraints that production and reserves remaining are nonnegative – yields the entire optimal time path of production that maximizes the present discounted value of profits.

Calibration

To calibrate the model to historical production paths for each field, we constrain production in the first period to be equal to historical production in the first period and introduce an adjustment cost, defined as follows.

(10) Adjustment cost: $A = c_{25}(Q(t) - Q(t-1))^2$

Adjustment costs capture important aspects of reality for oil production, but it is difficult to distinguish between several potential underlying drivers. First, physical limitations to oilfield development like a finite number of available drilling rigs (which was especially limiting during the initial boom of North Slope exploration and development) and a short working season may constrain the ability to ramp up production. Incurring additional cost can push these limits to some extent. Second, increasing or decreasing production rapidly may cause inefficiency as the project timeline becomes a constraining factor, causing higher cost for insufficient labor, materials, or equipment supply and management decisions in favor of expediency rather than cost minimization. Third, hedging behavior against the risk of uncertainty in reservoir characteristics may dictate a gradual ramp-up in oil production to allow gathering of additional reservoir information and revision of the development plan along the way. Fourth, reservoir engineering considerations not captured in our modeling may dictate gradual increase and lower peak-production rate than our "uncalibrated model" suggests if rapid initial production causes a loss in reservoir pressure that compromises ultimate recovery. In this case the adjustment cost parameter serves as a proxy for foregone future production.

Subtracting adjustment cost from revenue in Equation 2 yields the value function used in the calibrated model.

(11) Value function for the Calibrated Model:

$$V(S_i(t)) = (P(t)Q_i(t)(1 - R_{it} - T_{it}F_i(Q_i(t))) - A - C_i(Q_i(t),S_i(t))) + \beta V(S_i(t+1))$$

The parameter c_{25} and the discount rate are set for each field to calibrate the model to historical actual production. We use an iterative procedure over a coarse and fine mesh of adjustment cost and discount rate parameters to identify the "best-fit" combination based on minimizing the sum of squared errors between the simulated optimum production path and historical actual production path.[9] We refer to this result as the best-fit scenario.

For calibrating the models, we use the actual historical tax policy for Alaska state royalty and severance taxes and the reference price scenario. To evaluate historical production decisions, we consider whether the discount rate and adjustment cost parameters that best calibrated the model to historical production are reasonable.

RESULTS

The economically optimal oil production paths modeled with the calibrated models for all six of the North Slope units (excluding Badami) are shown in Figure 8, along with historical actual production data. With initial production constrained and adjustment costs imposed in the calibrated models, model results fit historical data well. The discount rate and adjustment cost used to calibrate the models for each unit are given in Table 6.

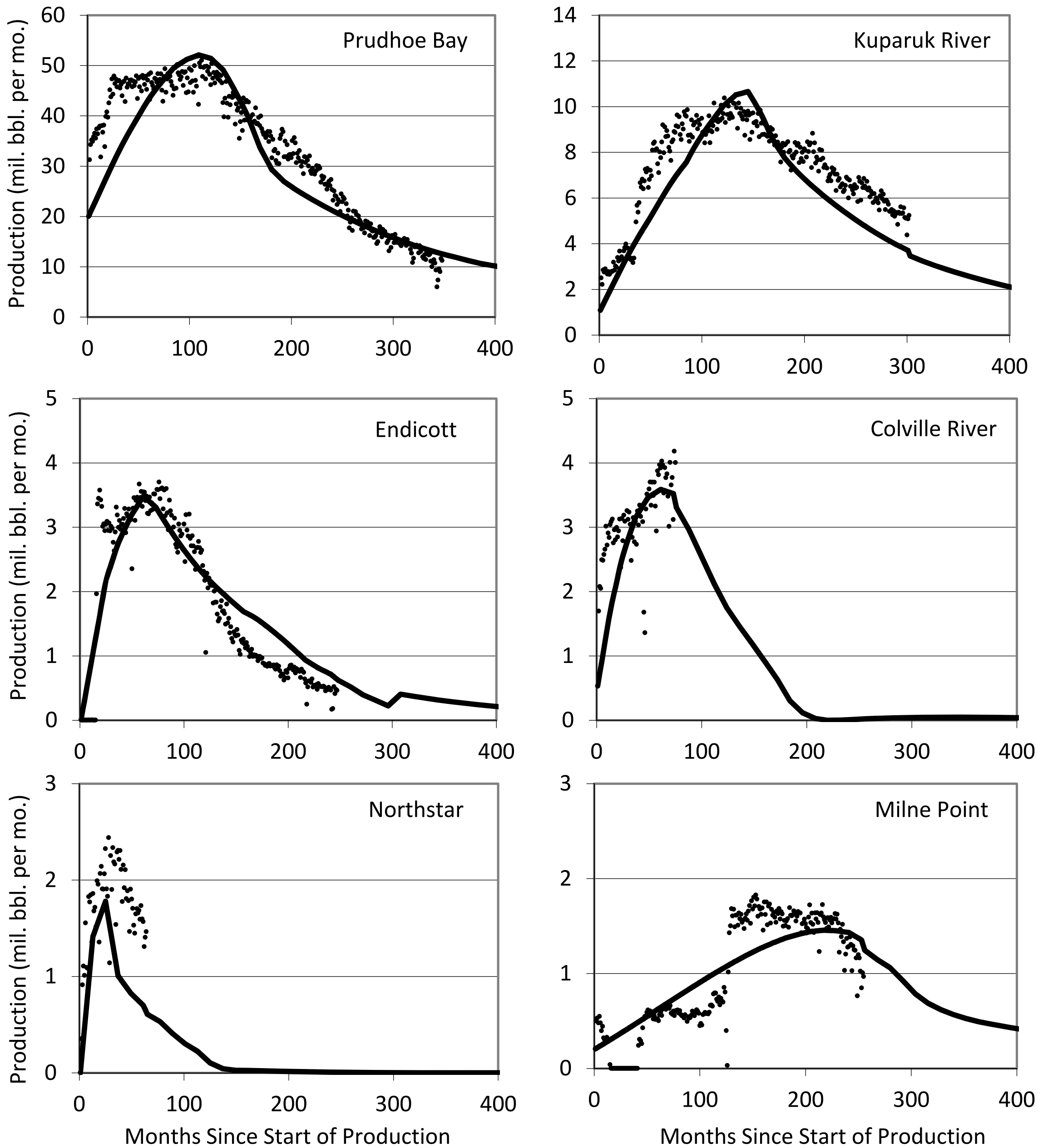

Figure 8: Historical production data and best-fit modeled optimal production path from the calibrated models for six units on the Alaskan North Slope.

North Slope Unit	Discount Rate	Adjustment Cost (c_{25})
Prudhoe Bay	7.4%	8*10^7
Kuparuk	8.6%	4*10^8
Milne Point	9.5%	6*10^9
Endicott	12%	7.5*10^7
Colville	20.5%	9*10^7
Northstar	46%	1.8*10^7

Table 6: Parameter values used in calibrated models. Initial production was set equal to historical initial production; adjustment cost and discount rate parameters were used to calibrate the model to the actual historical production path. The adjustment cost parameter c_{25} gives the cost in 1982-84 constant dollars per one million barrels-per-month change in production rate (e.g., $c_{25} = 8 \times 10^7$ implies $80 million adjustment cost to increase production capacity by 1 million barrels-per-month in one month). A real discount rate of 9 to 12 percent is considered reasonable for the petroleum industry (VanRensburg, 2000). Differences in adjustment cost across fields are likely due to a combination of relative size and differences in reservoir characteristics.

Sensitivity Analysis

We conduct sensitivity analyses with the un-calibrated and calibrated models to identify key parameters and evaluate the impact of uncertainty on simulated results. Parameter values used in these sensitivity analyses are summarized in Tables 7 and 8, with results for Prudhoe Bay shown in Figures 9 and 10 (similar results for the other six units are shown in the online Annex).

	Discount Rate	Wellhead Value	Taxes	Adjustment Cost
Reference Parameters	5%	Fixed, $20/bbl	none	none
Sensitivity Analyses				
Impact of High Price	5%	High P Scenario	none	none
Impact of Reference Price	5%	Ref. P Scenario	none	none
Impact of Low Discount Rate	2%	Fixed, $20/bbl	none	none
Impact of High Discount Rate	10%	Fixed, $20/bbl	none	none
Impact of Taxes, No ELF	5%	Fixed, $20/bbl	1	none
Impact of Taxes with ELF	5%	Fixed, $20/bbl	2	none

1. Historical Alaska state royalty and severance taxes, but *without* the ELF factor
2. Historical Alaska state royalty and severance taxes, *with* the ELF factor

Table 7: Parameter values used for sensitivity analysis of un-calibrated and calibrated model scenarios for Prudhoe Bay. Discount rates may differ for sensitivity analysis of other North Slope units (Table 8).

North Slope Unit	Low Discount Rate	High Discount Rate
Prudhoe Bay	2%	15%
Kuparuk	2%	15%
Milne Point	2%	15%
Endicott	5%	20%
Colville	10%	30%
Northstar	30%	60%

Table 8: Discount rates used in low- and high-discount-rate scenarios for sensitivity analyses of un-calibrated and calibrated modeling for six units on the North Slope.

We also test the sensitivity of model results to the Composite Cost Function we developed by shifting the constant returns plane up and down by +/- 50% for each unit. This change effectively alters the point at which decreasing returns to scale begin to increase cost (through the Wells Scalar), thereby changing the slope of the production cost function. Consequently, we refer to these sensitivity cases as the "steep" and "shallow" cost function scenarios.

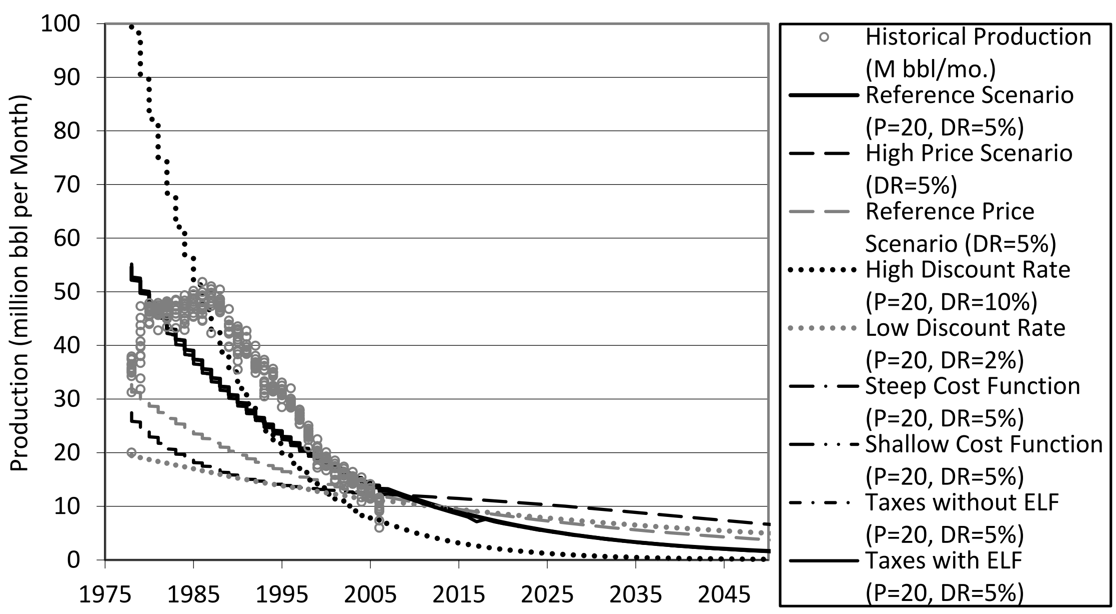

Figure 9: Sensitivity analysis for the un-calibrated model of Prudhoe Bay (i.e., unconstrained for initial production rate and without adjustment costs). Discount rate (DR) and oil price forecast (P) are given for each sensitivity case in parentheses.

Without initial production constrained or adjustment costs imposed in the uncalibrated model, results are negatively correlated with historical pre-peak production and positively correlated post-peak (Figure 9). The usual impact of discount rate is evident, with the higher rate (10%) shifting production into earlier periods as compared to the lower rate (2%). Comparison of the Reference Scenario (with price held constant at $20/barrel) to the High Price and Reference Price scenarios (both with price increasing over time) shows that an increasing price trend shifts production into later periods, all else equal. The modeled production path is most sensitive to discount rate, and is insensitive to changes in the cost function or ELF factor, which are visually indistinguishable from the Reference Scenario in Figure 9.

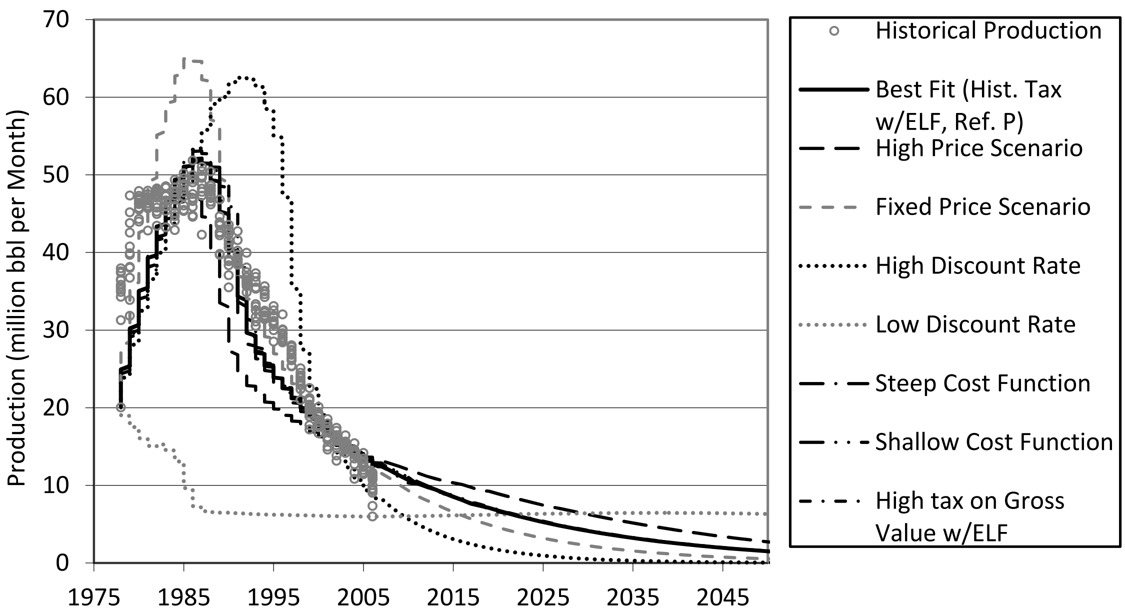

Figure 10: Sensitivity analysis of the calibrated model for Prudhoe Bay.

Sensitivities in the calibrated model are similar to the un-calibrated model. Production is shifted into earlier periods in the High Discount Rate scenario, producing higher peak production and precipitating an earlier end to production (Figure 10). The increasing price trend in the Best Fit and High Price scenarios shifts production into later periods, meaning the highest peak production occurs under the fixed price scenario. In other words, expectations for future price trends can impact the optimal production path as much as the selection of discount rate. As with the un-calibrated model, the modeled production path in the calibrated model is insensitive to changes in the cost function or ELF factor, which are visually indistinguishable from the best-fit scenario (Figure 10).

Tax Scenarios

Our modeling approach allows flexibility for incorporating almost any tax structure. We use the model to investigate the impact of several different tax structures on the optimal production path and net present value of profits and state tax revenue. These scenarios (Table 9) vary severance tax only, leaving royalty unchanged and leaving corporate income, property, and federal taxes omitted (all of which are relatively small for the units we modeled).

The baseline tax policy used in developing our "best fit" calibrated model was the policy that existed through 2006 under which historical production decisions were made. The severance tax was assessed on the gross wellhead value at a rate of 12.25% through June, 1981 and 15% thereafter, adjusted by the Economic Limit Factor for small and/or low-producing fields (Alaska Statute 43.55.150).

Our first hypothetical tax policy simulates a simple tax increase under the historical tax system by increasing the severance tax rate to 25% of gross wellhead value. Under such a policy, one would expect to see no change in the production path and the same net social benefit, but with tax revenue increased and profit decreased.[10]

This "first-best" policy does not distort the dynamic optimization of production and serves as the benchmark against which other policies are compared.

Our remaining hypothetical tax policies simulate combinations of severance tax rates and tax credits designed to encourage more rapid production by offsetting development expenses. The second hypothetical tax policy uses a severance tax rate of 25% of gross value with 20% credit for adjustment cost (up to but not exceeding the total tax burden). The tax rate remains relatively high in this scenario to offset the cost of credits refunded. Our third hypothetical tax policy repeats the simulation of the impact of 20% tax credits but with a lower tax rate on gross value (15%). Our fourth hypothetical tax policy repeats the simulation with 25% tax rate but with a higher credit rate (40%).

As with the first hypothetical tax policy, we expected to see no change in the optimal production path from changes in the tax rate, but expected to see the production path shifted to earlier periods by higher credit rates since some of the adjustment cost associated with more rapid ramp-up in production is "free" for the producer. Thus, we expected tax credits would motivate more rapid production, although at a cost to net social benefit due to distortion in the dynamic optimization (i.e., adjustment costs are partially borne by the government while production decisions are made by the unit operators).

Finally, our fifth hypothetical tax policy is an approximation of the new tax policy passed by the Alaska legislature in 2006 (amended in 2007). The Legislature's revision of the code for calculating severance tax (Alaska Statute 43.55) was a complex change from the former tax on the gross value of oil production (wellhead value) to a tax on net revenue, modified by a set of deductions and credits.[11] The new law essentially set a base tax rate of 25% of "tax value" (defined as gross value less allowable lease expense) that is modified by 20% to 40% credits for allowable expenditures (generally associated with exploration and development). We model this tax policy with a 25% tax rate on the net "tax value" defined as wellhead value less production cost, with 20% credit for adjustment cost. This approximation also facilitates direct comparison with our second hypothetical tax policy to investigate the impact of taxing the "net value" rather than gross value.

Scenario	Explanation	Tax Type	Severance Tax Rate / Credits (%)
Best Fit	Actual Historic Tax Policies	1	12-15 / 0*
Hyp. 1:	Higher Tax on Gross Value with ELF Adjustment & No Credits	1	25 / 0
Hyp. 2:	Higher Tax on Gross Value with Lower Credits	2	25 / 20
Hyp. 3:	Lower Tax on Gross Value with Lower Credits	2	15 / 20
Hyp. 4:	Higher Tax on Gross Value with Higher Credits	2	25 / 40
Hyp. 5:	Tax on Net with credits (Best Approximation of 2007 Policy)	3	25 / 20

*Severance tax rate was 12.25 percent through June, 1981 and 15 percent thereafter

Key to Tax Type	
1	State royalty and severance tax, with the ELF factor, no federal taxes
2	State royalty and hypothetical severance tax on gross WHV, no federal taxes
3	State royalty and hypothetical severance tax on net (WHV-cost), no federal taxes

Table 9: Tax policy scenarios simulated with the calibrated model of economically optimal dynamic oil production.

Results from simulation of these tax policies with the calibrated model of economically optimal dynamic oil production for Prudhoe Bay and the entire North Slope (all seven units) are shown in Figures 11 and 12 and Tables 10 and 11. Results are unique for each unit, but the interpretation of these results applies equally to all units. Results for other North Slope units are compiled in Annex 1 online.

A "first-best" tax policy does not distort the dynamic optimization of oil production, thereby maximizing the total surplus (defined as producer profit plus tax revenue). In our hypothetical tax policy scenarios, the first-best tax policy is approximated by the consistent tax rate on gross value in the first scenario (the ELF adjustment introduces minor deviation from first-best). The path induced by actual historical tax policy deviates slightly from this first-best policy due to the increase in severance tax rate in 1981 and the ELF adjustment factor, both of which acted to push production into earlier periods (Figures 11 and 12). Other hypothetical tax policies act to shift production into earlier periods as well by reimbursing a portion of adjustment costs, and also change the allocation of surplus between producer profit and tax revenue. But the total of profit and tax revenue – what we are calling the net social benefit – is highest under the first-best policy and is reduced by policies that induce shifts in the economically optimal production path (Tables 10 and 11).

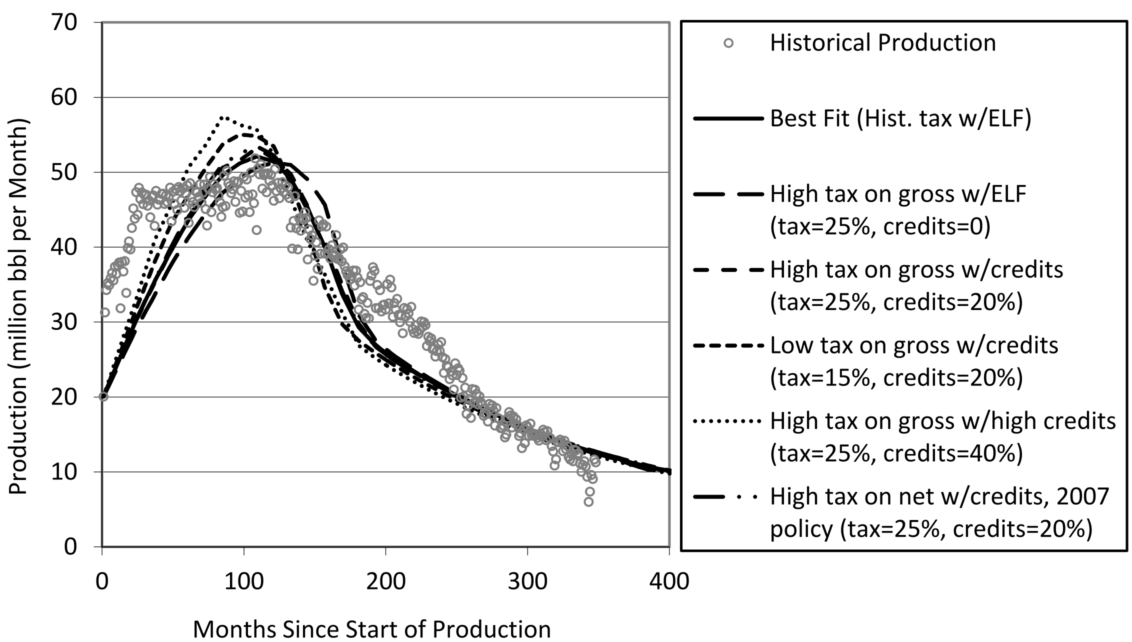

Figure 11: Historical Prudhoe Bay oil production data, modeled economically optimal production with historical tax policy (Best Fit), and modeled economically optimal production under several tax policy scenarios.

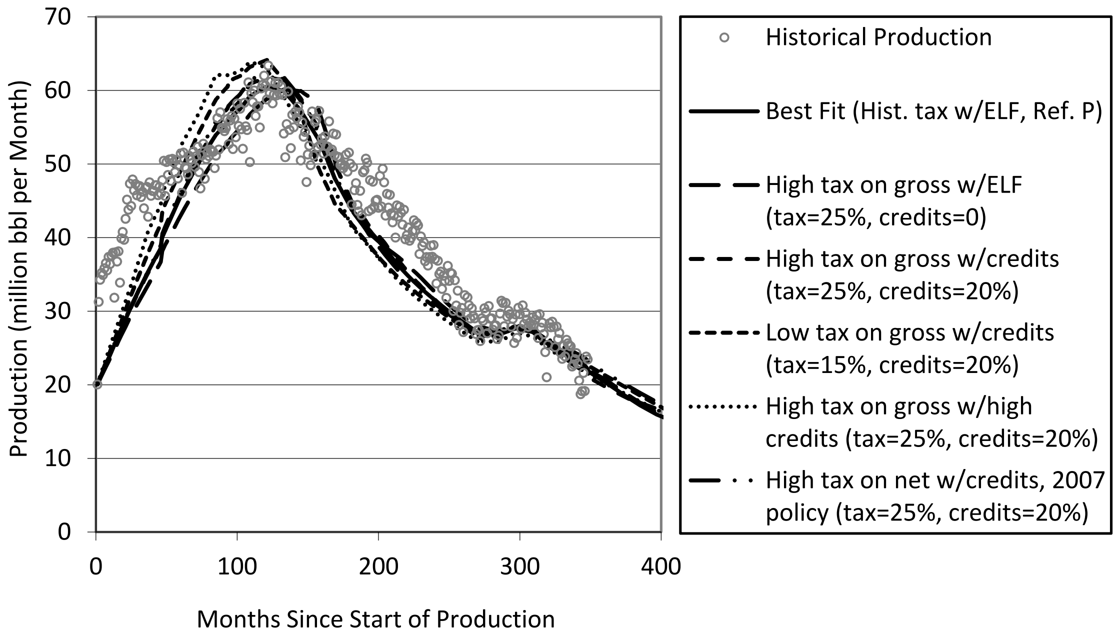

Figure 12: Historical total North Slope oil production data, modeled economically optimal production with historical tax policy (Best Fit), and modeled economically optimal production under several tax policy scenarios. The total North Slope production is the sum of production from seven independently-optimized production units.

Summary statistics for the main elements of the simulation of economically optimal oil production shown in Figure 11 are given in Table 10: production quantity and cost, wellhead value, producer profit, state taxes, adjustment cost, and state credits (for applicable tax scenarios). The present discounted values of producer profits and state taxes under each tax policy scenario are given in Table 11.

	Best Fit	Hypothetical Tax Scenarios					Historical Actual
		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	
Prod. End	2084	2084	2084	2083	2083	2083	2006
Production (millions of barrels)							
Mean	25.7	25.8	25.9	25.9	26.2	25.9	33.0
Maximum	52.1	51.3	53.2	55.0	57.5	53.5	51.9
(year)	(1987)	(1988)	(1987)	(1986)	(1985)	(1987)	(1986)
Minimum	0.25	0.26	0.25	0.26	0.25	0.26	6.00
(year)	(2084)	(2084)	(2084)	(2083)	(2083)	(2083)	(2006)
Std. Dev.	16.0	16.0	16.2	16.7	17.4	16.3	13.0
Production Cost ($/bbl)							
Mean	1.96	2.04	2.06	1.96	2.07	2.04	
Maximum	3.32	3.56	3.58	3.32	3.60	3.55	
(year)	(2084)	(2084)	(2084)	(2083)	(2083)	(2083)	
Minimum	0.43	0.43	0.44	0.45	0.46	0.44	
(year)	(1985)	(1985)	(1985)	(1984)	(1984)	(1985)	
Std. Dev.	0.98	1.05	1.06	0.97	1.06	1.05	
Wellhead Value ($/bbl)							
Mean	35.23	35.23	35.23	34.81	34.81	34.81	12.19
Maximum	79.93	79.93	79.93	78.68	78.68	78.68	27.90
(year)	(2084)	(2084)	(2084)	(2083)	(2083)	(2083)	(2006)
Minimum	12.11	12.11	12.11	12.11	12.11	12.11	5.05
(year)	(1978)	(1978)	(1978)	(1978)	(1978)	(1978)	(1999)
Std. Dev.	20.47	20.47	20.47	20.10	20.10	20.10	5.6
Producer Profit – for oil production from known fields, omitting exploration and overhead costs, net of taxes, including state credits ($ millions per year)							
Mean	1,266	1,063	1,070	1,236	1,058	1,100	
Maximum	5,444	4,697	4,719	5,799	5,384	4,859	
(year)	(1988)	(1989)	(1986)	(1987)	(1986)	(1987)	
Minimum	165	-1,226	-157	-444	-1,467	-185	
(year)	(2084)	(1992)	(1979)	(1991)	(1979)	(1979)	
Std. Dev.	1,187	1,081	1,030	1,236	1,112	1,047	
State Taxes – royalty and severance, net of state credits ($ millions per year)							
Mean	575	790	769	542	694	745	
Maximum	2,215	3,012	3,064	2,306	3,181	2,986	
(year)	(1987)	(1989)	(1987)	(1987)	(1987)	(1987)	
Minimum	67	92	90	67	89	89	
(year)	(2084)	(2084)	(2084)	(2083)	(2083)	(2083)	
Std. Dev.	565	776	723	513	657	704	
Net Social Benefit (Producer Profit + State Taxes)							
Mean	1,841	1,853	1,839	1,778	1,752	1,845	
Adjustment Cost ($ millions in one year)							
Mean	17	17	17	22	24	18	
Maximum	261	403	216	402	354	219	
(year)	(1992)	(1992)	(1992)	(1991)	(1981)	(1992)	
Minimum	0	0	0	0	0	0	
Std. Dev.	46	56	46	66	68	48	
State Credits ($ millions in one year)							
Mean			41	52	115	43	
Maximum			519	965	1701	526	
(year)			(1992)	(1991)	(1981)	(1992)	
Minimum			0	0	0	0	
Std. Dev.			111	157	326	114	

Table 10: Summary statistics for modeled simulation of economically optimal oil production from Prudhoe Bay under a variety of tax policies. Production is assumed to end when production rate falls below 0.5% of historical maximum production for the field or when producer profits become negative, whichever comes first.

In Table 11, the simulated production paths are compared to historical actual production with p-values for two-sample t-statistics testing the null hypothesis of equal means for historical and simulated production paths; correlation coefficients are given for comparison of these paths as well. Results indicate good fit of the "best fit" model to historical production, with little deterioration in fit for changes in the cost function or price scenario. The discount rate, however, alters the modeled production path significantly, thereby decreasing the fit to historical production. Hypothetical tax policies alter the production path as well, although to a lesser degree.

Table 11 also shows overall results for each tax policy scenario in terms of present discounted values of cash flows generated by the simulated production paths. The distribution between producer profit and government tax revenue can be changed with the tax rate without affecting the total if the tax structure is simple (i.e., no provisions like credits for adjustment cost that operate based on total tax liability). But tax policies that distort the dynamic optimization by shifting production into earlier periods with credits for adjustment costs deliver lower sums of profits and taxes.

Tax Policy Scenario	Two-sample t-test	Corr.	Results for Modeled Production Under Each Scenario, through 2175 PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Profits + Taxes
Prudhoe Bay: Sensitivity Analysis of Calibrated Model						
Best Fit (historical tax w/ELF, ref. price)	p=0.019	0.92	38,905	20,085	0	58,989
High Price Scenario	p=0.000	0.90	36,958	18,649	0	55,607
Fixed Price Scenario	p=0.042	0.92	65,479	35,440	0	100,919
Low Discount Rate Scenario (2%)	p=0.000	0.52	18,036	6,812	0	24,847
High Discount Rate Scenario (10%)	p=0.000	0.70	42,399	23,027	0	65,427
Steeper Cost Function Scenario	p=0.018	0.91	35,972	19,938	0	55,910
Shallower Cost Function Scenario	p=0.017	0.91	37,682	20,114	0	57,796
Prudhoe Bay: Tax Policy Scenarios						
1) Higher Tax on Gross w/ELF, no Credits	p=0.024	0.90	31,731	27,609	0	59,340
2) Higher Tax on Gross w/ Lower Credits	p=0.038	0.91	33,165	25,874	2,607	59,039
3) Lower Tax on Gross w/ Lower Credits	p=0.042	0.91	38,072	17,908	3,374	55,980
4) Higher Tax on Gross w/ Higher Credits	p=0.112	0.92	32,917	21,981	7,794	54,898
5) Tax on Net with Credits	p=0.041	0.91	33,727	25,054	2,689	58,782
Entire North Slope: Tax Policy Scenarios						
1) Higher Tax on Gross w/ELF, no Credits	p=0.024	0.91	37,567	32,020	0	69,587
2) Higher Tax on Gross w/ Lower Credits	p=0.038	0.92	38,011	31,791	3,315	69,802
3) Lower Tax on Gross w/ Lower Credits	p=0.042	0.91	44,082	22,048	4,246	66,130
4) Higher Tax on Gross w/ Higher Credits	p=0.112	0.92	38,013	27,105	9,364	65,118
5) Tax on Net with Credits	p=0.041	0.91	38,988	30,118	3,481	69,105

[i] including credits [ii] net of credits

Table 11: Present discounted values for modeled simulation of economically optimal oil production from Prudhoe Bay and the entire North Slope. P-values are given for two-sample t-statistics testing the null hypothesis of equal means for historical and simulated production paths; correlation coefficients are given for comparison of these paths as well. Profits are reported for oil production from known fields, omitting exploration and overhead costs. Values for the North Slope are the sum of the corresponding values for all seven of the independently-optimized production units. All profits, taxes, and credits are in millions of 1982-84 USD.

CONCLUSIONS

We present results from dynamic modeling of oil production in Alaska with a numeric value function maximization. Subject to uncertainty in the cost function estimation and calibration, the model can be used to simulate the impact of tax policies on economically optimal production paths, producer profits, and tax revenues, and can be used to evaluate the difference between model results and historical production (although interpretation as sub-optimal production versus modeling error remains elusive).

Our approach for constructing field-specific cost functions without direct production cost data produced reasonable results (Figure 6) and may be useful in other research where data are limited. Furthermore, model results were relatively insensitive to modifications to the cost function (Table 11 and Figures 9 and 10). Thus, although the cost function was one of the more difficult model components to determine, it was not among the most important factors for our results.

A constraint on initial production rate and an adjustment cost for rapid increases or decreases in production were necessary to calibrate the model to historical production (Figures 9 and 10). In general, a higher discount rate pushes production into earlier periods, higher adjustment cost delays production by slowing the rate of initial ramp-up, and price scenarios with trends of increase delay production by increasing the value of production in future periods.

Our results enable us to draw the following conclusions.

1. Producers have approximated dynamically optimal production

Calibration of our model to historical actual production provides some basis for evaluating whether producers have been dynamically optimal in their production decisions. VanRensburg (2000) has suggested that a real discount rate of 9 to 12 percent is reasonable for the petroleum industry; the average nominal discount rate used in the petroleum industry was 16% in 1985 and 14% in 2000 (ibid).[12] If the discount rate that best fits the model to historical production data is outside this reasonable range, it is suggestive of sub-optimal historical production, perhaps due to imperfect information rather than mismanagement.[13]

Four of the six fields modeled closely matched historic production with reasonable discount rates and adjustment costs, which is consistent with successful dynamic optimization by producers (Table 6). The relatively low best-fit discount rates for Prudhoe Bay and Kuparuk River may be commensurate with the relatively low risk of developing these large, well-defined “elephant” fields. By the same logic, a somewhat higher discount rate is appropriate for smaller, marginal, and hence riskier fields like Milne Point and Endicott.

The best-fit discount rates for two fields, Colville and Northstar, appear outside the realm of reasonable, which suggests historical management that was not dynamically optimal. The higher discount rate needed to calibrate the model to these historical production paths indicates historical production that was too fast, perhaps due to overly-optimistic resource evaluations, overly-pessimistic price forecasts, or a combination of these and other factors.

However, the production path is sensitive to the price scenario in a similar manner as the discount rate since both affect the present discounted value of future-period revenues. Since our reference price scenario forecasted wellhead value rising to more than $100 per barrel by 2100, a relatively high best-fit discount rate could also be interpreted as evidence of producers using a fixed price projection in their planning rather than as evidence of sub-optimal production. For example, the best-fit discount rate for Northstar with a constant price forecast of $20 per barrel was 20 percent.

The magnitudes of adjustment costs incurred for fitting modeled production to historical data were small (approximately one percent) relative to total producer profits and state tax revenue. Adjustment costs represent difficulty in turning oil production on and off. To the extent that the *percentage* change in production rate is what causes such difficulty, our finding of smaller adjustment cost parameters for larger fields is reasonable.[14]

2. Price trends affect optimal production paths

Our sensitivity analyses for price scenarios demonstrate the impact of a price *trend* on the optimal production path (Figures 9 and 10). A trend of increasing price tends to shift production to later periods while a constant price, regardless of level, pushes production into earlier periods, ceteris paribus. The implication is that both price *trends* and *levels* are necessary for adequate planning of optimal production. A policy of using fixed price forecasts in production planning ignores the importance of price *trends*, which can be more influential for results than the discount rate. Although oil producers have shied away from price forecasting in recent decades, our results illustrate the importance of at least including general price trends in scenarios for considering optimal production plans.

3. The structure of tax policy can influence dynamic optimality in oil production

We found that changing the tax rate alone before production begins does not change the optimal oil production path except for marginal fields that cease production. Introducing tax credits into the tax policy, however, *can* change the production path, but at the expense of net social benefit.

A fixed tax rate on gross revenue is the first-best policy because it does not distort the optimal production path. Thus, government can increase revenue, without altering the production path or net social benefit, by increasing the tax rate (Figures 11 and 12 and Table 11). But tax policies that introduce components to influence the production path (e.g., credits) result in lower net social benefit. Thus, government can also shift the production path with, for example, a system of tax credits, but at the expense of lower net social benefit.

These results are consistent with Kunce (2003), Helmi-Oskoui et al. (1992) and Uhler (1979), all of whom found that changes to the tax rate substantially change state tax revenue and producer profits but yield little or no change in the optimal time path of production. A change in tax rate does not change the producer’s dynamic optimization problem regardless of the magnitude or direction of change, unless the tax becomes high enough to cause production to cease. To this conclusion, however, we add the equally logical insight that severance tax policy *can* affect the optimal time path of production if distortionary components such as credits that modify the dynamic optimization problem are included. Consequently, the conclusion proposed by Kunce (2003) that, "states should be wary of arguments asserting that large swings in oil field activity can be obtained from changes in severance tax rates" should be qualified by the notion that the *structure* of a tax policy may make more impact on producer behavior than the tax rates or magnitude of revenue collection involved.

Our sensitivity analyses also revealed that the ELF, enacted with the intent to encourage continued production from marginal fields, has very little impact on the economically optimal dynamic production path, present discounted value of producer profits and state tax revenue, or date when modeled production ceases. However, the effect of the ELF factor on optimal production paths and NPV of profits and tax revenue is most apparent for Endicott and Northstar because the reduction in tax rate is relatively large for these marginal fields and makes a difference in profitability due to the fields' proximity to break-even operation. In contrast, the ELF factor has nearly zero impact on production paths or profits for large fields like Prudhoe Bay and Kuparuk River. These observations are reasonable since the ELF factor reduces the tax rate only when production rate is low, which occurs in earlier periods for small and/or marginal fields. Thus, it may be reasonable to say the ELF factor operated as it was designed to do.

The degree to which foresight is perfect or imperfect is also likely to affect the production path since production plans are based on expectations of future conditions. The implication here is that establishing expectations of future policy may be more important for influencing production decisions than changing current policy. Examination of perfect versus imperfect foresight is left to future work.

Finally, comparison of our approximation of the new Alaska tax policy enacted in 2006 (our fifth tax policy scenario) to an alternative with the same tax rate and credit rate (our second scenario) shows that assessing tax on the net tax value (gross value less allowable expenses) rather than gross value has negligible impact on the economically optimal production path but can shift allocation of the surplus toward producer profit. The economically optimal production paths for these two scenarios are visually indistinguishable in Figures 11 and 12, as are summary statistics for production in Table 10. The sum of producer profit and tax revenues also differs by less than one percent for these two scenarios (0.4% for Prudhoe Bay and 1% for the North Slope; Table 11). But the share of surplus collected as producer profits increases by 1.2 percentage points for Prudhoe Bay and 2.0 percentage points for the North Slope under the net tax value approach as compared to the equivalent tax on gross value.

Some notes on interpretation

The correct interpretation of the alternate production paths produced by our modeling is a retrospective of how things might have been under alternate conditions. We modeled the entire production path from startup to shutdown, presuming perfect information about future conditions, and we lack information on producers' ability to change production path once a field development plan has been implemented. Thus, our results are most applicable to the development of new fields rather than changing conditions for existing fields.

We have modeled the economically optimal production path for a known field, not the capital allocation for exploration and development across regions, which may be influenced by relative differences in tax level alone (not tax structure) and consequent differences in allocation of surplus between government revenue and producer profit.

The present discounted value of profits and state taxes should be evaluated relative to each other rather than as predictions of actual dollar amounts since their magnitude was influenced by the discount rate and adjustment cost factors and price scenario used.

The counter-intuitive result of lower PDV of profits with the “high-price” scenario is due to the lower initial prices in this scenario (Figure 7, Table 11). Thus, with discounting, the PDV of profits for the high-price scenario is less since profits in the early periods are less.

Research importance and policy implications

Better understanding of the dynamic landscape in Alaska’s oil industry may help avoid inefficiency in public policy and private investment decisions in the future. Private industry is considering investing tens of billions of dollars in Alaska over the next decade, and both state and federal governments are considering financial involvement as well. Potential projects include a major natural gas pipeline and development of oil and gas resources in the National Petroleum Reserve area (NPR-A) and outer continental shelf (OCS) areas offshore. As production from existing North Slope units declines, Alaska’s government continues to debate whether changes to the severance tax structure could encourage more exploration and development activity.

Our empirical analysis of production decisions enables examination of whether the dynamic optimum predicted by economic theory actually occurs in practice and how tax policy can change these production decisions. Modeling producer behavior at the field level with less complexity in our model structure than previous efforts enabled consideration of more complex tax policies (i.e., with credits for investment expenditures), which produced the finding that changes in tax policy *structure* can affect the optimal time path of oil production path while changes in tax *rate* do not.

We find that producers have approximated dynamically optimal production, that price trends affect optimal production paths, and that the structure of tax policy can influence dynamic optimality in oil production. Our research is of importance and relevance to industry practitioners who wish to dynamically optimize their oil production and who need to develop oil price scenarios for use in the production planning; to policymakers who wish to design policy to influence oil production decisions; and to researchers who wish to construct field-specific cost functions that incorporate engineering aspects of oil production without direct observations of production cost.

If policymakers wish to encourage more rapid oil production, then combinations of severance tax rates and tax credits designed to encourage more rapid production by offsetting development expenses would be effective. Higher credits rates would shift the production path to earlier periods since some of the adjustment cost associated with more rapid ramp-up in production is "free" for the producer. Thus, tax credits would motivate more rapid production, although at a cost to net social benefit due to distortion in the dynamic optimization (i.e., adjustment costs are partially borne by the government while production decisions are made by the unit operators).

Our flexible dynamic framework for considering the effect of policy on industry behavior can also be adapted to a variety of energy industries. For example, similar modeling of the Alaska natural gas industry may be important for understanding future domestic and low-carbon energy supplies, and for designing policies that influence energy production decisions, including policies that encourage the development of alternative sources of energy. The results may have implications for government tax and regulatory policy.

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[1] Although Prudhoe Bay was comprised of the East Operating Area (operated by ARCO) and the West Operating Area (operated by BP) prior to 2000 (BP, 2006), we treat it as a single field in our modeling.

[2] Another example of previous work considering the impact of tax policy on oil production industries is Pesaran's (1990) econometric model of offshore oil production in the UK, which was extended to include taxes by Favero (1992). However, the shadow value of oil in these analyses is not always positive, suggesting overestimation of the impact of taxation on profit (Kunce, 2003).

[3] Other "Pindyck-based simulation studies" that consider the effects of taxation on exploration and production include Yucel (1989) and Deacon (1993). These studies, however, were focused on assessing external validity of results rather than empirical case studies of particular oil industries or changes in tax policies (Kunce, 2003).

[4] The ELF adjustment factor was determined for each unit by the following formula in Alaska Statute 43.55.011. Since the ELF was designed to reduce the tax burden on "marginal" fields near their "economic limit," the ELF factor is lower for fields with low production volume.

$$\mathrm{ELF}(Q_i(t)) = \left(1 - \frac{300 * WELLS_{it} * DAYS_t}{Q_{it}}\right)^{\left(\frac{150{,}000}{Q_{it}/DAYS_t}\right)^{\left(\frac{460 * WELLS_{it} * DAYS_t}{300 * WELLS_{it} * DAYS_t}\right)}}$$

[5] The assumption that production cost is a decreasing function of stock size is common in the economic literature (see, for example, Farrow, 1985; Hartwick, 1982; Pindyck, 1978; Ruth and Cleveland, 1993).

[6] Although the data from Attanasi and Freeman were estimated for field size categories based on original field size, they should apply equally to newly-discovered fields and reserves remaining in producing fields. This can be seen from the dual perspective of a potential buyer of the producing field. While the newly-discovered field of a particular size may be expected to have a certain average cost per barrel for production of its lifetime, a producing field with equal quantity of reserves remaining should have the same average facilities investment cost of production for its remaining life because the costs of the existing facilities in place are amortized over their useful life and included in the purchase price.

[7] Drilling cost may also fluctuate with the quality of drilling sites, in response to oil price. If more marginal sites are drilled when oil price is high, then the first peak in Alaska drilling cost may correspond to the high prices caused by the oil crises of 1973 and 1979, the decline and trough in drilling cost from

1980 to the late 1990s may correspond to the relatively low oil prices of this period, and the recent increase in drilling cost may correspond to recent increases in oil prices.

[8] Changing well technology may have influenced the number of wells needed to produce at a certain rate, ceteris paribus, meaning our regressions may suffer from omitted variable bias. Well technology may also differ between fields due to reservoir differences. Lacking data on well technology in use on the North Slope, we considered including a time regressor to account for evolutionary change. But development of well technology has been lumpy (personal communication, Frank Kareeny, BP-Alaska, July, 2007). For example, coil tube drilling, which enables drilling multiple wells from the same pad, was developed in the early 1990s, water injection began in 1984 at Prudhoe Bay, and miscible gas injection began in 1987 with construction of the Central Gas Facility (CGF) and Central Compressor Project (CCP) (ibid). Consideration of well technology and systematic differences in well capacity is left to future work.

[9] A different adjustment cost parameter for increases in production than for decreases in production may be more realistic since adjustment up involves different actions than adjustment down. However, we used the simplifying assumption of symmetric adjustment cost because the rapid changes in production rate that cause significant adjustment costs occur only during initial ramp-up of production.

[10] In this paper, we use the term net social benefit to refer to the sum of producer profits and state tax revenue. This definition simplifies from a holistic assessment of economic impacts and assumes adequate regulation to effectively eliminate externalities.

[11] House Bill 2001, passed in special session in 2007 was called Alaska's Clear and Equitable Share (ACES). It made the following modifications to Alaska Statutes. Alaska Statute 43.55.011 specifies a 25% tax on the "tax value" under AS 43.55.160, with "floors" if west-coast price is less than $25 per barrel and "progressivity" if price increases above $92.50 per barrel. Alaska Statute 43.55.023 specifies 20% tax credits for allowable expenditures (generally exploration and development activities) and 25% tax credits for carried-forward annual loss. Alaska Statute 43.55.160 defines the production tax value as the gross value at the point of production (WHV) less lease expenditures (under Alaska Statute 43.55.165) – i.e., the tax is on net rather than gross revenue.

[12] A previous paper by Adelman (1993) suggested that a 10 percent discount rate is suitable for oil producing countries with diversified income sources (e.g., United States) while 20 percent or more is suitable for countries that are heavily reliant on oil-generated income.

[13] Of course, such interpretation may suffer from simultaneous testing of the hypothesis, model specification, and data, and could also reflect mistaken assumptions and model errors.

[14] Small changes in output across many wells at a large field can produce a large overall change in production rate while a similar change in production from a small field would require larger and more costly changes in output across a smaller number of wells.